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AN SU(5) MODEL WITH HIERARCHICAL FERMION MASSES
AND A PHANTOM AXION

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A B S T R A C T

We present an SU(5) model with hierarchical fermion structure and without strong CP violation. The Peccei-Quinn U(1) symmetry we need is tied to the symmetry needed to produce the observed fermion mass spectrum. It is spontaneously broken at 10^{15} GeV which makes the axion almost unobservable.

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1. One of the most severe problems facing unified gauge theories today is the so-called strong CP violation problem. Why is the famous, non-perturbatively created θ parameter at least smaller than 10^{-9} ? The most natural solution is to make θ the "unobservable" phase of an $U(1)$ global chiral symmetry. Since there are no massless quarks around, Peccei and Quinn suggested¹⁾ that the theory should necessarily be invariant under a built-in $U(1)$ symmetry, called here $U(1)_{PQ}$. This $U(1)_{PQ}$ symmetry will eventually be spontaneously broken, say at some scale M and since $U(1)_{PQ}$ is also explicitly broken by the Adler-Bell-Jackiw anomaly, a pseudo-Goldstone boson called axion²⁾ will be formed with mass

$$m_A \approx \frac{f_n m_n}{M} \quad (1)$$

and a pseudoscalar coupling

$$t_A \approx \frac{t_n}{M} \quad (2)$$

Peccei and Quinn identified the scale M with the scale M_W of unification of weak and electromagnetic interactions. Negative experimental searches³⁾ and astrophysics⁴⁾ entail that

$$M > 10^9 \text{ GeV} \quad (3)$$

Recently Dine, Fischler and Srednicki (DFS)⁵⁾ made the observation that a natural candidate for M is the grand unification scale $M_X \sim 10^{15} \text{ GeV}$. An explicit realization of this idea in $SU(5)$ has been given in Ref. 6). In this note we present an $SU(5)$ model where the DFS trick is naturally realized since the highly desired $U(1)_{PQ}$ symmetry is part of the global symmetry that is needed in order to produce a hierarchical fermion mass spectrum.

2. Before getting to the specific example, let us discuss in general terms how we believe that the $U(1)_{PQ}$ appears and why it is broken at superhigh energies. As is well known, grand unified theories do not solve the family problem. We need something more, so we should not be surprised if, at some energies above or equal to $M_X (\sim 10^{15} \text{ GeV})$, something like $G_{\text{world}} \rightarrow G_{\text{GUT}} \times G_{\text{family}} \rightarrow G_{\text{GUT}} \times U(1)^n$ is happening. To us, the family problem and the hierarchical fermion mass spectrum are closely interrelated, if not two views of the same coin. So by buying this bigger symmetry, we assume that we produce

three families and hierarchical fermion masses. Then, it is easy to see that some combination of the n $U(1)$ s may be identified with the $U(1)_{PQ}$ which will be spontaneously broken at 10^{15} GeV thus creating a PHANTOM AXION and exorcising sizeable strong CP violation.

3. We next move to a specific example materializing the above scenario. The cornerstone of the model is the Yukawa coupling, given by

$$\begin{aligned} \mathcal{L}_Y = & a \bar{5}_3 10_3 \bar{5}_H + b 10_3 10_3 5_H + c \bar{5}_2 10_3 \bar{5}'_H + d 10_2 10_2 5'_H \\ & + e 10_3 10_2 \bar{5}_H + f 10_3 10_2 \bar{5}'_H + g 5_1 5_3 \bar{10}_H + h.c. \end{aligned} \quad (4)$$

Here, $5_i, 10_i$ denote the usual three fermion generations and the fields with a subscript H are scalars. The above couplings a, b, c, d follow uniquely if one requires that the first and second generations get only two-loop and one-loop masses, respectively, and charm is not massless at the one-loop level, while the third generation has a direct mass. The couplings to the other Higgs fields (notice that $\langle 10 \rangle = \langle 50 \rangle = 0$ because of colour - and charge conservation) are the minimal ones in order to obtain the above masses, e.g., m (first generation): m (second generation): m (third generation) = $\alpha^2:\alpha:1$.

$\mathcal{L}_Y$ has, in addition to the local $SU(5)$ symmetry, a global $U(1)^4$ symmetry, given by

$$\begin{aligned} U(1)_A : \quad & 5_2 \rightarrow e^{i\alpha} 5_2 \quad 5_3 \rightarrow e^{i3\alpha} 5_3 \quad 10_3 \rightarrow e^{i\alpha} 10_3 \\ & 10_H \rightarrow e^{i3\alpha} 10_H \quad 5_H \rightarrow e^{-2i\alpha} 5_H \quad 50_H \rightarrow e^{i\alpha} 50_H \quad 50'_H \rightarrow e^{i\alpha} 50'_H \end{aligned} \quad (5)$$

