TURBULENT DIFFUSION AND THE SOLAR NEUTRINO PROBLEM

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ABSTRACT

This lecture presents a summary of the problem and includes the solutions found by Maeder and the author. The first part gives an outline of the structure of the Sun, especially the existence of a convective zone, and the different sources of energy, with predictions of the solar neutrino flux and earlier solutions of the solar neutrino problem. It is then shown how various kinds of instability existing inside rotating stars can generate a mild turbulence. A proof of the existence of this turbulence is looked for in the abundances of chemical elements at the surface of stars: abundances at variance with gravitational sorting; surface abundance of lithium and beryllium in the Sun; surface abundances of lithium in field stars; surface abundances of carbon isotopes in evolved stars. The last section shows that a turbulent diffusion coefficient $\varnothing_t \approx 0.1 \text{Re}^{\text{critical}}$, where $\text{Re}^{\text{critical}}$ is the critical Reynolds number for which turbulence sets in, leads to a consistent solar model with a low neutrino flux. As a conclusion a comment is made on the gallium experiment, which can discriminate between the structural effects of the Sun and a possible neutrino oscillation.
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1. **INTRODUCTION**

As a first approximation we can consider a star, and especially the Sun, as a sphere.

A star like the Sun, as a whole, can be considered as a fluid in hydrostatic equilibrium. Departures from equilibrium are present, but can be neglected in a first approximation and have a negligible influence on the bulk of the Sun.

Models of the Sun are devised to describe its internal structure: temperature, density, and chemical composition, as a function of the distance to the centre. They must satisfy the major constraints, that is to say, they must lead, at \( t = 4.6 \times 10^9 \) years (the age of the Sun), to the present values of the luminosity \( (L = 3.6 \times 10^{26} \text{ W}) \) and of the radius \( (R = 6.96 \times 10^8 \text{ m}) \), for the given mass of the Sun \( (M = 1.989 \times 10^{30} \text{ kg}) \).

Computation of a solar model has been carried out several times, a number of improvements of the physical input being made in the computation scheme, which concern the equation of state, the absorption coefficients (determining the relation between the temperature gradient and the heat flow), and the rate of energy generation (improvements of the cross-sections). Figure 1 displays, for a specific model, the distribution of temperatures and densities in the Sun. As we shall see later, the model depends on the choice of the chemical composition.

![Graph](image)

**Fig. 1** The temperature and density of the matter as a function of the mass ratio \( M_r / M \). The central values of \( T \) and \( \rho \) are \( T_c = 13.48 \times 10^7 \text{ K} \) and \( \rho_c = 89.12 \).

1.1 **The convective zone**

A peculiar situation exists close to the surface of the Sun. A thermally driven instability (with a high Rayleigh number) generates a convective zone. Its depth is not known with precision at the present stage of the theory of compressible fluids with a high Rayleigh number.

We see the convective zone through the surface radiative zone. From the observed velocities and the estimates of the scale (the correlation length) of the turbulent motion present in the convective zone, it is possible to have an estimate of the Reynolds number \((10^{12} \text{ or about})\) and an estimate of the turbulent diffusion coefficient, defined as
\[ D_{\text{turb}} = \frac{1}{3} (\xi v). \]

\( D_{\text{turb}} \) is of the order of \( 10^{12} \) to \( 10^{13} \). The time scale of diffusion mixing throughout the convective zone is very short (compared with the solar lifetime). It is of the order of 1 year.

Other stars also have a convective zone. Their characteristic properties depend essentially on the gravity and the heat flux. For non-evolved stars, where the main parameter is the mass of the star, the characteristic properties of the surface convective zone depend on the mass of the star.

1.2 Energy sources

The main sources of energy of the Sun result from the conversion of hydrogen into helium, according to the pattern given in Table 1.

As far as the rate of energy production is concerned, an interpolation formula, valid in the range of temperature close to the central temperature of the Sun, is

\[ L \sim T^4. \]

This is due to the fact that the rate of energy production of the Sun is determined by the slowest of all these reactions, the proton reaction. Owing to the relatively low potential barrier, the rate of the reaction does not depend much on the temperature.

Table 1

Nuclear reactions of the hydrogen cycle

<table>
<thead>
<tr>
<th>Reactions</th>
<th>Q (MeV)</th>
<th>Maximum energy of the neutrinos (MeV)</th>
<th>Branching ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^1\text{H} + (^1\text{H}\rightarrow (^2\text{H} + e^+ + \nu)</td>
<td>1.442</td>
<td>0.42</td>
<td></td>
</tr>
<tr>
<td>(^1\text{H} + (^1\text{H} + e^- \rightarrow (^2\text{H} + \nu)</td>
<td>2.464</td>
<td>1.44 a)</td>
<td></td>
</tr>
<tr>
<td>(^2\text{H} + (^1\text{H} + (^3\text{He} \rightarrow \gamma)</td>
<td>5.493</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(^3\text{He} + (^3\text{He} \rightarrow (^4\text{He} + 2^1\text{H})</td>
<td>12.859</td>
<td></td>
<td></td>
</tr>
<tr>
<td>or</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(^3\text{He} + (^4\text{He} \rightarrow (^7\text{Be} + \gamma)</td>
<td>1.587</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(^7\text{Be} + e^- \rightarrow (^7\text{Li} + \nu)</td>
<td>0.860</td>
<td>0.860</td>
<td>90</td>
</tr>
<tr>
<td>(^7\text{Li} + (^1\text{H} \rightarrow (^2\text{He})</td>
<td>17.346</td>
<td>0.583</td>
<td>10</td>
</tr>
<tr>
<td>or</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(^7\text{Be} + (^1\text{H} \rightarrow (^8\text{B} + \gamma)</td>
<td>0.133</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(^8\text{B} \rightarrow (^6\text{Be}^* + e^+ + \nu)</td>
<td>17.980</td>
<td>14.06</td>
<td></td>
</tr>
<tr>
<td>(^8\text{Be}^* \rightarrow (^4\text{He})</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) Monokinetic.
However, this is not true of other reactions, which have a higher Coulomb potential barrier. The rate of production of the neutrinos from the beryllium itself depends on the concentration of $^3\text{He}$, which is itself temperature dependent. Iben$^1$ gives

