Ref.TH-3141-CERN

MAGNITUDE OF THE $O(\alpha_s^2)$ CORRECTIONS TO JET PRODUCTION
IN e^+e^- ANNIHILATION

Z. Kunszt^{*)}

CERN -- Geneva

ABSTRACT

I give a very short review of the existing theoretical calculations and present some new results on the thrust distributions in e^+e^- annihilation. I point out that the magnitude of the corrections relevant to the experimental determination of α_s is about 35%.

^{*)} On leave from the L. Eötvös University, Budapest.

Ref.TH.3141-CERN

20 August 1981

Jet production in e^+e^- annihilation gives one of the most convincing pieces of evidence for the existence of quarks and coloured gluons. In particular, the three-jet production rate gives a measure of α_s , the running coupling constant of QCD¹⁾. The experimental determination of α_s requires a knowledge of the $O(\alpha_s^2)$ corrections since α_s is uniquely defined only at this order.

In the last few years there has been a considerable effort to calculate these $O(\alpha_s^2)$ contributions. The purpose of this paper is to briefly review the existing theoretical work, to present some new results on thrust distributions and to point out that the corrections relevant for the experimental determination of α_s are about 35%.

The theoretical calculation consists of three parts:

- i) a determination of the four-jet matrix elements which are singular in the soft and collinear region,
- ii) a calculation of the loop corrections and
- iii) the introduction of a certain pole term in order to cancel analytically the collinear and soft singularities present in the loop and the four jet contributions respectively.

The original four-jet matrix element calculations²⁾ have now been verified by three other groups³⁾. A recent major development is due to Ellis, Ross and Terrano⁴⁾ (ERT), who calculated the loop corrections, gave an explicit pole term and demonstrated the analytical cancellation of the $1/\epsilon^2$ and $1/\epsilon$ singularities of the loop corrections and the four-jet contributions. Their analytical formula for the loop corrections was recently confirmed by Kramer et al.⁵⁾. Ellis, Martinelli and Petronzio⁶⁾ calculated the $O(\alpha_s^2)$ corrections to the large p_T spectrum of the Drell-Yan process, where the same loop diagrams enter, but in the crossed channel. Performing crossing they confirmed the result of ERT.

The pole term was slightly modified in Ref. 7) and this modified pole term was used for the numerical evaluation of the thrust distribution^{*)}. It was argued that this is the appropriate method for the subsequent introduction of jet fragmentation. The numerical evaluation of the relevant distributions is difficult because certain integrable singularities still remain in the regularized four-jet matrix elements even after the $1/\epsilon^2$ and $1/\epsilon$ singularities are eliminated by the subtraction of the pole term.

*) The numbers in Ref. 7) have $\approx 15\%$ Monte Carlo integration errors. Furthermore, due to a numerical error α_s is larger by about 10% than indicated in the paper. I thank R.K. Ellis and D.A. Ross for correspondence on this point.

Another important development was the calculation of Gaemers, Oldham and Vermaseren⁸⁾ (GOV), who evaluated the loop corrections numerically and arranged the cancellation of the collinear and soft singularities by the introduction of a small gluon mass. The comparison of their results with others is only possible numerically. GOV calculated thrust distribution and C distribution^{*)} and their result is in qualitative agreement with the numerical evaluation done in Refs. 7) and 11), where the formulae of ETR have been used. In order to make a quantitative check, one must calculate the same quantities with the same input parameters. Recently, Ellis and Ross¹²⁾ also evaluated the thrust distribution in the region $t = 0.5 - 0.975$ with the parameters used by GOV.

