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POINT-LIKE PARTICLES AND OFF-SHELL
SUPERSYMMETRY ALGEBRAS

Lars Brink^{*)}

and

Michael B. Green^{†)}

CERN -- Geneva

ABSTRACT

The reparametrization invariant action, describing supersymmetric particles, is shown to have a non-local symmetry and consequently an enlarged, $N = 2$, supersymmetry invariance with a central charge. The construction of multiplets of states only involves physical operators - there are no auxiliary states in the theory, even though the supersymmetry algebra applies off-shell. The free field action is described in terms of superfields which are Clifford operators. Comments are made about analogous string theories.

^{*)} Permanent address: Institute of Theoretical Physics, Göteborg, Sweden.

^{†)} Permanent address: Physics Dept., Queen Mary College University of London, London, England.

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Motivated by recent work on point particles¹⁾⁻³⁾ and strings⁴⁾ we have investigated the quantum mechanics of free massless particles described by reparametrization invariant, supersymmetric actions in various space-time dimensions, D.

For several of these models (when $D > 4$) the only known²⁾⁻⁵⁾ quantization procedure involves sacrificing manifest Lorentz invariance. We have found also that there is an extra non-local symmetry of the action that has hitherto been unnoticed. (This also occurs when $D = 4$.) This gives rise to an additional supersymmetry realized in a non-local manner resulting in an invariance of the action under transformations forming an $N = 2$ supersymmetry algebra with a central charge. We have constructed the smallest multiplets of states for these models when $D = 4, 6$ and 10 together with their off-shell supersymmetry transformations. When interpreted in four space-time dimensions by dimensional reduction these models describe N extended supersymmetry with $N = 2, 4$ and 8 respectively. In our construction there are no auxiliary states - the off-shell states are explicitly related to the on-shell ones. The extra supersymmetry (and the central charge) plays a crucial role in providing constraints between the states that ensure the consistency of this formulation. Although both the constraints and the supersymmetry transformations are non-local, the free action constructed from fields creating these states in the "light-like frame" is local. This action is described succinctly by introducing superfields which are functions of variables in a Clifford algebra instead of the usual Grassmann variables. Our superfields only contain physical component fields.

These results will be surveyed in this paper. Further details, with emphasis on the particularly interesting $D = 10$ model, are contained in a separate publication⁶⁾.

The theories we consider are defined by the supersymmetric reparametrization invariant action^{1),2)} (describing a massless particle)^{*)}:

$$I = \frac{1}{2} \int d\tau V^{-1} (\dot{x}^\alpha - \frac{i}{2} \bar{\theta} \gamma^\alpha \dot{\theta} + \frac{i}{2} \dot{\bar{\theta}} \gamma^\alpha \theta)^2 \quad (1)$$

where V is a one dimensional ("einbein") field and x^α is the D dimensional position co-ordinate. The spinor θ^a is taken to satisfy a Weyl condition $\theta = R\theta$ (where the right-hand Weyl projector $R = 1/2(1 + \gamma^{D+1})$) when $D = 4$ and 6 , and both a Weyl and a Majorana condition when $D = 10$. The action (1) is manifestly invariant under the $N = 1$ supersymmetry transformation:

*) Following Refs. 2) and 4), our conventions are $A \cdot B \equiv A^i B_i + A^L B_L - A^0 B^0 \equiv A^i B^i - A^+ B^- - A^- B^+$. Also $\{\gamma^\alpha, \gamma^\beta\} = -2\eta^{\alpha\beta}$ and $\gamma^{\alpha_1 \alpha_2 \dots \alpha_M}$ is the anti-symmetrized product of γ matrices normalized so that $\gamma^{12 \dots M} = \gamma^1 \gamma^2 \dots \gamma_M$.