$$\Phi_7 \sim T_5^2.$$ 

The rate of production of neutrinos from boron depends on the rate of production of $^6\text{B}$, which is itself related to the rate of production of $^7\text{Be}$. Iben gives

$$\Phi_8 \sim T_5^{2.8}.$$ 

1.3 Predicted flux and experimental results

The idea of detecting the solar neutrinos is due to Fowler$^4)$. The flux of neutrinos can be measured by the reactions of the incident neutrinos on a certain kind of atom. The usual unit, the SNU (solar neutrino unit), measures the number of reactions per second and for $10^{36}$ atoms. The first estimates$^3-5$ of the neutrino flux were quite optimistic (Table 2). Fowler himself used an optimistic estimate of the cross-section of the reaction $^7\text{Be}(p,\gamma)^8\text{B}$ and his evaluation of the neutrino flux corresponds to something like 1000 SNU. The first results of the experiment carried out by Davis turned out to be quite a deception, giving $1 \pm 1$ SNU. The experiment of Davis has been described several times [see especially Bahcall$^5$]). It takes place in Homestake goldmine, Lead, South Dakota, at a depth of 4850 feet. The detector consists of 600 m$^3$ of C$_2$H$_5$Cl$_4$: the last result obtained by Davis is $2.2 \pm 0.4$ SNU [quoted by Bahcall et al.$^7$].

It should be noticed (Fig. 2) that the main contribution to the detected flux comes from the $^8\text{B}$ neutrinos (Table 1). The main idea in order to put the predicted flux into agreement with the observed flux is to bring down the central temperature of the Sun. Bahcall et al.$^7$ on their side have finally, for a standard model, brought the predicted solar neutrino flux to 7.5 SNU, still much above the observed flux.

In a standard model, the problem of the adjustment of the theoretical neutrino flux to the observed neutrino flux depends on the existence of free parameters. The free parameters at our disposal are first the parameters of the chemical composition. We can adjust the helium and the concentration of heavy elements in the Sun, in order to fit the major constraints ($L$, $R$, at $t = 4.6 \times 10^9$ years) and the solar neutrino flux.

However, it is not possible to fit the major constraints with the adjustment of the chemical composition only. Roughly speaking, the chemical composition determines the luminosity, and the phenomenological parameter which describes the efficiency of the convective transport in the convective zone determines the radius.

Iben$^1$ discussed the possibility of adjusting the computed solar neutrino flux by a proper choice of the chemical composition. A solar model, with a low helium content, and a low content of heavy elements, also produces a low neutrino flux (Fig. 3). Table 3 gives the physical characteristics of a sequence of models [Christensen-Dalsgaard et al.$^8,9$] with a decreasing helium content. It should be noticed that, in order to adjust the computed radius to the solar radius, it is necessary to assume a low value of the phenomenological parameter which describes the efficiency of the convective transport, therefore associating with the low $Y$ and $Z$ content a shallow convective zone.
These results are in conflict with the observations and with the theoretical interpretation of the surface abundance of the heavy elements in the Sun. The helium content of the Galaxy is higher and it is difficult to understand how the Sun would happen to have such a low helium content. Furthermore, in order to explain the presence of metals in the solar spectrum, in the case of the low Z model, it is necessary to assume that the solar surface is 'dirty'. Accretion of metals from the interstellar matter might provide the necessary pollution of the solar surface. However, metals falling on the solar surface get mixed and then drift inside the Sun.

Table 2
Predicted neutrino flux $10^{-36} \text{ s}^{-1} (^{37}\text{Cl})^{-1}$ (SNU)

<table>
<thead>
<tr>
<th>Solar model</th>
<th>Flux</th>
<th>X</th>
<th>Y</th>
<th>Z</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sears$^1$)</td>
<td>38</td>
<td>0.683</td>
<td>0.292</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>0.708</td>
<td>0.272</td>
<td>0.020</td>
</tr>
<tr>
<td>Pochoda and Reeves$^6$)</td>
<td>40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bahcall and Shaviv$^5$)</td>
<td>22</td>
<td>0.707</td>
<td>0.266</td>
<td>0.027</td>
</tr>
</tbody>
</table>

Fig. 2 The contribution of the different reactions to the neutrino flux and the relative sensitivity of detectors for neutrino capture [after Bahcall$^6$]
Fig. 3 The relationship between the initial helium abundance \( Y \) and \( \Sigma \phi_i \phi_i \) (total counting rate), \( \phi_t \phi_t \) (the contribution of \(^8\)B neutrinos), \( \phi_L \phi_L \) (the contribution of \(^7\)Be neutrinos), \( T_C \) (the Sun's central temperature), and \( \rho_C \) (the Sun's central density). All \( \phi_i \phi_i \) are given in units of \( 3 \times 10^{-36} \) s\(^{-1}\) per \(^{37}\)Cl atom. The age of the Sun is supposed to be \( 4.5 \times 10^9 \) y; no mixing is permitted [after Iben\(^{13}\)].

Table 3

Properties of solar models [Christensen-Dalsgaard et al.\(^{8,4}\)]

<table>
<thead>
<tr>
<th></th>
<th>Model A</th>
<th>Model B</th>
<th>Model C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial heavy element abundance ( Z_0 )</td>
<td>0.02</td>
<td>0.004</td>
<td>0.001</td>
</tr>
<tr>
<td>Initial hydrogen abundance ( X_0 )</td>
<td>0.73</td>
<td>0.81</td>
<td>0.84</td>
</tr>
<tr>
<td>Initial helium abundance ( Y_0 )</td>
<td>0.25</td>
<td>0.186</td>
<td>0.159</td>
</tr>
<tr>
<td>Efficiency parameter of convection</td>
<td>1.35</td>
<td>1.11</td>
<td>0.62</td>
</tr>
<tr>
<td>Depth of present convective zone (D/R(_\odot))</td>
<td>0.240</td>
<td>0.172</td>
<td>0.042</td>
</tr>
<tr>
<td>Total mass of present convective zone</td>
<td>( 1.0 \times 10^{-2} )</td>
<td>( 1.7 \times 10^{-3} )</td>
<td>( 1.6 \times 10^{-6} )</td>
</tr>
<tr>
<td>Neutrino flux (SNM)</td>
<td>5.2</td>
<td>2.3</td>
<td>1.7</td>
</tr>
</tbody>
</table>
The velocity of drifting has been determined by Aller and Chapman; it is produced by the drag due to gravity and pressure gradient, and is ruled by the microscopic diffusion coefficient \( D_{\text{mic}} \); it can be written as

