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GRAND UNIFIED AXION NOT OBSERVABLE ?

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ABSTRACT

It is a characteristic of grand unified theories with an axion of the type suggested by Dine, Fischler and Srednicki that they predict a very small neutron electric dipole moment because the QCD θ vacuum parameter is suppressed. The minimal SU(5) axion model of Wise, Georgi and Glashow yields a cosmological baryon-to-photon ratio in the same order as that in the minimal SU(5) model with one $\underline{5}$ of Higgs, which is much smaller than required by astrophysics. More complicated grand unified axion models can avoid this problem.

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Combining QCD with the Kobayashi-Maskawa¹⁾ mechanism of CP violation in the standard $SU(2) \times U(1)$ model²⁾ leads to a strong CP problem due to the non-perturbative θ vacuum³⁾ parameter of QCD. A possible resolution of this problem invokes the existence of a global chiral $U(1)$ symmetry, broken only by non-perturbative QCD effects. The original version of this mechanism due to Peccei and Quinn⁴⁾ entailed the existence⁵⁾ of a light pseudoscalar particle, called the axion a , whose couplings to fermions $\sim m_f/v$ and whose mass $\sim m_\pi f_\pi/v$, where v is the vacuum expectation value of the Higgs field breaking $SU(2) \times U(1)$ symmetry. Such an axion has long seemed to be excluded by several experiments⁶⁾, while a recent reincarnation⁷⁾ has only served to stimulate new sport negative results⁸⁾. Dine, Fischler and Srednicki⁹⁾ have proposed a recycled axion whose couplings and mass are scaled by $1/V$, where V is a relatively large $SU(2)$ singlet Higgs vacuum expectation value. Consistency with models of stellar evolution requires that $V > O(10^9)$ GeV, and it is natural to surmise⁹⁾ that $V \approx O(10^{15})$ GeV, the presumed scale of grand unification. We term such a grand unified axion $\mathcal{A}$ with couplings $\sim m_f/V$ and mass $\sim f_\pi m_\pi/V$ a "hypoaxion". It has been thought to be essentially unobservable^{9),10)} because of its very small couplings to quarks and leptons.

In this paper we propose an indirect experimental way of determining whether a hypoaxion exists. We claim¹¹⁾ that in generic GUTs which are complicated enough to generate a baryon-to-photon ratio (n_B/n_γ) of the observed magnitude $\geq 2 \times 10^{-10}$, the neutron electric dipole moment $d_n \geq O(5 \times 10^{-28})$ e-cm, which should be observable in the relatively near future. But in GUTs with a Peccei-Quinn⁴⁾ symmetry d_n is probably much smaller and practically unobservable. This argument applies to a wide class of hypoaxion models. As a final remark we observe that the minimal $SU(5)$ hypoaxion model of Wise, Georgi and Glashow¹⁰⁾ does not give a cosmologically acceptable value of (n_B/n_γ) , but rather a value comparable with that found¹²⁾ in the minimal $SU(5)$ model with just one $\underline{5}$ of Higgs fields. This objection does not apply to more complex hypoaxion models¹³⁾.

We first discuss d_n as an indirect discriminator between hypoaxion and other GUTs. We have argued previously¹¹⁾ that in generic GUTs where the baryon-to-photon ratio (n_B/n_γ) comes about through the decays of superheavy Higgs bosons, there are contributions to θ renormalization and hence to d_n ¹⁴⁾:

$$\Delta d_n \approx 3 \times 10^{-16} \theta \text{ e-cm} \quad (1)$$

which have a diagrammatic structure analogous to that responsible for the C- and CP-violating decays of the superheavy Higgs bosons. Barring bizarre

cancellations, but we never promised you a rose garden, we therefore argued that

$$d_n \gtrsim O(2.5 \times 10^{-18}) \times \left(\frac{n_B}{n_\gamma}\right) \times E \quad \text{e-cm} \quad (2)$$