In this paper I present some new calculations with the same parameters as ER and GOV. A convenient way of giving the result is to plot the functions $A_0(t)$ and $A_1(t)$ as proposed by GOV⁸⁾. $A_0(t)$ and $A_1(t)$ are defined as follows^{**)}

$$\frac{1}{G_0} \frac{dG}{dt} = \frac{\alpha_s}{\pi} A_0(t) + \left(\frac{\alpha_s}{\pi}\right)^2 A_1(t) \quad (1)$$

where t denotes thrust,

$$G_0 = \frac{4\pi\alpha^2}{3Q^2} \cdot \frac{11}{3} \quad (2)$$

and

$$\alpha_s = \frac{2\pi}{3.833 \ln \frac{Q^2}{\Lambda^2} + 2.522 \ln \ln \frac{Q^2}{\Lambda^2}} \quad (3)$$

Q is the mass of the virtual photon. In Fig. 1 I have plotted the curves I have found for $A_0(t)$ and $A_1(t)$. They are in complete numerical agreement with those obtained by GOV⁸⁾, CW¹¹⁾ and ER¹²⁾. Furthermore, for the quantity

$$H_{0.85} = \frac{\int_{0.5}^{0.85} A_1(t) dt}{\int_{0.5}^{0.85} A_0(t) dt} \quad (4)$$

I obtain $H_{0.85} = 17.6 \pm 0.3$ which should be compared with results $H_{0.85} = 17.2 \pm \pm 0.2$ of GOV and $H_{0.85} = 17.6 \pm 0.3$ of ER. This agreement is very impressive and I conclude that both the analytic result of ERT⁴⁾ and the numerical evaluation of GOV⁸⁾ are correct^{***)}.

*) C distribution⁹⁾ is a kind of topological distribution similar to thrust¹⁰⁾.

**) I always assume five flavours.

***) The histogram $A_1(t)$ of Fig. 1 is in disagreement with the curve derived from Fig. 2 of Ref. 13).

Let me now discuss the size of the corrections. ERT⁴⁾ found large corrections for C distributions; Refs. 7), 8), 11) and 12) found large corrections for thrust distribution. However, relatively large α_s was used ($\alpha_s \approx 0.16-0.17$) and the corrections have been evaluated in regions where the four-jet contributions are kinematically enhanced. Therefore, the large corrections found by them does not indicate a priori any major difficulty for the perturbative description of jet production in e^+e^- annihilation in general. I illustrate this remark in Fig. 2 where I have plotted the thrust distribution for three body and four-body phase space distribution. One can see that the four-jet phase space has a maximum of about 0.85 while the three-body phase space is peaked at $T = 1$. Therefore the corrections are kinematically enhanced in the region $T = 0.5 - 0.85$.

Another simple observation concerns the magnitude of the thrust cut-off. A thrust value as high as $t = 0.975$ still represents a three-jet-like configuration. In particular, with $T_C = 0.975$ at $Q = 35$ GeV, the minimal invariant mass of two final partons is ≈ 5 GeV. As a matter of fact, the experimentalists^{14),15)} have a thrust cut-off in the three-jet matrix element around $T_C = 0.96$. Varying T_C in the region $0.98 - 0.95$, the change is compensated by a corresponding change in the properties of the jets. The three-jet cross-section with $T_C = 0.975$ is almost ten times larger than the cross-section with $T_C = 0.85$ (≈ 120 pbarn vs ≈ 14 pbarn at $Q = 35$ GeV, $\Lambda_{\overline{MS}} = 250$ MeV). Therefore the bulk of the data used for the measurement of α_s was in the region $t = 0.975 - 0.85$.

After these remarks I list three effects which all decrease the magnitude of the corrections. In the end I obtain a total correction relevant to the experimental determination of α_s of about 35%.

- a) A new and very reliable experimental determination of $\Lambda_{\overline{MS}}$ was obtained recently using the $O(\alpha_s^4)$ corrections¹⁶⁾ to the total width of the T ¹⁷⁾. It was found that $\Lambda_{\overline{MS}} = 100 \pm \frac{34}{25}$ MeV. As it was mentioned above in Refs. 4), 7), 8) and 11) calculating C and thrust distributions $\Lambda_{\overline{MS}} = 500 - 300$ MeV was used. Obviously their result is decreased if we use $\Lambda_{\overline{MS}} = 100$ MeV. For the sake of illustration I give three values of the running coupling constant with five flavours

$$\begin{aligned}\alpha_s(Q=35, \Lambda=.5) &= 0.166 \\ \alpha_s(Q=35, \Lambda=.25) &= 0.144 \\ \alpha_s(Q=35, \Lambda=.1) &= 0.123\end{aligned}\tag{5}$$

We can see that the value of α_s decreases by $\approx 34\%$ if we change $\Lambda = 0.5$ GeV to $\Lambda = 0.1$ GeV.