\[
v_{\text{drift}} = - \frac{D_{\text{mic}}}{H(A,Z,i)},
\]

where the scaling length \( H(A,Z,i) \) depends on the atomic characteristics \( A \) and \( Z \), and on the degree of ionization \( i \). The scaling length is related to the pressure scale height \( H = \frac{d\tau}{d \log P} \) by the relation

\[
H(A,Z,i) = H^1 \left[ (2A - i - 1) + (2.65 f i^2 - 0.804 f i) \left( \frac{d \log T}{d \log \rho} \right) \right],
\]

where \( f \) is a numerical constant of the order of unity \([\text{Michaud and Montmerle}]\). The time scale of depletion of the convective zone is given by

\[
t_{\text{depl}} = H_{\text{conv. zone}} \cdot H(A,Z,i) \cdot D_{\text{mic}}^{-1}.
\]

If microscopic diffusion was the only way of depleting the surface convective zone, Michaud noticed that, for transport processes to have negligible effects in the convective zone, the mass of the convective zone should be at least of the order of \( 3 \times 10^{-3} \) to \( 10^{-2} \) M\(_\odot\). Furthermore, as mentioned by Christensen-Dalsgaard et al., transport by fingering \([\text{Joss}]\), Ulrich\(^{19}\) may proceed even faster. It seems quite difficult for these reasons to accept the solar model with a low \( Z \) and \( Y \) content and a shallow convective zone.

### 1.4 Mixing

It is quite clear that mixing provides an alternative explanation to the low solar neutrino flux. Bringing some hydrogen into the central region would increase, for the same temperature, the rate of energy generation. However, the Sun is a strange thermodynamical machine. If the amount of fuel available becomes larger, the Sun hardly changes its luminosity, but adjusts its central temperature in order to decrease the rate of consumption of the fuel.

Owing to the high sensitivity to the temperature of the \( ^7\text{Be} \) and \( ^8\text{B} \) neutrino flux, a small change in chemical composition can eventually decrease the central temperature of the Sun and bring down the solar neutrino flux to the observed amount.

With \( L \approx X^2 T^4 \equiv \text{constant} \), we get \( T \propto X^{-7/2} \) and \( ^8\text{B} \propto X^{-10} \). An 18\% increase in \( X \) brings down the \( ^8\text{B} \) neutrino flux by a factor of 5.4.

The idea of mixing was considered first by Ezer and Cameron, discussed by Iben\(^1\), and Shaviv and Beaudet\(^6\) calculated mixed solar models, their parameter being the fraction of the solar mass which was assumed to be completely mixed. This brings down, for standard initial composition, the solar neutrino flux at most by a factor of 3.4, not quite enough to explain the result of Davis. Bahcall et al.\(^17\) calculated in a similar way mixed solar models and found a possible reduction by at most a factor of 5. Shaviv and Salpeter\(^18\) also calculated mixed models with various assumptions concerning the detailed balance of the nuclear species and found a possible reduction by at most a factor of 3.5. However, despite the suggestion made by Ezer and Cameron that the observed spin-down of the Sun might
generate a redistribution of angular momentum inside the Sun, Bahcall et al.\textsuperscript{17}) reject the mixed solar models on the grounds that they cannot find any instability powerful enough to realize the mixing.

This brings us back to the physics of mixing. We shall consider in the following what is the astrophysical basis for a theory of mixing inside the Sun and stars.

2. TURBULENCE

Various instabilities can be present in a star. Let us just sketch these different instabilities:

2.1 Thermal instabilities

These are of the same nature as the well-known Rayleigh-Bénard instabilities. When the heat flow which is forced through a fluid in equilibrium in a gravitational field is larger than that which can be carried by thermal conduction, the fluid starts moving. Defining the Rayleigh number as

$$Ra = \frac{g \alpha \ell}{K\nu} \beta$$

[where $g$ is the gravity, $\alpha = 1/T$ is the dilatation coefficient, $\ell$ is the vertical scale, $K$ is the thermometric conductivity, $\nu$ is the kinematic viscosity, and $\beta = \left| (\text{grad} T)_{\text{star}} - (\text{grad} T)_{\text{ad}} \right|$], it is known that when this number exceeds slightly the critical value, $Ra = 1000$, the fluid becomes turbulent. This is the case of the convective zone, which has been described above.

2.2 Instabilities associated with rotation

There is no complete theory of the instabilities associated with rotation for the reason that the complete set of eigenfunctions and eigenvalues associated with a given law of rotation $\Omega(\theta, z)$ (where $\theta$ is the distance to the rotation axis) is not known. There are a number of instabilities which are found for certain classes of perturbations. As a consequence, the fact that a condition of stability in $\Omega(\theta, z)$ is fulfilled for a given class of perturbations does not mean that the condition of stability is not violated for another class of perturbations.

Experiments on fluids in between rotating cylinders or on turntables are enlightening, but do not answer all possible questions which can be raised about rotating stars. Nevertheless, we can mention the following:

i) If the angular momentum squared $(\Omega \varpi)^2$ decreases outwards ($\Omega$ circular frequency, $\varpi$ distance to the axis), the motion is unstable. This is the classical Rayleigh criterion, which can be obtained by the simple method of exchange of stability.

ii) In a shear flow, if the buoyancy force exceeds the acceleration of the motion obtained by a particle crossing the shear flow, the fluid is stable. This is known as the Richardson criterion and is usually written:

$$Ri = \left[ \frac{g}{C_p} \frac{ds}{dz} / \left( \frac{dv}{dz} \right)^2 \right] > \frac{1}{4}.$$
This concerns non-axisymmetric disturbances. The question, to be discussed later, is how to apply the Richardson criterion to the case of differential rotation. For the time being, we are concerned with the effect of heat exchange during the fluid motion. It is known that when the heat exchange by radiation is included, the buoyancy force is considerably reduced (depending on the size of the fluid "eddies", as the size determines the rate at which the heat is exchanged between the eddies and the surrounding medium). The new criterion which has to be considered, called the flux-Richardson criterion due to Townsend, can be written:

$$\text{Ri}_F = \left( \frac{\rho}{C_p} \right) \left( \frac{dS}{dz} \right) \left( \frac{\epsilon_{\text{cool}}}{\alpha \epsilon} \right) > 1 .$$

This is equivalent to the introduction of a much smaller critical Richardson number; this new criterion is much more easily violated. In fact, the flux-Richardson criterion explains the presence of turbulence in terrestrial atmospheric shear flow.