$$\delta x^\alpha = \frac{i}{2} \bar{\epsilon} \gamma^\alpha \theta - \frac{i}{2} \bar{\theta} \gamma^\alpha \epsilon \quad (2a)$$

$$\delta \theta = \epsilon \quad (2b)$$

generated by the supersymmetry charge⁷⁾:

$$Q_1 = i \gamma \cdot p \theta, \quad \bar{Q}_1 = -i \bar{\theta} \gamma \cdot p \quad D = 4, 6 \quad (3a)$$

$$Q_1 = 2i \gamma \cdot p \theta \quad D = 10 \quad (3b)$$

where p^α is the momentum conjugate to x^α and ϵ is an anticommuting parameter. Although the action (1) is unique when $D = 10$ another term may be added inside the bracket when $D = 4$ or 6 ⁷⁾:

$$\ell \bar{\theta} \gamma^\alpha \dot{\theta} \quad (4)$$

[When $D = 4$, θ may be chosen to be a Majorana spinor instead of a Weyl spinor in which case the extra term is that given in Ref. 2).] For simplicity we shall only consider theories with $\ell = 0$ in this paper.

The case $\ell = 0$ is rather special since there are then second class constraints between the co-ordinates which do not, therefore, have the canonical Poisson bracket structure needed to proceed to a conventional quantum theory¹⁾⁻³⁾. Canonical quantization can be achieved for $D > 4$ by defining new variables in a manner which abandons manifest Lorentz invariance^{2), 5)}. For example, the variables which do possess simple commutation relations when $D = 10$ are:

$$q^\alpha = x^\alpha + i n_\beta p_\beta \frac{\bar{\theta} \gamma^{\alpha\beta\delta} \theta}{2[n \cdot p + \sqrt{-p^2}]} \quad (5a)$$

$$s^a = -e^{-\gamma/2} \frac{[\sqrt{-p^2} \theta + \gamma \cdot n \gamma \cdot p \theta]^a}{[\sqrt{-p^2} + n \cdot p]^{1/2}} \quad (5b)$$

$$T^a = -i(\gamma \cdot n S)^a = i e^{-\lambda/2} \frac{[\sqrt{-p^2} \gamma \cdot n \theta - \gamma \cdot p \theta]^a}{[\sqrt{-p^2} + n \cdot p]^{1/2}} \quad (5c)$$

$$[q^\alpha, q^\beta] = [q^\alpha, s^a] = [q^\alpha, T^a] = 0 \quad (5d)$$

where n^α is the vector $(-\cosh\lambda, 0, \sinh\lambda)$. [For other D there are minor changes in normalizations.] In the "light-like frame", $\lambda \rightarrow \infty$, Eqs. (5a) - (5c) reduce to those of Ref. 2.

a) A non-local $N = 2$ supersymmetry exposed

It is instructive to rewrite the theories in the phase space form

$$I = - \int d\tau \left[\sqrt{\frac{p^2}{2}} - p_\alpha \left(\dot{x}^\alpha - \frac{i}{2} \bar{\theta} \gamma^\alpha \dot{\theta} + \frac{i}{2} \dot{\bar{\theta}} \gamma^\alpha \theta \right) \right] \quad (6)$$

The action is now manifestly invariant under the substitution

$$\theta \rightarrow \frac{i \gamma \cdot p}{\sqrt{-p^2}} \theta \quad (7)$$

When expressed in terms of the original action [Eq. (1)] by integrating over the momentum variable p this corresponds to a non-local symmetry (which we shall call "duality"). Notice that this transformation changes the handedness of the Weyl spinor θ . Correspondingly there is an additional supersymmetry generator, Q_2 :

$$Q_2 = \frac{i \gamma \cdot p}{\sqrt{-p^2}} Q_1 = -\sqrt{-p^2} \theta \quad D = 4, 6 \quad (8a)$$

$$= -2\sqrt{-p^2} \theta \quad D = 10 \quad (8b)$$

with opposite handedness to Q_1 .