- b) The correction for the higher T region are smaller. If we calculate $H_{0.975}$ instead of $H_{0.85}$ [see Eq. (4)] we find^{*)}

$$H_{0.975} = 13.3 \pm .6 \quad (6)$$

implying an additional 32% decrease in the size of the correction.

- c) It is partly a question of definition how to compare the size of the correction with the leading order result. It seems to me that a reasonable definition is to consider the ratio

$$R(t) = \frac{\frac{\alpha_s}{\pi} A_0(t) + \left(\frac{\alpha_s}{\pi}\right)^2 A_1(t)}{\frac{\alpha_s^{(0)}}{\pi} A_0(t)} \quad (7)$$

where $\alpha_s^{(0)}$ is the "leading order" running coupling constant

$$\alpha_s^{(0)} = 1 / (3.833 \ln \frac{Q^2}{\Lambda^2}) \quad (8)$$

ERT used α_s/π in the denominator of (7) which gives $\approx 15\%$ increase in the size of the correction. Fortunately this ambiguity is small compared to the effects (a) and (b).

It is quite conceivable that the observations listed above are relevant for the experimental determination of α_s and a very reasonable estimate of the size of the correction is given by the ratio

$$\bar{R} = \frac{\int_{0.5}^{0.975} dt \left[\frac{\alpha_s}{\pi} A_0(t) + \left(\frac{\alpha_s}{\pi}\right)^2 A_1(t) \right]}{\int_{0.5}^{0.975} dt \frac{\alpha_s^{(0)}}{\pi} A_0(t)} \quad (9)$$

evaluated with $\Lambda_{\overline{MS}} = 100$ MeV. At $Q = 35$ GeV and $n_f = 5$ I obtained the value^{**)}

$$\bar{R} = 1.34 \pm 0.04 \quad (10)$$

^{*)} ER obtained $H_{0.975} = 12.8 \pm 0.3$ (private communication).

^{**)} With $T_C = 0.95$ I obtain $\bar{R}' = 1.41 \pm 0.04$. The error bar represents the MC integration errors.

To characterize the behaviour of the corrections in more detail, I have plotted the behaviour of $[R(t)-1]$ in Fig. 3^{*)}. We see a gradual increase in the size of the correction with decreasing t . The very large corrections at small values of t are the kinematical effect explained by the phase space distributions of Fig. 2.

We may try to estimate the experimental value of α_s observing that in a rough approximation we can say that when the experimentalists fit their data with three jet Monte Carlo programmes they use an effective $\bar{\alpha}_s$

$$\bar{\alpha}_s \approx \alpha_s \left(1 + 14 \frac{\alpha_s}{\pi} \right)$$

With $\Lambda_{\overline{MS}} = 100$ MeV we obtain

$$\bar{\alpha}_s = 0.19$$

which has to be compared with the experimental value obtained, e.g., by TASSO (with only three-jets in the Monte Carlo fit)¹⁵⁾

$$\bar{\alpha}_s = 0.19 \pm .03$$

Finally, I would like to make a comment on four jet production. One may try to define the experimental four-jet events at the parton level as events which have in any pair of the final partons an invariant mass larger than 5 GeV. Applying the formalism described in Ref. 7), with $T_c = 0.95$ and $\Lambda_{\overline{MS}} = 100$ MeV, I find that the fraction of these four-jet events is

$$\frac{\sigma_{4j}}{\sigma_0} = (5.5 \pm 1.1) \%$$

in agreement with the experimental data^{14),15)}. In Fig. 3 I plotted the ratio (dotted histogram) $(d\sigma/dt)^{4j} (m_j \geq 5 \text{ GeV}) / (d\sigma/dt)^{(4j+3j)}$ which illustrates the size and the relative importance of the four-jet contributions.

In conclusion, the QCD corrections to jet production in e^+e^- annihilation are sizeable but not overwhelmingly large. The formulae and calculations which give these corrections have been available for more than a year^{4),7)}. It would be important to take them into account in the experimental analysis of jet production in e^+e^- annihilation.