There is little difference between plane parallel motion and axisymmetric shear flow as long as the radial scale of the shear is small compared to the radius of rotating layers. This is naturally the case in the earth's atmosphere. From the experiments on rotating fluids, it is quite clear that the important quantity is the angular momentum $\Omega r^2$. In other words, if $\Omega r^2$ increases outwards, a very large shear in $\Omega$ is necessary, even if the flux-Richardson criterion is violated, in order to have the generation of turbulence. On the other hand, if $\Omega r^2$ is decreasing outwards, and if the flux-Richardson criterion is violated, the shear in $\Omega$ can be sufficient to generate a turbulence.

iii) A similar criterion, also called the Richardson criterion, gives the condition for the persistence of turbulence when it has been established. This criterion can be written [see Eliassen and Kleinschmidt19]:

$$\text{Ri} = \left( \frac{g}{\Theta} \right) \left( \frac{\partial \Theta}{\partial z} \right) \leq \frac{K}{K_c} ,$$

where $g$ is the gravity, $\Theta = (P_0/P)^{(\gamma - 1)/\gamma}$ (with $\gamma = c_p/c_v$ being a measure of the entropy), $\partial \Theta/\partial z$ is the measure of the shear, $K_c$ is the eddy conductivity, and $K$ is the eddy viscosity.

When the heat exchange by radiation is included, the condition for the persistence of turbulence is equivalent to the violation of the flux-Richardson criterion.

iv) Another dynamical instability exists in a rotating star. If the angular velocity is not constant on cylinders,

$$\frac{\partial \Omega}{\partial z} \neq 0 ,$$

then the fluid is unstable.

v) Finally, it should be mentioned that in a stably stratified rotating star, as suggested by Büsse20, the tendency of the low-amplitude turbulence is always towards achieving the state of coinciding surfaces of constant entropy and constant potential. But, as Büsse says, "since this state is not attainable, a continuously changing 'weather' must be expected, just as in our own atmosphere".
2.3 Diffusion

Considering the various sources of instabilities which have been briefly described above, it is very likely that in a large variety of circumstances, a low-amplitude turbulence is present in a star.

In the presence of turbulence, transport processes take place through a mechanism of random walk. To make things simple, we can define a turbulent diffusion coefficient

\[ D_T = \frac{1}{3} \langle k v \rangle, \]

where \( \langle k v \rangle \) is the average of the product of a length \( k \) and a correlated velocity \( v \), characteristic of the turbulence. On the other hand, a Reynolds number \( Re \) can be defined by

\[ Re = \frac{L v}{\nu}, \]

where \( L \) is some maximum characteristic length, \( v \) the corresponding correlated velocity, and \( \nu \) the kinematic viscosity.

If we consider a power law spectrum for the energy density of the turbulent flow, we have

\[ E(k) \, dk = C k^{-n} \, dk, \quad k > k_{\text{max}}. \]

This defines the mean square velocity at the scale \( k \):

\[ v^2 = \frac{C}{n - 1} k^{1-n}, \]

from which we derive after averaging,

\[ D_T = \frac{2}{3} \frac{n - 1}{5n - 1} Re. \]

This defines a pseudo-Reynolds number

\[ Re^* = \frac{2}{3} \frac{n - 1}{5n - 1} Re. \]

If \( Re \) takes the critical value, we have for \( n = \frac{5}{3} \)

\[ Re^* \approx \frac{1}{9} \, (Re)_{\text{critical}} = 100 \text{ to } 200. \]

The exact value depends on the choice of \( (Re)_{\text{critical}} \) and of the (unknown) degree of intermittency.

2.4 Astrophysical estimates: Am versus A stars

The group of A stars is defined by a certain range of surface gravity and of surface temperature. Strange spectral differences exist among A stars. Some of these stars, called normal A stars, present normal abundances of the metals. Another group, called metallic A stars (the standard astronomical notation for the spectral class of these stars is Am) shows an excess of abundance of certain metals. As we shall explain in the following, the presence of turbulent diffusion is necessary in order to understand the existence of a class of stars in which a normal abundance of the elements is observed.
If the heavy elements are stored in a convective zone of equivalent thickness $H$ and are dripping through the lower boundary at a velocity $v_{\text{drift}}$ (given in Section 1.3), we have a time scale of depletion:

$$t_{\text{depl}} = \frac{H_{\text{conv}} H(A,Z,i)}{D_{\text{mic}}}.$$ 

This results from the equation of diffusion, written in a simplified form

$$\text{div} \left( - \frac{D_{\text{mic}}}{H(A,Z,i)} \frac{\partial}{\partial t} X \right) = \frac{\partial X}{\partial t},$$

with the boundary condition

$$- \frac{D_{\text{mic}}}{H(A,Z,i)} X = H \frac{\partial X}{\partial t}.$$ 

If we add the turbulent diffusion, we have to write

$$\text{div} \left( D_{T} \text{grad} X - \frac{D_{\text{mic}}}{H(A,Z,i)} \frac{\partial}{\partial t} X \right) = \frac{\partial X}{\partial t},$$

with the boundary condition

$$D_{T} \text{grad} X - \frac{D_{\text{mic}}}{H(A,Z,i)} X = \frac{\partial X}{\partial t},$$

from which it turns out that the time scale of depletion is now increased in the ratio of the diffusion coefficients,

$$t_{\text{diff}} = \frac{H_{\text{conv}} H(A,Z,i)}{D_{\text{mic}}} \frac{D_{T}}{D_{\text{mic}}}.$$ 

[Schatzman\textsuperscript{21}]. Applied to late type A stars, this gives $t_{\text{depl}} = 10^3$ to $10^4$ years, for a stellar lifetime of the order of $10^7$ to $10^8$ years. This implies a ratio

$$\frac{D_{T}}{D_{\text{mic}}} = 10^3$ to $10^5,$

from which we derive a value of $D_{T},$

$$D_{T} = 10^3$ to $10^4$$

and a pseudo Reynolds number

$$\text{Re}^* \approx 100.$$ 

2.5 Abundances of nucletarily processed elements: the solar case

To make things simple, imagine a deep convective zone, reaching at its bottom a temperature of about $2 \times 10^6$ K. Since the $^7\text{Li}$ burning takes place at $2.4 \times 10^6$ K, the burning region is just below the bottom of the convective zone.