where E is a possible factor accounting for entropy generated after baryon creation. Cosmological arguments suggest that $(n_B/n_\gamma) \gtrsim 2 \times 10^{-10}$ which leads us to expect $d_n \gtrsim O(5 \times 10^{-28}) E \text{ e-cm}$. We have in fact further argued¹¹⁾ that in most GUTs, there are contributions to θ and hence to d_n occurring in lower orders of non-strong perturbation theory, so that perhaps

$$d_n \gtrsim O(6 \times 10^{-26}) \times E \quad \text{e-cm} \quad (3)$$

By contrast, the conventional CP-violating $SU(2) \times U(1)$ Kobayashi-Maskawa model¹⁾ makes a contribution to d_n which is¹⁵⁾ at most $O(10^{-30 \pm 1}) \text{ e-cm}$, while other candidate models¹⁶⁾ of CP violation in the weak interactions are probably excluded¹⁷⁾ by the upper bound on $|\epsilon'/\epsilon|$ in neutral K decays.

The estimates (2) and (3) do not apply to GUTs with a hypoaxion, in which the aforementioned bizarre cancellations are fated to occur. In such models there is a global Peccei-Quinn⁴⁾ $U(1)$ symmetry which is only broken by non-perturbative QCD effects - or possibly by effects associated with gravitation? We expect the QCD θ parameter to be zero^{18),19)} up to contributions vanishing when $\Lambda_{\text{QCD}} \rightarrow 0$. It seems likely^{18),19)} that the dominant contribution to θ may arise from a mismatch in phases between the effective chiral symmetry breaking $(\bar{\psi}\psi)$ and $(\bar{\psi}\psi)^2$ operators appearing when heavy non-strong degrees of freedom are integrated out. In this case one expects¹⁹⁾

$$\theta \approx O(\alpha^2) \Lambda_{\text{QCD}}^3 m_q / m_w^4 \times (\text{product of phases and mixing angles}) \quad (4)$$

which is certainly $\lesssim O(10^{-12})$. The coefficient and phase of one such $(\bar{\psi}\psi)^2$ operator, namely $(\bar{s}d)^2$, are known phenomenologically from the $K_S - K_L$ system. They suggest that a reasonable estimate of Eq. (4) for θ may be more like

$$\theta \approx \left| \frac{\ln M_{12}}{m_K} \right| \approx O(10^{-17}) \quad (5)$$

Inserting this estimate into Eq. (1) gives a contribution to d_n which is if anything smaller than the previous¹⁵⁾ perturbative Kobayashi-Maskawa estimate, which is anyway too small to be seen in the foreseeable future.

We see one possible way¹⁰⁾ of getting a contribution to θ which is larger than (4,5). It is generally believed that Nature abhors global symmetries, and the Peccei-Quinn $U(1)$ is surely no exception. We already know that it is broken by non-perturbative gauge theory effects, and may also suspect¹⁰⁾ that it could be broken by (non-perturbative ?) phenomena occurring at the Planck scale. For example, one could imagine²⁰⁾ such effects generating fermion-fermion-multiHiggs or (Higgs)^{n>4} interactions which did not respect the $U(1)_{PQ}$ symmetry. These could give rise to a hypoaxion mass larger than otherwise expected, and a contribution to θ of order $(\text{phases}) \times (m_X/m_{\text{Planck}})^{n-4}$ which could in principle be much larger than (4.5). For this reason, it may well be necessary that any such interactions either respect the $U(1)_{PQ}$ symmetry or participate in a mechanism akin to Weinberg's "vacuum relaxation"¹⁹⁾, in order to suppress θ below the experimental limit, while perhaps being larger than Eq. (5).