The full supersymmetry algebra [having both charges Q_1 and Q_2 is obtained by using the commutation relations in Refs. 2) or 7)]. Thus we have when $D = 4$ or 6 (the $D = 10$ case differs only by trivial normalizations):

$$\{Q_1^a, \bar{Q}_1^b\} = -(\gamma \cdot p R)^{ab} \quad (9a)$$

$$\{Q_2^a, \bar{Q}_2^b\} = -(\gamma \cdot p L)^{ab} \quad (9b)$$

$$\{Q_1^a, \bar{Q}_2^b\} = i \sqrt{-p^2} (L)^{ab} \quad (9c)$$

$$\{Q_2^a, \bar{Q}_1^b\} = -i \sqrt{-p^2} (R)^{ab} \quad (9d)$$

together with the Lorentz algebra under which the Q 's transform as spinors. [In Eqs. (9a) - (9d) the left-hand Weyl projection operators $L = (1 - \gamma^{D+1})/2$. The algebra has a central charge which vanishes on-shell ($p^2=0$), reminiscent of schemes proposed in analogous field theories⁸⁾. The above arguments apply equally well in the presence of the term (4) with non-zero ℓ .

b) The quantum states and the off-shell supersymmetry algebra

The equation of motion defining the physical (on-shell) states is equivalent to the first class constraint $p^2 = 0$ derived from the action (1). As usual, we may define off-shell states by relaxing this constraint (and thus, for example, obtain the free particle propagator).

We choose to work with variables appropriate to the light-like frame defined earlier. In this case their commutator algebra becomes:

$$\{S^a, \bar{S}^b\} = (\gamma + R)^{ab} \quad \{T^a, \bar{T}^b\} = (\gamma + L)^{ab} \quad (10)$$

$$\{S^a, T^b\} = \{S^a, \bar{T}^b\} = \dots = 0 \quad (11)$$

with

$$\gamma_- S = \gamma_- T = \bar{S} \gamma_- = \bar{T} \gamma_- = 0 \quad (12)$$

When $D = 4$ or 6 we also have

$$\{S^a, S^b\} = \{T^a, T^b\} = 0 \quad (13)$$

The case $D = 10$ is very special since S and T are Majorana (as well as Weyl) spinors and Eq. (13) does not hold. In this paper we shall describe the $D = 6$ model leaving details of $D = 4$ and the particularly interesting $D = 10$ models to a separate paper⁶⁾.

From Eqs. (10) - (13) we may identify $\bar{S}$ and $\bar{T}$ as "creation" operators with S and T the corresponding "annihilation operators" in a space with a "ground state" $|0\rangle$ which we shall choose to be a Lorentz scalar satisfying:

$$S^a |0\rangle = T^a |0\rangle = 0 \quad (14)$$

Notice that when $D = 6$ $\bar{S}^a$ and $\bar{T}^a$ each have two independent complex components due to the imposition of both the Weyl condition and the γ_- condition Eq. (12).

A multiplet of states can be obtained by repeated operation on $|0\rangle$ with the spinorial operators $\bar{Q}_1^a, \bar{Q}_2^a$ which are obtained from Eqs. (3), (8) and (5) (in the limit $\lambda \rightarrow \infty$):

$$\bar{Q}_1^a = \frac{1}{\sqrt{2p^+}} (-i \bar{S} \cdot p + \sqrt{-p^2} \bar{T})^a, \quad \bar{Q}_2^a = \frac{1}{\sqrt{2p^+}} (-i \bar{T} \cdot p - \sqrt{-p^2} \bar{S})^a \quad (15)$$

We shall denote the Majorana conjugate of a spinor ψ by $\tilde{\psi}$ defined by:

$$\tilde{\psi} = C \bar{\psi}^T \quad (16)$$

and

$$\tilde{\bar{\psi}} = -\psi^T C \quad (17)$$

(recall that the charge conjugation matrix C is symmetric when $D = 6$).