Since the $^9\text{Be}$ burning takes place at a temperature of $3.5 \times 10^6$ K, the $^9\text{Be}$ burning region is much deeper.
Let us now consider more closely the problem. Considering a diffusion equation

\[ D \frac{\partial^2 \phi}{\partial z^2} = \frac{\partial \phi}{\partial t} \]

with the corresponding boundary conditions (a) and (b):

a) At the bottom of the convective zone, the rate at which the chemically homogeneous convective zone is losing its \(^7\text{Li}\) is determined by the flow of \(^7\text{Li}\) through the lower boundary of the convective zone:

\[ \rho D \frac{\partial \phi}{\partial z} = \rho \frac{(H_p)_{\text{CE}}}{\tau} \phi , \]

where \((H_p)_{\text{CE}}\) is the equivalent thickness of the convective zone, and \(\tau\) the decay time of the concentration \(\phi\).

b) At the level of the burning region, the transition region between a finite \(^7\text{Li}\) concentration and a vanishingly small \(^7\text{Li}\) concentration is so thin that we can as well write for the boundary condition

\[ \phi = 0 . \]

The eigenvalue problem for \(\tau\) is given by the solution of

\[ \frac{(h/H_p)_{\text{CE}}}{h(Dr)^{-1/2}} = \frac{h(Dr)^{-1/2}}{T_{\text{BlLi}} - T_{\text{CE}}} \]

where \(h\) is the distance of the bottom of the convective zone to the burning region.

Consider the case \(P = K_0^y\), which is a fairly satisfactory approximation. The equation of hydrostatic equilibrium gives a relation between the temperature \(T_{\text{CE}}\) at the bottom of the convective envelope, the temperature of the lithium burning region \(T_{\text{BlLi}}\), and the distance of the bottom of the convective zone to the lithium burning regions:

\[ \frac{\gamma}{\gamma - 1} \frac{T_{\text{BlLi}} - T_{\text{CE}}}{T_{\text{CE}}} = gh_{\text{Li}} . \]

Introducing the equivalent height

\[ (H_p)_{\text{CE}} = \left( \frac{P}{gD} \right)_{\text{CE}} = \frac{RT_{\text{CE}}}{g\nu} , \]

we obtain the relationship

\[ \frac{h_{\text{Li}}}{(H_p)_{\text{CE}}} = \frac{\gamma}{\gamma - 1} \frac{T_{\text{BlLi}} - T_{\text{CE}}}{T_{\text{CE}}} \]

and a similar relation for the beryllium burning region

\[ \frac{h_{\text{Be}}}{(H_p)_{\text{CE}}} = \frac{\gamma}{\gamma - 1} \frac{T_{\text{BBBe}} - T_{\text{CE}}}{T_{\text{CE}}} . \]

The constraint on the lithium abundance and on the beryllium abundance gives

\[ \exp \left( -\frac{t_{\text{Li}}}{t_{\text{Li}}} \right) = 2.5 \times 10^{-3} \quad \text{and} \quad \exp \left( -\frac{t_{\text{Be}}}{t_{\text{Be}}} \right) = 0.5 , \]
from which we derive the characteristic decay times:

\[ \tau_{\text{Li}} = 0.167 \, \tau_\odot \]
\[ \tau_{\text{Be}} = 1.443 \, \tau_\odot \]

The solution of the eigenvalue problem gives

\[ T_{\text{PCE}} = 2.19 \times 10^6 \, \text{K} \]
\[ D = 735 \, \text{cm}^2 \, \text{s}^{-1} \]

With \( \nu_{\text{mol}} = 8.6 \) we have

\[ \text{Re}^* = 85 \]

The distances of \( h_{\text{Li}} \) and \( h_{\text{Be}} \) to the bottom of the convective zone are, respectively

\[ h_{\text{Li}} = 2 \times 10^9 \, \text{cm} \]
\[ h_{\text{Be}} = 12.4 \times 10^9 \, \text{cm} \]

the equivalent depth of the hydrogen convective zone being (see Fig. 4)

\[ H_{\text{PCE}} = 8.3 \times 10^9 \, \text{cm} \]

Fig. 4 A sketch of the structure of the outer layers of the Sun, showing the distance of the bottom of the convective zone to the regions of Li and Be burning.
2.6 Abundances of nearly processed elements: Li in field stars

The abundance of lithium in main sequence field stars has been measured by Boesgaard. There is a large dispersion in the measured abundance of lithium for a given spectral type. However, this can be explained by the dispersion of the ages of main sequence field stars. Then we have to consider the general behaviour of the concentration as a function of the spectral type, and this has to be explained by the intrinsic spectral properties.

After a time $t$, the lithium concentration is lowered by lithium burning in a proportion $\exp(-t/\tau)$.

Considering the average of the concentration for stars of various ages, we can write

$$\langle X \rangle = \langle X_0 \rangle \frac{\tau_{Li}}{\tau_{MS}} \left[ 1 - \exp \left( - \frac{T_{MS}}{\tau_{Li}} \right) \right],$$

where $\tau_{Li}$ is the characteristic time for lithium destruction by diffusion to the lithium burning level, and $\tau_{MS}$ is the lifetime on the main sequence. We can write approximately

$$\tau_{Li} = \frac{1}{Re \nu_{mol} h_{Li}^2},$$

where $h_{Li}$ is the distance from the bottom of the convective zone to the level of lithium burning.

Three different effects determine the final lithium concentration:

i) the lifetime on the main sequence as a function of spectral type;

ii) the variation of the viscosity (molecular and radiative) as a function of spectral type;

iii) the variation of the depth of the convective zone as a function of spectral type, which determines almost entirely the distance of diffusion $h_{Li}$. The results are displayed in Fig. 5.

