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ASPECTS OF A SUPERLIGHT GRAND UNIFIED AXION

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A B S T R A C T

We discuss the implications of a recently proposed minimally extended SU(5) model that incorporates the superlight axion mechanism to solve the strong CP problem. We prove that in the absence of instanton effects, the pseudoscalar nature of the axion coupling to fermionic matter is not modified by higher order weak radiative corrections including CP violation, thus leading to the induced scalar coupling of axion to matter $\sim f_\pi/M_U \cdot \theta$. We also note that the axion coupling to light Higgs bosons is suppressed in a similar manner by the heavy mass, M_U in the absence of instanton effects, thus making the GUT axion practically invisible. Finally, we note that in this model the baryon asymmetry of the Universe cannot be generated in lowest order - a problem which can be solved by the introduction of extra Higgs mesons.

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1. The Peccei-Quinn symmetry (PQ symmetry) is an appealing mechanism for solving the strong CP problem¹⁾. In its original version, however, where the symmetry is broken at the scale of the standard weak interactions, it predicts a light particle, the axion²⁾, which is not seen^{*)},³⁾. This conflict is apparently resolved by the recent suggestion of Dine, Fischler and Srednicki (DFS)⁴⁾: they propose that the PQ symmetry be broken at a much higher energy scale M , in which case the axion mass and its coupling to normal matter is inversely proportional to M , so becoming invisible. The same authors observe that this modified PQ mechanism is most likely to be implemented in a grand unified scheme. This seems in fact logically necessary since otherwise, in the pure $SU(3) \times SU(2) \times U(1)$ theory, one would simply trade the tuning of the θ parameter related to strong CP violation for the hierarchy puzzle $M \gg M_W$.

In this letter we study several implications of the DFS idea. We will illustrate our ideas using the minimally extended $SU(5)$ model proposed by Wise, Georgi and Glashow⁵⁾ that incorporates the DFS idea. In this case the superlight axion has a mass of order 10^{-8} eV. As a result, it is important to find out the strength and the nature of its coupling to light particles at low energies. It has been noted^{4),5)} that the axion coupling to light fermions at the tree level is pseudoscalar in nature and has a strength $\sim f_\pi/M_U$, where f_π is the pion decay constant and M_U is the grand unification mass ($\sim 10^{15}$ GeV). In the non-relativistic limit therefore, it gives rise to spin dependent couplings between matter with couplings, much smaller than the presently known bounds⁶⁾. However, it is conceivable that in the presence of weak CP and P violating couplings present in the Lagrangian, one could generate a scalar coupling of axions to fermions. Such couplings would give rise to a modification of the gravitational potential GMm/r ($ae^{-(r/\lambda_a)}$), where λ_a is the Compton wavelength of the axion (~ 20 meters in this case) and could be detectable if a is not too small. In the first part of the letter we study this problem and show that, in the absence of instanton effects, the Goldstone boson nature of the axion forbids the appearance of scalar fermion couplings to all orders in perturbation theory in weak interactions. This implies that a scalar fermion coupling (if any) will arise only due to the presence of instanton effects and is therefore of order $(f_\pi/M_U) \times \theta \lesssim 10^{-25}$, if $\theta \lesssim 10^{-9}$.

Next, we point out that, again in the absence of instanton effects, the Goldstone boson nature of the axion prevents it from appearing in the Higgs potential. However, in the rest of the Lagrangian it appears only with derivative coupling and its couplings are suppressed by the superheavy mass M_U , thus making it invisible even in Higgs boson interactions.

^{*)} Note however that H. Faissner et al. (Aachen preprint, 1981) have claimed to have observed effects which can be interpreted as an axion.

Finally, we note that the baryon asymmetry of the Universe^{*)} in this model cannot arise in the lowest order in Higgs couplings and may therefore not be large enough to account for observations.

2. In this section, we briefly review the extended SU(5) model of the super-light axion. In addition to the usual left-handed fermion multiplets $\psi_{\bar{5}}$ and ψ_{10} , two complex Higgs fields H_1 and H_2 transforming as a $\underline{5}$ and a complex field Σ transforming as $\underline{24}$ under SU(5) are included. The Yukawa Lagrangian is taken to be:

$$\mathcal{L}_Y = g_1 \psi_{\bar{5}}^T C^{-1} \psi_{10} H_1^\dagger + g_2 \psi_{10}^T C^{-1} \psi_{10} H_2 + h.c. \quad (1)$$

where C is the Dirac charge conjugation matrix and family indices are understood (g_1, g_2 are matrices in family space). $\mathcal{L}_Y$, together with the standard gauge interaction Lagrangian, possesses a $U(1)_A \times U(1)_B$ global symmetry defined as follows:

$$\begin{aligned} U_A(1): \quad \psi_{\bar{5}} &\rightarrow e^{-i\alpha} \psi_{\bar{5}}; \quad \psi_{10} \rightarrow e^{i\alpha} \psi_{10}; \quad H_2 \rightarrow e^{-2i\alpha} H_2 \\ U_B(1): \quad \psi_{\bar{5}} &\rightarrow e^{i\beta} \psi_{\bar{5}}; \quad H_1 \rightarrow e^{i\beta} H_1 \end{aligned} \quad (2)$$