The states we obtain are:

A complex scalar

$$|B\rangle \equiv |0\rangle \quad (18a)$$

Two Weyl spinors

$$|\bar{\Lambda}_1^a\rangle \equiv \frac{1}{\sqrt{2}} \bar{Q}_1^a |0\rangle = \frac{1}{2\sqrt{p^+}} (-i \bar{S} \gamma \cdot p + \sqrt{-p^2} \bar{T})^a |0\rangle \quad (18b)$$

$$|\bar{\Lambda}_2^a\rangle \equiv \frac{1}{\sqrt{2}} \bar{Q}_2^a |0\rangle = \frac{-1}{2\sqrt{p^+}} (i \bar{T} \gamma \cdot p + \sqrt{-p^2} \bar{S})^a |0\rangle \quad (18c)$$

A complex vector

$$|A^i\rangle \equiv \frac{1}{2\sqrt{2}} \bar{S} \gamma^i - \tilde{T} |0\rangle \quad (18d)$$

$$|A^-\rangle \equiv -\frac{i\sqrt{-p^2}}{2p^+} \frac{[\bar{S} \gamma - \tilde{S} - \bar{T} \gamma - \tilde{T}]}{2\sqrt{2}} |0\rangle + \frac{p_-}{p^+} |A^i\rangle \quad (18e)$$

$$|A^+\rangle \equiv 0$$

A complex scalar

$$|C\rangle \equiv \frac{(\bar{S} \gamma - \tilde{S} + \bar{T} \gamma - \tilde{T})}{4\sqrt{2}} |0\rangle \quad (18f)$$

A state which is a complex scalar on shell

$$|C'\rangle \equiv \frac{(\bar{S} \gamma - \tilde{S} - \bar{T} \gamma - \tilde{T})}{4\sqrt{2}} |0\rangle \quad (18g)$$

Two Weyl spinors

$$|\bar{X}_1^a\rangle \equiv -\frac{1}{8\sqrt{p^+}} [i \bar{S} \gamma \cdot p \bar{T} \gamma - \tilde{T} - \sqrt{-p^2} \bar{T} \bar{S} \gamma - \tilde{S}]^a |0\rangle \quad (18h)$$

$$|\bar{X}_2^a\rangle \equiv -\frac{1}{8\sqrt{p^+}} [i \bar{T} \gamma \cdot p \bar{S} \gamma - \tilde{S} + \sqrt{-p^2} \bar{S} \bar{T} \gamma - \tilde{T}]^a |0\rangle \quad (18i)$$

A complex scalar

$$|D\rangle \equiv \frac{1}{16} (\bar{S} \gamma - \tilde{S})(\bar{T} \gamma - \tilde{T}) |0\rangle \quad (18j)$$

These states are guaranteed to transform correctly under Lorentz transformations by the manner of construction since $\bar{Q}_1$ are spinors. The superficial lack of covariance is due to our special co-ordinate frame which has resulted in the states being automatically defined in the $|A^+> = 0$ gauge.

All the $2^4 = 16$ complex states that can be created by $\bar{S}$ and $\bar{T}$ are physical states. We thus have the same number of states off-shell as on-shell. Although this is unusual, consistency is guaranteed by some equally unusual relationships between the states. Thus the longitudinal component $|A^->$ is related to the on-shell scalar state $|C'>$ by

$$|A^-> = \frac{p_j}{p^+} |A^j> - i \frac{\sqrt{-p^2}}{p^+} |C'> \quad (19)$$

i.e.,

$$p_\mu |A^\mu> = i \sqrt{-p^2} |C'> \quad (20)$$

with

$$|A^+> = 0$$

Also:

$$|\bar{\Lambda}_2^a> = \left[|\bar{\Lambda}_1> \left(-\frac{i \gamma \cdot p}{\sqrt{-p^2}} \right) \right]^a \quad (21)$$

$$|\bar{X}_2^a> = \left[|\bar{X}_1> \left(-\frac{i \gamma \cdot p}{\sqrt{-p^2}} \right) \right]^a \quad (22)$$

These last conditions follow directly from the dual symmetry [Eq. (7)].