![Fig. 5 The surface abundance of Li as a function of spectral type for stars of the main sequence. The points are abundance averages according to Boesgaard.](image-url)
2.7 Abundances of nuclearly processed elements: carbon isotopes

It is well known that, inside the stars, the following groups of reactions can take place (Bethe's cycle):

\[
\begin{align*}
^{12}\text{C} + ^1\text{H} & \rightarrow ^{13}\text{N} + \gamma \\
^{13}\text{N} & \rightarrow ^{13}\text{C} + \text{e}^+ + \nu \\
^{13}\text{C} + ^1\text{H} & \rightarrow ^{14}\text{N} + \gamma \\
^{14}\text{N} + ^1\text{H} & \rightarrow ^{15}\text{O} + \gamma \\
^{15}\text{O} & \rightarrow ^{15}\text{N} + \text{e}^+ + \nu \\
^{15}\text{N} + ^1\text{H} & \rightarrow ^{12}\text{C} + ^4\text{He} \\
^{15}\text{N} + ^1\text{H} & \rightarrow ^{16}\text{O} + \gamma \\
^{16}\text{O} + ^1\text{H} & \rightarrow ^{17}\text{F} + \gamma \\
^{17}\text{F} & \rightarrow ^{17}\text{O} + \text{e}^+ + \nu \\
^{17}\text{O} + ^1\text{H} & \rightarrow ^{18}\text{F} + \gamma \\
^{18}\text{F} & \rightarrow ^{18}\text{O} + \text{e}^+ + \nu \\
^{18}\text{O} + ^1\text{H} & \rightarrow ^{15}\text{N} + ^4\text{He} .
\end{align*}
\]

The cycle is completed only in the centre of stars heavier than the Sun. Inside the Sun itself, the energy released by the carbon cycle is negligible compared with that of the proton-proton chain.

To understand the effect of turbulent diffusion, let us first consider the distribution of the various isotopes in a 1 $\text{M}_\odot$ star at the end of its lifetime on the main sequence. Going inwards, we find first a region where $^{12}\text{C}$ is destroyed and $^{13}\text{C}$ is built. Immediately inside, owing to a higher temperature, we find a region where $^{12}\text{C}$ and $^{13}\text{C}$ are converted into nitrogen. The curves of Fig. 6 show the concentrations of various isotopes, as calculated by Dearborne et al.\textsuperscript{23).} When the concentration of $^{13}\text{C}$ is plotted as a function of the radius (Fig. 7) a remarkably steep gradient of concentration of $^{13}\text{C}$ is visible. Turbulent diffusion will transport $^{13}\text{C}$ outwards, out of the region where it is nuclearly processed.

---

Fig. 6 Composition of a 1 $\text{M}_\odot$ star and a 2 $\text{M}_\odot$ star near the end of their main sequence evolution (without mixing) (according to Dearborne et al.\textsuperscript{23})
3. TRY THE WHOLE SUN?

3.1 Building a solar model with turbulent diffusion

The fact that turbulent diffusion plays a similar role at different depths, from the bottom of the convective zone (T = 2.2 \times 10^6 K) to the region where $^{12}$C is nuclearly processed (T = 10 \times 10^6 K), suggests very strongly that one should try to calculate a solar model with turbulent diffusion throughout the whole Sun.

The work has been carried out by Schatzman and Maeder\(^{26}\). The program for the diffusion process has been included by Maeder in his stellar evolution scheme [Maeder\(^{27}\)]. The basic equations are written in the form used by Iben\(^{28}\), where the reaction rates are written:

$$\frac{dX_i}{dt} = R_{ij} = \frac{2.62 \times 10^{22}}{1 + \delta_{ij}} \frac{X_i X_j}{A_i A_j} (A_i Z_j)^{-1} S \xi^2 e^{-\tau},$$

where

$$\tau = 42.48 \left( \frac{z_{i}^{2} z_{j}^{2}}{z_{i} + z_{j}} \right)^{1/3} \frac{1}{T_6},$$

In these equations, $R_{ij}$ is the rate per unit mass at which nuclei of type $i$ react with nuclei of type $j$ [Fowler et al.\(^{29}\)]. Here $A_i$ is the atomic mass, $Z_i$ the atomic number, $A = A_i A_j (A_i + A_j)^{-1}$ the reduced mass, S the centre-of-mass cross-section in keV barns, and
\[ \delta_{ij} = 1 \] for identical nuclei and zero otherwise. It is assumed that the reaction of destruction of deuterium is so fast that deuterium has always its local equilibrium concentration. Then, for hydrogen and helium \( (^3\text{He} \text{ and } ^4\text{He}) \), with concentrations in mass \( X_1, X_3, \) and \( X_4 \), we write the equation of diffusion with chemical reactions:

\[
\begin{align*}
\frac{\partial \rho}{\partial t} & = \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \rho D \frac{\partial X_1}{\partial r} + \rho A_1 m_1 \left(-3R_{11} + 2R_{33} - R_{44}\right) \\
\frac{\partial \rho}{\partial t} & = \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \rho D \frac{\partial X_3}{\partial r} + \rho A_3 m_3 \left(R_{31} - 2R_{33} - R_{44}\right) \\
\frac{\partial \rho}{\partial t} & = \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \rho D \frac{\partial X_4}{\partial r} + \rho A_4 m_4 \left(R_{44} + R_{34}\right).
\end{align*}
\]

We recall here the expression for \( D \),

\[ D = \Re^* \left( v_{\text{mol}} + v_{\text{rad}} \right), \]

with \( v_{\text{mol}} \) already given by Schatzman\(^{16}\),

\[ v_{\text{mol}} = 2.21 \times 10^{-15} \frac{T^{5/2}}{\rho \ln A} \frac{1 + 7X}{8}, \]

and \( v_{\text{rad}} \) having the classical value

\[ v_{\text{rad}} = 6.728 \times 10^{-26} \frac{T^n}{\kappa \rho^{\frac{1}{2}}}, \]

where \( \kappa \) is the Rosseland mean of the absorption coefficient. The nuclear reaction rates are those of Fowler et al.\(^{29}\). The equations of diffusion are solved with the following boundary conditions:

a) at the centre,

\[ \frac{\partial X_1}{\partial r} = 0 \quad \text{for} \quad r = 0; \]

b) at the lower boundary of the surface convective zone,

\[ -4\pi r^2 \rho D \frac{\partial X_1}{\partial r} = \frac{\partial}{\partial t} \frac{4\pi r^2}{P/g} X_1 \quad \text{for} \quad r = R_{\text{HCZ}}, \]

where \( P/g \) is the mass per unit surface at the bottom of the hydrogen convective zone.