Barring this exotic possibility, detection of d_n in the next few years would be indirect evidence against an axion model, or indeed any other theory with a Peccei-Quinn $U(1)$ symmetry. Conversely, non-observation of d_n at a level significantly below the order-of-magnitude "bound" (2) would be circumstantial evidence that the GUT does indeed possess some hitherto inapparent symmetry, very possibly of the $U(1)_{PQ}$ type. Independently of measurements of d_n , if one became convinced that a large entropy factor E must have been created after baryon number generation, e.g., during the Weinberg-Salam phase transition²¹⁾, one might become convinced already that the bound (2) was violated, in which case one could be forced into a hypoaxion model.

Broaching the subject of cosmology, it is reasonable to ask whether the hypoaxion could produce any observable cosmological effects. Despite the weakness of its interactions, its presence would affect the rate of expansion during nucleosynthesis^{22),23)}. There is an upper limit from ^4He abundance on the ratio ξ of the Universe's actual expansion rate, corresponding roughly to the addition of one extra two-component light neutrino²³⁾:

$$\xi^2 - 1 < 0.15 \quad (6)$$

By comparison, if the hypoaxion decoupled thermally from the rest of the Universe at a temperature T_A : $m_X > T_A > m_W$, one would expect²²⁾

$$\xi^2 - 1 \approx 4.4 \times 10^{-3} \quad (7a)$$

whereas if $m_X < T_A$ one would expect²³⁾

$$3^2 - 1 \leq 2.5 \times 10^{-3} \quad (7b)$$

Both these estimates are way below the allowable limit (6), and it seems unlikely that such a small indirect effect of the hypoaxion could ever be inferred or excluded by cosmological observation.

Now that we are on the subject of cosmology, we close our paper with a discussion of baryon number generation in the minimal SU(5) hypoaxion model of Wise, Georgi and Glashow (WGG)¹⁰⁾. This model contains two $\underline{5}$ of Higgs $\underline{H}_1$ and $\underline{H}_2$ and a complex $\underline{24}$ of Higgs Σ . The hypoaxion is the anti-Hermitian SU(3) $\times$ SU(2) singlet from Σ , while there are two colour triplet mass eigenstates $\underline{H}$ and $\tilde{\underline{H}}$ which are arbitrary orthonormal complex superpositions of the colour triplet fields $\underline{H}_i$ in $\underline{H}_i$ ($i=1,2$):

$$\left. \begin{aligned} \underline{H} &= a \underline{H}_1 + b \underline{H}_2 \\ \tilde{\underline{H}} &= b^* \underline{H}_1 - a^* \underline{H}_2 \end{aligned} \right\} |a|^2 + |b|^2 = 1 \quad (8a)$$

Similarly there are two SU(2) doublet mass eigenstates $\underline{H}$ (which is hierarchically light) and $\tilde{\underline{H}}$ (which is heavy). They are in general different orthonormal complex superpositions of the SU(2) doublet fields $\underline{H}_i$ in $\underline{H}_i$ ($i=1,2$):

$$\left. \begin{aligned} \underline{H} &= \alpha \underline{H}_1 + \beta \underline{H}_2 \\ \tilde{\underline{H}} &= \beta^* \underline{H}_1 - \alpha^* \underline{H}_2 \end{aligned} \right\} |\alpha|^2 + |\beta|^2 = 1 \quad (8b)$$

To realize the U(1) Peccei-Quinn⁴⁾ symmetry, only $\underline{H}_1$ couples $\underline{5}$ to $\underline{10}$ fermions with a coupling matrix A , while only $\underline{H}_2$ couples $\underline{10}$ fermions to $\underline{10}$ fermions with a symmetric coupling matrix K . At this point we begin to smell trouble because we recall that the Georgi-Glashow²⁴⁾ (GG) minimal version of the SU(5) model with only one $\underline{5}$ of Higgs fields also had only two coupling matrices to fermions. This paucity prevented the GG model from obtaining a non-vanishing (n_B/n_Y) through diagrams of lower than 8th order¹²⁾, which gave an unacceptably small number. Our only hope of getting $(n_B/n_Y) \neq 0$ from a lower order fermion trace in the WGG model¹⁰⁾ is to pick up extra phase factors from the complex superposition parameters (a,b,α,β) in Eqs. (8). It is immediately obvious that no such factor is forthcoming from the 4th order trace of Fig. 1 [see also Ref. 18)]. It is easy to prove that the a 's and b 's do not provide a complex factor for any diagram of the generic type of Fig. 2, where there is a