Equations (21) and (22) constrain the spinors so that they may be expressed entirely in terms of their on-shell components $|\bar{\Lambda}_1>_{\gamma_-}$ and $|\bar{\Lambda}_2>_{\gamma_-}$:

$$|\bar{\Lambda}_1^a> = \left[|\bar{\Lambda}_1>_{\gamma_-} \gamma_- \gamma \cdot p + i \sqrt{-p^2} |\bar{\Lambda}_2>_{\gamma_-} \gamma_- \right]^a / 2p^+ \quad (23a)$$

$$|\bar{\Lambda}_2^a> = \left[|\bar{\Lambda}_2>_{\gamma_-} \gamma_- \gamma \cdot p - i \sqrt{-p^2} |\bar{\Lambda}_1>_{\gamma_-} \gamma_- \right]^a / 2p^+ \quad (23b)$$

and similarly for $|\bar{X}_1>$ and $|\bar{X}_2>$.

The supersymmetry transformations are obtained by operating on these states with

$$\delta = \bar{\epsilon}_1 Q_1 + \bar{\epsilon}_2 Q_2 + \bar{Q}_1 \epsilon_1 + \bar{Q}_2 \epsilon_2 \quad (24)$$

where ϵ_1 and ϵ_2 are anticommuting constant parameters.

To make contact with field theory we shall write these rules in terms of fields that create the above states. Our notation will be that a lower case greek or roman letter represents the field corresponding to the state with the same upper case letter. Furthermore we will consistently impose a reality condition on the vector field a^μ :

$$a^\mu = a^{\mu*} \quad (25)$$

which implies

$$c = c^*, \quad c' = -c'^*, \quad \chi_1 = \tilde{\lambda}_1, \quad \chi_2 = \tilde{\lambda}_2, \quad d = -b^* \quad (26)$$

This leads to an irreducible representation that contains sixteen fields. The off-shell transformations generated by δ are:

$$\delta b^* = \sqrt{2} (\bar{\epsilon}_1 \lambda_1 + \bar{\epsilon}_2 \lambda_2) \quad (27a)$$

$$\begin{aligned} \delta \lambda_1 = & -\frac{1}{\sqrt{2}} (\gamma \cdot p \epsilon_1 - i \sqrt{-p^2} \epsilon_2) b^* + \frac{1}{2} (\gamma \cdot p \tilde{\epsilon}_1 - i \sqrt{-p^2} \tilde{\epsilon}_2) c \\ & - \frac{i}{2} \Gamma^\mu (\sqrt{-p^2} \tilde{\epsilon}_1 - i \gamma \cdot p \tilde{\epsilon}_2) p_{\mu\nu} a^\nu \end{aligned} \quad (27b)$$

$$\begin{aligned} \delta \lambda_2 = & -\frac{1}{\sqrt{2}} (\gamma \cdot p \epsilon_2 + i \sqrt{-p^2} \epsilon_1) b^* + \frac{1}{2} (\gamma \cdot p \tilde{\epsilon}_2 + i \sqrt{-p^2} \tilde{\epsilon}_1) c \\ & - \frac{i}{2} \Gamma^\mu (-\sqrt{-p^2} \tilde{\epsilon}_2 - i \gamma \cdot p \tilde{\epsilon}_1) p_{\mu\nu} a^\nu \end{aligned} \quad (27c)$$

$$\delta c = \bar{\epsilon}_1 \tilde{\lambda}_1 + \tilde{\lambda}_1 \epsilon_1 + \bar{\epsilon}_2 \tilde{\lambda}_2 + \tilde{\lambda}_2 \epsilon_2 \quad (27d)$$

$$\delta a^\mu = \bar{\epsilon}_1 \Gamma^\mu \tilde{\lambda}_2 + \bar{\epsilon}_2 \Gamma^\mu \tilde{\lambda}_1 + \tilde{\lambda}_2 \Gamma^\mu \epsilon_1 + \tilde{\lambda}_1 \Gamma^\mu \epsilon_2 \quad (27e)$$