It is not necessary here to give any further details concerning the procedure which has been used for calculating the neutrino flux. It is important, however, to give the model composition which has been taken:

- hydrogen concentration: \( X = 0.73 \),
- helium concentration: \( Y = 0.25 \),
- other elements: \( Z = 0.02 \).

We turn now to the main results, which have been obtained with a pseudo-Reynolds number \( \Re^* = 0, 100, \text{and} 200 \).
3.2 Internal distribution of the elements

The distribution of the main isotopes in the Sun is considerably influenced by turbulent diffusion. Figure 8 illustrates the internal distribution of $X_H$ and of $X_3$ (mass fraction of $^3$He) at the age $t = 4.57 \times 10^9$ y (taken here as a typical value of the solar age, cf. Section 4) for the three values of $Re^*$ considered. The central hydrogen content in the present Sun is much higher when diffusion is included: while the central $X_H$ is only 0.359 in the standard case, it is as large as 0.676 with $Re^* = 200$ and 0.569 with $Re^* = 100$. This excess of central $X_H$ evidently results from the transport inwards due to a positive abundance gradient. Outside the central regions, the differences are smaller and of opposite sign. On the whole, turbulent diffusion makes the $X_H$ distribution much flatter. Thus, the diffusive transport may considerably increase the reservoir of nuclear fuel potentially available for stellar evolution, owing to the replenishment of the central regions by turbulent transport.

![Graph showing the hydrogen $^1$H and $^3$He concentrations in a 1 $M_\odot$ star at the age of $4.6 \times 10^9$ y according to Schatzman and Maeder (5)]. The curves are plotted for various values of the parameter $Re^*$ which describes the efficiency of the turbulent diffusion.

The internal distribution of $^3$He presents in the standard model a peaked distribution which is illustrated in Fig. 8. The peak is shaped by two effects: at high $T$, there is a very rapid destruction of the large amounts of $^3$He created; while in the outer layers, with low $T$, there is no $^3$He destruction. During the evolution, the peak moves outwards and increases in size. At $t = 4.57 \times 10^9$ y, the maximum reaches $2.8 \times 10^{-3}$ at $M/M = 0.58$. 
With turbulent diffusion, with \( \text{Re}^* = 100 \) and 200, the distribution of \(^3\text{He}\) below \( \text{M}_p/\text{M} = 0.4 \) is still mainly determined by the rapid equilibrium between creation and destruction as in the standard case. Above \( \text{M}_p/\text{M} = 0.4 \) (\( T < 8.3 \times 10^5 \) K), the transport by turbulent diffusion tends to uniformize the \(^3\text{He}\) distribution: the peak is reduced and \(^3\text{He}\) is spread out through the star and its abundance considerably increased in the external layers. The surface ratio \(^4\text{He}/^7\text{He}\) is very much increased during solar evolution if turbulent diffusion proceeds: at \( t = 4.57 \times 10^9 \) y the surface abundance in \(^3\text{He}\) is \( 8.5 \times 10^{-4} \) for \( \text{Re}^* = 200 \) and \( 3.8 \times 10^{-4} \) for \( \text{Re}^* = 100 \). Thus, we see that an accurate determination of the enrichment of the solar surface in \(^3\text{He}\) could provide most valuable information on the importance of the diffusion in solar evolution.

Let us now consider the effects of turbulent diffusion on the main observable quantities: solar luminosity, neutrino flux, and \(^3\text{He}/^7\text{He}\) surface ratio.

The inclusion of turbulent diffusion leads to a decrease of the luminosity at a given solar age. At \( t = 4.57 \times 10^9 \) y, the standard model computed with \( Y = 0.25 \) and \( Z = 0.02 \) has a luminosity given by \( \log L/L_0 = 0.048 \); for \( \text{Re}^* = 100 \) and 200, these values are 0.009 and -0.003, respectively. Although the standard case appears to give a luminosity too high by 11.7%, it is the opinion of Maeder and Schatzman that no serious arguments based on these differences can be put forward in favour of any model, since changes of the helium content \( \Delta Y = +0.01 \) give the changes \( \Delta \log L = +0.028 \). For the metal content \( Z \), one has for \( \Delta Z = +0.001 \) a change in luminosity by \( \Delta \log L = -0.008 \). Thus, minor changes in \( Y \) and \( Z \) could indeed bring any of the three solar models considered into perfect agreement with the observed solar luminosity.

Let us now consider the effect of turbulent diffusion on the solar neutrino flux.

Figure 9 shows the variation with time, for the three considered models, of the values of the central temperature and of the solar neutrino flux \( N_\nu \). The turbulent diffusion leads

Fig. 9 The evolution of the central temperature \( T_C \) and of the neutrino flux as a function of time for different values of \( \text{Re}^* \) [according to Schatzman and Maeder\(^{99}\)]
to a much smaller neutrino flux during the evolution. In particular, at $t = 4.57 \times 10^5$ y, the standard model with $Re^* = 0$, $Y = 0.25$, and $Z = 0.002$ gives a neutrino flux of 11.6 SNU, which quite closely corresponds to the value of 4.6 SNU found by Bahcall$^{11}$ for $Y = 0.21$, $Z = 0.02$, when a factor of about 2.4 is incorporated for taking into account the difference in the $Y$ and $Z$ content [cf. Iben$^8$]. Now, at the same age, the models with $Re^* = 100$ and 200 give, respectively, 2.38 and 1.43 SNU, i.e. a reduction of the solar neutrino flux by a factor of 4.9 and 8.1. Early tests made without radiative viscosity (which has small effects) indicate that for $Re^* = 50$, we should obtain about 4.5 SNU, i.e. a reduction by a factor of 2.6. Thus, we conclude that turbulent diffusion reduces the solar neutrino flux by a factor of 2.6 to 8.1 for values of $Re^*$ in the range of 50 to 200, for the composition that we have chosen; and even more easily for the composition adopted by Bahcall, solar models with turbulent diffusion may bring the computed solar neutrino flux into the range of the observed one, i.e. of about 2.2 SNU.