$$\begin{aligned} U(1)_B : \quad & 10_2 \rightarrow e^{i\beta} 10_2 \quad 5_2 \rightarrow e^{i2\beta} 5_2 \\ & 5'_H \rightarrow e^{-2i\beta} 5'_H \quad 50_H \rightarrow e^{i\beta} 50_H \end{aligned} \quad (6)$$

$$U(1)_C : \quad 5_1 \rightarrow e^{i\gamma} 5_1 \quad 10_H \rightarrow e^{i\gamma} 10_H \quad (7)$$

$$U(1)_D : \quad 10_1 \rightarrow e^{i\delta} 10_1 \quad 50'_H \rightarrow e^{i\delta} 50'_H, \quad (8)$$

fields not occurring behave neutrally under the respective $U(1)$ s. The four $U(1)$ s protect the lighter fermions from getting a direct mass and ensure B-L conservation. Notice that all $U(1)$ s are anomalous^{*}).

In order that the light quark masses are non-zero, the $U(1)$ s must be broken. In Ref. 7) we considered only breaking by soft terms in the Higgs potential; in this paper we will modify this. The minimal terms needed to break the chiral symmetries and to obtain satisfactory Cabibbo mixings⁸⁾ are

$$\begin{aligned} \mathcal{L}^3 = & A (\bar{10}_H S_H S'_H) + B (10_H 10_H \bar{50}_H) \\ & + C (S_H 50_H' 75_H) + D (S'_H 50_H 75_H^\dagger) + h.c. \end{aligned} \quad (9)$$

where we have introduced a (complex) 75-dimensional Higgs multiplet⁷⁾. The four terms then transform under (5)-(8) as follows:

$$\begin{aligned} U(1)_A : & A \rightarrow e^{i5\alpha} A \quad B \rightarrow e^{i5\alpha} B \quad C \rightarrow e^{i(-1+\bar{a})\alpha} C \quad D \rightarrow e^{i(1-\bar{a})\alpha} D \\ U(1)_B : & -2 \quad -1 \quad \bar{b} \quad -1-\bar{b} \\ U(1)_C : & -1 \quad 2 \quad \bar{c} \quad -\bar{c} \\ U(1)_D : & 0 \quad 0 \quad 1+\bar{d} \quad -\bar{d} \end{aligned} \quad (10)$$

where $\bar{a}$, $\bar{b}$, $\bar{c}$, $\bar{d}$ are the transformation properties of the 75 under the $U(1)$ s [e.g.; $U(1)_A : 75 \rightarrow e^{i\bar{a}\alpha} 75$].

One might ask whether (9) must not be amended by additional terms, transforming like A, B, C, D under the $U(1)$ s acting as renormalization counterterms. For example, take a term $\bar{5}_H \bar{50}_H 75_H^\dagger$, which transforms like D. It can be constructed from a one loop-diagram involving D and the invariant term $50_H 50_H 5_H 5_H^\dagger$. But since it is linear in D its renormalization is multiplicative and need therefore not be added to $\mathcal{L}^3$ ⁹⁾ as an extra counterterm. We have verified that $\mathcal{L}^3$ is indeed complete.

In Ref. 7) the 75_H was taken to be real ($\bar{a} = \bar{b} = \bar{c} = \bar{d} = 0$). Then all $U(1)$ s are explicitly broken by $\mathcal{L}^3$. But if 75_H is complex we may ask if there exists an unbroken $U(1) = xU_A + yU_B + zU_C + sU_D$ with appropriate x, y,

^{*}) This is so although the $U(1)$ s are not "chiral" in the usual sense.

z, s. Using rules (10) we find^{*)} this to be the case if

$$x=-1 \quad y=1 \quad z=3 \quad s=1 \quad (11a)$$

$$-\bar{a} + \bar{b} + 3\bar{c} + \bar{d} = -2 \quad (11b)$$

Equation (11b) indicates that the 75 transforms non-trivially under $U(1)$, in fact $75_H \rightarrow e^{-2i\phi} 75_H$. For further use we notice that $U(1)$ is also anomalous.

In the following we take the 75 to satisfy (11b). Before continuing we determine if there exist terms such as $(5'_H 5'_H) 75_H$, $(5_H \bar{5}'_H) 75^2$ ^{**) in the Higgs potential (see below). The first term is not allowed because of (11b). Depending on the choice of $\bar{a}$, $\bar{b}$, $\bar{c}$, $\bar{d}$, the second may be allowed ($\bar{a} = 1$, $\bar{b} = -1$, $\bar{c} = \bar{d} = 0$) or not. We will consider both cases.}

We now see that $U(1)$ can be taken as PQ symmetry. (For a different realization see below). That is, it can be used to rotate the vacuum angle way in the tree Lagrangian.