We demand that the Higgs potential $V(H_1, H_2, \Sigma)$ be such that this symmetry is respected, provided a non-trivial transformation property is also given to the field Σ . A possible potential is [apart from terms that are trivially invariant under (2) regardless of how Σ transforms]:

$$V = a H_2^\dagger \Sigma^2 H_1 + b H_2^\dagger H_1 \text{Tr} \Sigma^2 + h.c. \quad (3)$$

demanding Σ to transform as follows:

^{*)} This point has been independently realized and discussed by Ellis et al.⁷⁾.

$$\begin{aligned} U_A(1) : \Sigma &\rightarrow e^{-i\alpha} \Sigma \\ U_B(1) : \Sigma &\rightarrow e^{-\frac{i\beta}{2}} \Sigma \end{aligned} \quad (4)$$

Note that a and b are in general complex parameters, as are the Yukawa coupling matrices g_i .

The fact that the $U(1)$ s are anomalous allows the strong CP parameter θ to be rotated away exactly at the tree level and with sufficient precision⁸⁾ in higher orders. Spontaneous symmetry breaking will give rise to vacuum expectation values of the colourless neutral components $\phi_{1,2}$ of $H_{1,2}$ and the $SU(3) \times SU(2) \times U(1)$ singlet ϕ_0 in Σ :

$$\langle \phi_{1,2} \rangle = v_{1,2} e^{i\gamma_{1,2}}; \quad \langle \phi_0 \rangle = v_0 e^{i\gamma_0} \quad (5)$$

with

$$v_0 \sim M_U; \quad v^2 = v_1^2 + v_2^2 \sim \frac{M_W^2}{g^2} \quad (6)$$

After spontaneous symmetry breaking, the non-anomalous combination $(X_A - 2X_B)$ of the generators of the global symmetry $U(1)_A, U(1)_B$ will give rise, together with the broken local generator $(Q - T_{3L})$, to the unbroken global generator

$$\frac{4}{5} (Q - T_{3L}) - \frac{1}{5} (X_A - 2X_B) \sim B - L \quad (7)$$

as in the minimal $SU(5)$ model. Any other combination of X_A and X_B will play the role of the PQ symmetry.

This combination can be fixed by requiring that the massless Higgs-Kibble ghost does not couple to the charge X associated with PQ symmetry⁴⁾:

$$X = \frac{1}{2} \frac{v_2^2}{v^2} X_A + \frac{v_1^2}{v^2} X_B \quad (8)$$

Its spontaneous breakdown would be associated with a massless Goldstone boson, the axion, if it were not for the instanton anomalies. Its actual mass is $m_A \simeq f_\pi (m_\pi/M_U)$. Defining the shifted fields

$$\begin{aligned}\varphi_{1,2} &= (v_{1,2} + \eta_{1,2}) e^{i(\gamma_{1,2} + \frac{\xi_{1,2}}{v_{1,2}})} \\ \varphi_0 &= (v_0 + \eta_0) e^{i(\gamma_0 + \frac{\xi_0}{v_0})}\end{aligned}\tag{9}$$

the axion field can be identified with:

$$A = N_A (X_0 v_0 \xi_0 + X_1 v_1 \xi_1 + X_2 v_2 \xi_2)\tag{10}$$

where X_i are the quantum numbers X of the various Higgs fields $\phi_{0,1,2}$ and N_A is the normalization factor. Using Eqs (2), (4) and (8) and the fact that $v_1, v_2 \ll v_0$ we obtain:

$$A \simeq -\xi_0 + \frac{2v_1 v_2}{v^2 v_0} (v_2 \xi_1 - v_1 \xi_2)\tag{11}$$

Its coupling to fermions (of mass m) goes through the components $\xi_{1,2}$ and is $\sim m/M$ except for the nucleons, in which case it is $\sim f_\pi/M_U$ since then it couples predominantly through π^0 and η components. Taking a value for $M_U \sim 10^{15}$ GeV, we get typically $m_A \sim 10^{-8}$ eV and a coupling $\sim 10^{-16}$. We will now show that all the axion coupling, i.e., to fermions as well as its coupling to light Higgs bosons, are suppressed by a heavy mass, thus making it practically invisible.

3. In view of its superlight mass, the nature under parity of the axion coupling to matter is of crucial importance. A priori there exist two possibilities: this coupling could either be pseudoscalar or scalar. If it is of the pseudoscalar type, in the non-relativistic limit it will lead to spin dependent forces with couplings $\sim 10^{-16}$, which is far below the present experimental limit ($< 10^{-3}$) on such strengths. On the other hand, if it is of the scalar type, it will introduce significant modifications to gravitational forces for distances $R \lesssim 1/m_a$ ($1/m_a \approx 20$ meters); experimental limits on such modifications to gravity do exist⁹⁾ at the level of 10%. It is therefore important to study any possible existence of induced scalar interactions of the axions.