In the above:

$$p_{\mu\nu} = \frac{p_\mu p_\nu}{p^2} + \eta_{\mu\nu} \quad (28)$$

and

$$\Gamma^\mu = \gamma^\mu + \gamma_- \frac{p^\mu}{p^+} \quad (29)$$

The Γ^μ matrices play a central role in ensuring that the supersymmetry transformations are consistent with the gauge condition $a^+ = 0$ ⁴⁾.

The spinors λ_1 and λ_2 satisfy conditions conjugate to Eqs. (23a) and (23b) and a^- satisfies a condition like (20) relating it to an on-shell scalar field c' . When $p^2 \rightarrow 0$ the transformations (27a) - (27e) reduce to familiar ones.

It is now easy to obtain the $N = 4$ Yang-Mills representation in four dimensions by dimensional reduction of Eqs. (27a) - (27e). In $D = 4$ the creation operators $\bar{S}$ and $\bar{T}$ each have one independent complex component. In total they can create $2^2 = 4$ complex states when acting on $|0\rangle$, thus creating the $N = 2$ Wess-Zumino multiplet ⁶⁾.

For $D = 10$ it is not possible to construct creation and annihilation operators since Eq. (13) is not valid. However, an alternative method for constructing the off-shell representation using S and T (which each have eight real components) leads to a multiplet with $2^8 = 256$ states, appropriate to $N = 2$ supersymmetry.

Other representations of these theories can be obtained by operating with $\bar{Q}$'s on a ground state which transforms as a higher spin representation of the Lorentz group.

c) The free action and Clifford valued superfields

It is straightforward to construct a supersymmetric free action out of the physical components of the fields in the multiplet. The equations of motion are all simply given by the constraint equation $p^2 = 0$ which is obtained from the Klein-Gordon action density

$$\mathcal{L} = - (b p^2 b^* + \frac{1}{2} c p^2 c + \frac{1}{2} a^i p^2 a^i + \frac{1}{2} c' p^2 c') \quad (30)$$

$$-\frac{1}{\sqrt{2}} \left(\frac{\gamma - \lambda_1}{\sqrt{p^+}} \right)^\dagger p^2 \left(\frac{\gamma - \lambda_1}{\sqrt{p^+}} \right) - \frac{1}{\sqrt{2}} \left(\frac{\gamma - \lambda_2}{\sqrt{p^+}} \right)^\dagger p^2 \left(\frac{\gamma - \lambda_2}{\sqrt{p^+}} \right) \quad (30)$$

cont.

which is invariant under the transformations of Eq. (27). Notice that this action density contains no $\sqrt{-p^2}$ factors. This can be rewritten in a more apparently covariant way as

$$\mathcal{L} = - \left(\partial_\mu b^* \partial^\mu b + \frac{1}{2} \partial_\mu c \partial^\mu c + \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\lambda}_1 \gamma \cdot \partial \lambda_1 + i \bar{\lambda}_2 \gamma \cdot \partial \lambda_2 \right) \quad (31)$$

in the gauge $a^+ = 0$ supplemented by the constraints on a^- , λ_1 and λ_2 described above. $F_{\mu\nu}$ is the Abelian field strength in terms of a_μ .