When the 75_H and then the 5_H , $5'_H$ get a non-vanishing vacuum expectation value we expect a massless (Goldstone) boson associated with the breaking of the $U(1)$, which is a mixture of neutral components of the 75_H , 5_H and $5'_H$. Since the vacuum expectation value of the 75_H is much bigger than the ones of the 5_H , $5'_H$, we expect it to lie predominantly in the 75_H with a small coupling to the 5_H and $5'_H$ of order $M_W/M_{GUT} \sim 10^{-13}$, as pointed out by DFS⁵⁾. At this stage the terms

$$K (\bar{5}' 5)_1 (75^2)_1 \quad \text{and} \quad K' (\bar{5}' 5)_{24} (75^2)_{24} \quad (12)$$

are important. It is easy to see that the following combinations of neutral scalar^{***)} field form mass eigenstates (in an obvious notation):

^{*)} Of course, x , y , z , s are given only up to a common factor.

^{**) There are, of course, two terms of this form in $SU(5)$: $(\bar{5}' 5)_1 (75^2)_1$ and $(\bar{5}' 5)_{24} (75^2)_{24}$ in an obvious notation.}

^{***)} We write the neutral component of a Higgs multiplet as $\phi = e^{i\beta} (v + \eta + i\xi)$ where v is a real number, β the phase of the vacuum expectation value and η and ξ real fields.

$$\textcircled{1} \quad \frac{U_5 \bar{f}_5 + U_5' \bar{f}_5'}{\sqrt{U_5^2 + U_5'^2}} \rightarrow \text{eaten by } Z\text{-boson} \quad (13a)$$

$$\textcircled{2} \quad \frac{1}{N} \left(\frac{-U_5' \bar{f}_5 + U_5 \bar{f}_5'}{\sqrt{U_5^2 + U_5'^2}} + \frac{2 U_5 U_5'}{\sqrt{U_5^2 + U_5'^2} \cdot V_{75}} \bar{f}_{75} \right) \quad \text{massive state} \quad (13b)$$

with mass $= \frac{\sqrt{2|\bar{k}|} \cdot V_{75} \cdot \sqrt{U_5^2 + U_5'^2}}{\sqrt{U_5 U_5'}}$

where $\bar{k}$ is given by that linear combination of k and k' in which the doublet pieces of $5_H, 5_H'$ enter and

$$N^2 = 1 + \frac{4 U_5^2 U_5'^2}{V_{75}^2 (U_5^2 + U_5'^2)} \quad (13c)$$

$$\textcircled{3} \quad \frac{1}{N} \left(\frac{-U_5' \bar{f}_5 + U_5 \bar{f}_5'}{(U_5^2 + U_5'^2)} \cdot \frac{2 U_5 U_5'}{V_{75}} - \bar{f}_{75} \right)$$

This last state will be identified with the axion, for when the instanton effects are taken into account, it is not strictly massless but has a mass $m \sim \Lambda_{\text{QCD}}^2 / M_X \sim 10^{-15}$ GeV. This bears out the above expectations and ensures that the axion's coupling to matter is negligible.

As was pointed out before, the terms (12) are only possible if $\bar{a} = 1$, $\bar{b} = -1$, $\bar{c} = \bar{d} = 0$. Then both terms occur and in general one phase is allowed since only one of the k , or a suitable linear combination can be made real by appropriately choosing the 75_H . If the above assignment of $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ is not met, the terms (12) vanish at tree level; but it is easy to see that they are generated in higher order. (See, for example, the figure.) Notice that in this case there is only one term of the form $(75_H^2) 5_H^+ 5_H'$, as seen from a group theoretical analysis of the figure. Of course, in even higher order, both terms (12) may be generated, which can have interesting consequences for the phenomenology of CP violation. While the size of the coupling corresponding to the figure is reduced, say by a factor $\sim 10^{-2}$, the above results about the mass and coupling of the axion are altered only slightly.

In the above analysis we have taken B of the four U(1)s to be broken softly. In the light of the above discussion it is easy to see how to construct a situation in which all U(1)s are broken spontaneously without observable (Goldstone bosons). For example, we can modify $\mathcal{L}^3$ as follows

$$\mathcal{L}^3 \rightarrow \mathcal{L}^{3'} = A S_H S_H' \bar{10}_H \phi_i^+ + \dots \quad (14)$$

where the ϕ_i are complex singlets and the 75 might be taken real. If the ϕ_i transform under the U(1)s just opposite to the four couplings in Eqs (10), then the U(1)s are good symmetries. (Any of them could be taken as PQ symmetry.) Spontaneous symmetry breaking, due to $\langle \phi_i \rangle \approx M$, gives four essentially uncoupled massless scalars, thus preserving all nice results of Ref. 7) and of this paper.

Finally, we wish to comment on the cosmological baryon generation. Unlike the model presented in Ref. 6), the model described here leads, in general, to baryon generation in accord with observations.

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FIGURE CAPTION

Effective $5_H^+ 5_H' 75_H 75_H$ coupling.