*) The MC fluctuations in a given bin are still large $\approx 15\%$. The dashed histogram is the corresponding result of R.K. Ellis and D.A. Ross, private communication.

ACKNOWLEDGMENTS

I am grateful to R.K. Ellis, K.J.F. Gaemers, J.A.M. Vermaseren and D.A. Ross for useful discussions and for supplying me with some of their unpublished results. I thank P. Langacker for reading the manuscript. I also thank the Theoretical Physics Division at CERN for the hospitality extended to me.

REFERENCES

- 1) J. Ellis, M.K. Gaillard and G.G. Ross, Nucl. Phys. B111 (1976) 253.
- 2) A. Ali, J.G. Körner, Z. Kunszt, E. Pietarinen, G. Kramer, G. Schierholz and J. Willrodt, Nucl. Phys. B167 (1980) 454.
- 3) K.J.F. Gaemers and J.A.M. Vermaseren, Zeit. f. Physik C7 (1980) 81;
R.K. Ellis, D.A. Ross and A.E. Terrano, Nucl. Phys. B178 (1981) 42.
- 4) R.K. Ellis, D.A. Ross and A.E. Terrano, Phys. Rev. Lett. 45 (1980) 1226; Nucl. Phys. B178 (1981) 421.
- 5) K. Fabricius, G. Kramer, G. Schierholz and I. Schmitt, DESY preprint 81/035 (1981).
- 6) R.K. Ellis, G. Martinelli and R. Petronzio, CERN preprint TH-3079 (1981) and private communication.
- 7) Z. Kunszt, Phys. Lett. 99B (1981) 429.
- 8) J.A.M. Vermaseren, K.J.F. Gaemers and S.J. Oldham, CERN preprint TH-3002 (1980) to be published in Nucl. Phys. B.
- 9) G. Parisi, Phys. Lett. 74B (1978) 65;
J.F. Donoghue, F.E. Low and S.Y. Pi, Phys. Rev. D20 (1979) 2759;
G.C. Fox and S. Wolfram, Nucl. Phys. B149 (1979) 413.
- 10) E. Farhi, Phys. Rev. Lett. 39 (1977) 1237.
- 11) L. Clavelli and D. Wyler, Bonn preprint BONN HE-81-3 (1981).
- 12) R.K. Ellis and D.A. Ross, CERN preprint TH-3131 (1981).
- 13) K. Fabricius, I. Schmitt, G. Schierholz and G. Kramer, Phys. Lett. 97B (1980) 431.
- 14) R. Brandelik et al., Phys. Lett. 94B (1980) 437;
W. Bartel et al., Phys. Lett. 91B (1980) 142 and DESY report 81/09 (1981);
C. Berger et al., Phys. Lett. 97B (1980) 459;
D.P. Barber et al., Phys. Rev. Lett. 43 (1979) 830 and MIT-LNS report no. 115 (1981).
- 15) S.L. Wu, XX International Conference on High Energy Physics, (1980) Wisconsin, AIP Conference Proceedings No. 68 p. 604 ed. H.G. Wolf (1981).
P. Söding and G. Wolf, DESY preprint 81/013 (1981);
P. Duinker, DESY preprint 81/012 (1981).

- 16) P.B. Mackenzie and G. Peter Lepage, Cornell University preprint CLNS/81-498 (1981).
- 17) M.S. Alam et al., (CLEO Collaboration), submitted to the 1981 International Symposium on Lepton and Photon Interactions at High Energies, Bonn.
See also J.J. Muller et al., Phys. Rev. Lett. 46 (1981) 1181.

FIGURE CAPTIONS

- Figure 1 : The functions $A_0(t)$ and $A_1(t)$ defined by Eq. (1).
- Figure 2 : Thrust distributions of the three-body and four-body phase space.
- Figure 3 : (a) The quantity $R(t) - 1$ [see Eq. (7)], solid histogram. The dashed histograms are the corresponding results of R.K. Ellis and D.A. Ross.
(b) The $[d\sigma^{(4j)}/dt/d\sigma^{(3j+4j)}/dt]$ ratio, dotted histogram.

FIG. 1

FIG. 2

FIG. 3