It is also interesting to introduce a superfield Φ which is a function of the c number co-ordinates p^+ , x^+ , $x^\perp$ and the operators S , $\bar{S}$, T and $\bar{T}$. The expression $\Phi|0\rangle$ can be expanded in a series of operators:

$$\begin{aligned} \Phi|0\rangle = & \left[B b^* + \frac{i}{\sqrt{p^+}} (\bar{S} \lambda_1 + \bar{T} \lambda_2) + C c + A^\mu p_{\mu\nu} a^\nu \right. \\ & \left. - \frac{i}{\sqrt{p^+}} (\bar{S} \gamma - \tilde{S} \bar{T} \tilde{\lambda}_1 + \bar{T} \gamma - \tilde{T} \bar{S} \tilde{\lambda}_2) + D b \right] |0\rangle \end{aligned} \quad (32)$$

Under the supersymmetry transformation Eqs. (27a) - (27e) $\Phi|0\rangle$ varies so that:

$$\delta \Phi|0\rangle = \delta \Phi|0\rangle \quad (33)$$

$\Phi|0\rangle$ describes only the physical degrees of freedom of the theory (both on-shell and off-shell). Unlike for conventional superfields there are no problems with field dimensions since $\bar{S}^a$ and $\bar{T}^a$ are dimensionless. The free action [Eq. (30)] is given simply by

$$\mathcal{L} = - \langle 0 | \Phi^\dagger p^2 \Phi | 0 \rangle \quad (34)$$

Demanding that $\Phi|0\rangle$ transforms as a Lorentz scalar is sufficient to ensure the correct constraints on the complete spinors λ_1 and λ_2 and between a^- and c' .

We do not yet know whether this superfield formulation is the most appropriate to describe the interacting theory. Let us note, however, that imposing the condition that a product of several superfields be a Lorentz scalar leads to a modification of the constraints between the spinors and on the longitudinal vector. Clearly, such a modification must arise for the interacting theory.

d) The supersymmetric string

A natural generalization of the Nambu-Goto string action to the supersymmetric case when $D = 10$ is the reparametrization invariant supersymmetric action^{*)}

$$I = \frac{1}{2\pi\alpha'} \int d^2\zeta \sqrt{-g} g^{\mu\nu} (\partial_\mu x^\alpha - i\bar{\theta}\gamma^\alpha\partial_\mu\theta)(\partial_\nu x_\alpha - i\bar{\theta}\gamma_\alpha\partial_\nu\theta) \quad (35)$$

where $x^\alpha \equiv x^\alpha(\sigma, \tau)$ is the position co-ordinate and $\theta^a \equiv \theta^a(\sigma, \tau)$ a Majorana-Weyl spinor defined on the world sheet of the string parametrized by $\xi^\mu = (\sigma, \tau)$. This action may be rewritten in phase space co-ordinates. Once again there is an extra symmetry under the interchange of Eq. (7) where p is now the total momentum on the string. Hence this theory also has $N = 2$ supersymmetry, suggesting that the spectrum may be associated with the closed string sector of the conventional string model. The zero slope limit ($\alpha' \rightarrow 0$) would then correspond to $N = 8$ supersymmetry when compactified to four space-time dimensions. Unlike for the point-like particle when $D = 10$ it is possible to add a term to Eq. (35):

$$L \int d^2\zeta \sqrt{-g} g^{\mu\nu} \partial_\mu \bar{\theta} \partial_\nu \theta \quad (36)$$

which also allows the $N = 2$ symmetry.

We have not yet considered the spectrum of these string theories.

e) Comments

We have found that the quantum mechanics of massless particles described by the action (1) has $N = 2$ supersymmetry. There are only enough operators to create the physical states which means that the off-shell states automatically satisfy constraints that eliminate extra components. Although the constraints are non-local, such non-locality does not appear in the free action due to a

^{*)} This action has also been considered independently by J.H. Schwarz, B. Julia and M. Roček.

subtle cancellation. We have explicitly considered the massless states.

However, the addition of a mass term to the action (1) [as in Ref. 2)] does not affect the extra supersymmetry. The massive multiplets can therefore be constructed by the same procedure as in this paper.

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