Finally, consider the enrichment in $^3$He at the solar surface.

It was shown above how diffusion may produce an enrichment in $^3$He in the outer solar layers. Figure 10 illustrates how the ratio $X_3/X_4$ of the abundances in $^3$He and $^4$He (in mass) at the base of the external convective zone varies with time for the various chosen values of $Re^*$. During the first billion years for $Re^* = 200$, about two billion years for $Re^* = 50$ (deduced from initial test computations), the surface ratios $^3$He/$^4$He undergo almost no change; then a rapid enrichment in $^3$He at the solar surface proceeds to take place. The ratios ($X_3/X_4$)$_S$ for $Re^* = 200$ and 100 appear to be rather too large in view of

\[ \frac{X_3}{X_4} \]

Fig. 10 The surface abundance ratio $^3$He/$^4$He as a function of time for different values of $Re^*$ [according to Schatzman and Maeder$^6$].
the maximum possible ratio of about $6 \times 10^{-6}$, and a value of Re* closer to 50 could be envisaged. The reservoir represented by the external convective zone is quite small ($\leq 2\%$ of the total stellar mass) and it does not intervene in this comparison. More important could be the problems related to the helium isotope fractionation in the solar-wind source which could influence such comparisons.

A very interesting point is that a moderate diffusion (with Re* between 50 and 100) may produce the necessary enrichment from the very low pre-main-sequence concentration to the much higher concentration observed at present (see Section 2). This interpretation was firstly advocated by Schatzman\textsuperscript{12}) and then by Boschler and Geiss\textsuperscript{11}) for interpreting the relatively high $^3\text{He}$ concentration observed in the solar wind. Now, a very remarkable fact is that the constraint on Re* put by the isotopic ratio at the surface of the Sun demands not only the same mechanism, but also similar values of Re*, in the range of 50 to 100, for interpreting those very different solar observations which are the solar neutrino flux and the surface $^3\text{He}$ abundance.

4. CONCLUSIONS

As a main point, we note that turbulent diffusion reduces the solar neutrino flux by a factor of 2.6 to 8.1, relative to standard models for the critical Reynolds number Re* between 50 and 200; simultaneously, diffusion enhances the surface ratio of $^3\text{He}/^4\text{He}$ and may thus explain the excess of the present solar value with respect to the pre-solar ratio. However, the observed data appear to exclude values of Re* as large as 200 and support values in the range 50-100. It is very encouraging that the same mechanism with quite a substantial physical justification [cf. Schatzman\textsuperscript{16})] is able to satisfy such very different requirements as the solar luminosity, the solar neutrino flux, and the $^3\text{He}/^4\text{He}$ surface enrichment in the Sun.

Recent experiments carried out in order to determine the mass of the neutrino have produced a revival of the hypothesis of Pontecorvo\textsuperscript{14}) of neutrino oscillations.

The possibility of the mixing of the states of the three kinds of neutrinos [see, for example, the bibliography given by Bahcall et al.\textsuperscript{7})] would bring down the measurable flux of $\nu_e$ from the Sun. However, owing to the broad energy spectrum of the solar neutrinos, the phase mixing as calculated by Bahcall and Frautschi\textsuperscript{15}) reduces the neutrino flux only by a factor of two. There would still be a discrepancy, as emphasized by Bahcall et al.\textsuperscript{7}), which would then easily be matched by the effect of turbulent diffusion! To conclude this point, it should be noticed that the mixing of neutrino states is not yet fully established [see, for example, the review article of Lubkin\textsuperscript{16})].

The final test will naturally be given by the experiment. The possibility of capturing the solar neutrinos by other elements has been discussed for a long time [see, for example, Bahcall\textsuperscript{17}), and bibliography there]. At present an experiment using gallium, to be carried out jointly by Heidelberg, Philadelphia, Princeton, and Rehovoth, is under preparation.

The threshold for the capture of neutrinos on $^{71}\text{Ga}$ is 0.231 MeV, much lower than the threshold for capture on $^{37}\text{Cl}$. Gallium gives the possibility of measuring the neutrinos coming from the pp reaction. This is very important, as the number of neutrinos coming
Table 4  
Neutrino flux (SNU)

<table>
<thead>
<tr>
<th>Element</th>
<th>pp</th>
<th>pep</th>
<th>$^7\text{Be}$</th>
<th>$^8\text{B}$</th>
<th>Others</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{37}\text{Cl}$</td>
<td>0</td>
<td>0.23</td>
<td>0.81</td>
<td>3.46</td>
<td>0.16</td>
<td>4.66 a)</td>
</tr>
<tr>
<td>$^{71}\text{Ga}$</td>
<td>65</td>
<td>2.4</td>
<td>21.8</td>
<td>1</td>
<td>3.1</td>
<td>93</td>
</tr>
</tbody>
</table>

a) Revised [Bahcall et al.7] to 7.5 SNU.

from the pp reaction is directly related to the total luminosity of the Sun. Bahcall\textsuperscript{37} gives, for example, the values of Table 4, where the neutrinos from the pp and pep reactions represent 70\% of the total detectable flux.

At present experiments are being carried out with 4.6 t of a solution of GaCl\textsubscript{3}; extraction of Ga is quite successful, reaching more than 90\% of the chemical yield. The plans are to build a reactor containing 50 t of gallium, with the expectancy of counting 1 pp and pep neutrinos per day.

It is quite clear that, if the detected flux equals the predicted flux, this would mean that there is no neutrino oscillation, and that the neutrino deficiency in the experiment of Davis is entirely due to structural effects in the Sun.

If, however, a deficiency in the pp or pep neutrino flux is detected, this would mean that only part of the neutrino deficiency in the experiment is due to structural effects in the Sun. It should be noticed that this would not disprove the existence of turbulent diffusion in the Sun, but eventually would make it easier to reconcile the solar properties with the relatively high values of the helium concentration $Y$ in stars which is usually accepted on the basis of evolutionary arguments.
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