single fermion trace with the colour triplet Higgses not interacting between their emission and absorption. If one emits n $\underline{H}$ and m $\tilde{\underline{H}}$ Higgs from $\underline{5}-\underline{10}$ -Higgs vertices, and absorbs n' $\underline{H}$ and m' $\tilde{\underline{H}}$ Higgs on $\underline{5}-\underline{10}$ -Higgs vertices, while emitting k $\underline{H}$ and l $\underline{H}$ from $\underline{10}-\underline{10}$ Higgs vertices and absorbing k' $\underline{H}$ and l' $\tilde{\underline{H}}$ on $\underline{10}-\underline{10}$ -Higgs vertices, then one has the obvious sum rules

$$k' = k + n - n' \quad , \quad l' = l + m - m' \quad , \quad m' = n + m - n' \quad (9)$$

The resulting over-all prefactor from the complex mixing parameters (8a) is just

$$(b)^{n+l+m-m'} (b^*)^{n'+l} (a)^{m+k} (a^*)^{k+n-n'+m} \quad (10)$$

which is clearly real.

To get a complex factor multiplying the product of A and K coupling matrices we should therefore either include Higgs self-interactions, and/or include exchanges of $SU(2)$ Higgs doublets. It is not difficult to verify that while the a 's and b 's provide a complex factor of (a^*b^*) coming from the fourth order fermion trace in Fig. 3, this is countered by a factor of (ab) coming from the four-Higgs vertex, and one can then see easily that the same real catastrophe befalls more complicated diagrams with self-interactions of $\underline{H}$ and $\tilde{\underline{H}}$. [This is most easily seen by working in the original $(\underline{H}_1, \underline{H}_2)$ basis.] On the other hand, the 6th order diagram of Fig. 4, where one colour triplet Higgs and one $SU(2)$ doublet Higgs is exchanged, does acquire a complex factor from the product of complex mixing parameters from Eq. (8). However, there is a mirror image diagram with the triplet and doublet exchanges interchanged, which has an equal and opposite imaginary part so that the sum is real and no (n_B/n_Y) is generated.

The above arguments demonstrate that there is no diagram before 8th order which contributes to (n_B/n_Y) in the minimal WGG $SU(5)$ hypoaxion model¹⁰⁾. Baryon number generation is therefore probably comparable to that estimated¹²⁾ in the minimal $SU(5)$ model with a single $\underline{5}$ of Higgs, which is far too small unless there is an implausible fourth generation of heavy "light" fermions with mass $O(m_W)$. However, one can easily construct¹³⁾ richer $SU(5)$ hypoaxion models which give (n_B/n_Y) large enough, e.g., with two $\underline{5}$ of Higgses coupled to each of the $\underline{5} - \underline{10}$ and $\underline{10} - \underline{10}$ combinations of fermions, so this is not a mortal blow to the hypoaxion concept.

We hope this paper has indicated that grand unified axion models may not be immune to experimental demolition.

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FIGURE CAPTIONS

- Fig. 1 : Fourth order fermion trace which is a candidate for generating a baryon asymmetry.
- Fig. 2 : Generic class of diagrams with one fermion trace and no Higgs self-interactions.
- Fig. 3 : Candidate sixth order diagram featuring a Higgs self-interaction.
- Fig. 4 : Candidate sixth order diagram featuring the exchange of a Higgs triplet and a Higgs doublet.

Fig. 1

Fig. 2

Fig. 3

Fig. 4