In the following we will prove that in the absence of instanton effects, there exists no scalar coupling of the axion to fermions to all orders in weak interactions. The proof proceeds in two steps: first we show that, minimizing the Higgs potential at the tree level implies the vanishing of all phases in Eq. (9). To see this, we write down an effective Higgs potential in the presence of instanton contributions^{*)}, 10)

$$V(\varphi_1, \varphi_2, \varphi_0) = V_0 + V_{PQ}(\varphi_1, \varphi_2, \varphi_0) + \tilde{A}^2 (\text{Arg det } m(\varphi_i))^2 \quad (12)$$

where V_0 refers to the part of the potential that does not contain any phase of the vacuum expectation value; V_{PQ} denotes the part that is invariant under PQ symmetry, but involves the phases in the vacuum expectation value [the part contained in Eq. (3)]; the last term parametrizes the instanton contribution and, if non-vanishing at the minimum, will generate a θ . The minimization of this potential gives rise to the following relations between the phases in Eq. (9):

$$\gamma_2 - \gamma_1 + 2\gamma_0 = 0 \quad (13a)$$

$$\text{Arg det } m(v_1 e^{i\gamma_1}, v_2 e^{i\gamma_2}, v_0 e^{i\gamma_0}) = 0 \quad (13b)$$

Equation (13a) comes from the invariance of V_{PQ} whereas Eq. (13b) implies $\theta_{\text{eff}} = 0$ to this approximation. In lowest order, $\text{arg det } m = \gamma_2 - \gamma_1$ so that

^{*)} We have used a suitable generalization of the approach of Ref. 10) to include the effects of weak interactions.

we get $\gamma_1 = \gamma_2 = \delta = 0$. This concludes the first part of the proof, which says that at the tree level the axion coupling to fermions is purely pseudoscalar as can be seen by looking at Eq. (1).

Let us now proceed to show that in the absence of instanton contributions [i.e., $A = 0$ in Eq. (12)], this result is not modified by the inclusion of all higher order weak corrections bearing in mind that such corrections prior to symmetry breaking respect PQ symmetry. To show this, let us concentrate only on the part of the Lagrangian involving ϕ_1, ϕ_2, ϕ_0 and the up and down quarks. The most general coupling of fermions to ϕ s to all orders in weak interactions can be written as: (we work only with one generation)

$$\mathcal{L}_Y = \bar{u}_R u_L G_2(\varphi_1, \varphi_2, \varphi_0) + \bar{d}_R d_L G_1(\varphi_1, \varphi_2, \varphi_0) + h.c. \quad (14)$$

where

$$G_2 = g_2 \varphi_2 + \tilde{G}_2(\varphi_1, \varphi_2, \varphi_0)$$

and

$$G_1 = g_1 \varphi_1^* + \tilde{G}_1(\varphi_1, \varphi_2, \varphi_0) \quad (15)$$

where in Eq. (15) the first term in $\tilde{G}_1$ represents the tree level contribution and $\tilde{G}_1$ are the contributions from the higher orders. Note that, in general, $\tilde{G}_1$ will involve complex Higgs couplings in Eq. (3) and could, therefore, a priori lead to effective induced scalar coupling of axions to fermions^{*)}. It is clear that $\mathcal{L}_Y$ in Eq. (14) respects the PQ symmetry and as we now show, this prevents the appearance of an induced scalar coupling. In this discussion, we will ignore the induced effective Higgs couplings which are momentum dependent, since for diagonal scalar coupling such terms yield zero anyway. For this purpose, let us rewrite $\mathcal{L}_Y$ as follows:

^{*)} Note that Eq. (3) leads to scalar meson couplings of the type: $V \sim (a + 15/2b) \phi_0^2 T_1^+ T_1 + (9/4a + 15/2b) \phi_0^2 D_1^+ D_1 + h.c.$ where we have written $H_1 = \begin{pmatrix} T_1 \\ D_1 \end{pmatrix}$, T_1 = colour triplet and $D_1 = \text{SU}(2)_L$ doublet. By redefining the phase of ϕ_0 , $9/4a + 15/2b$ can be made real; but the coupling $c = a + 15/2b$ still remains complex. This is the reason to suspect that induced scalar couplings might arise once c enters the mass matrix.

$$\begin{aligned} \mathcal{L}_Y = & g_2 \bar{u}_R u_L \varphi_2 F_2(\varphi_1, \varphi_2, \varphi_0) \\ & + g_1 \bar{d}_R d_L \varphi_1^* F_1(\varphi_1, \varphi_2, \varphi_0) + h.c. \end{aligned} \quad (16)$$

By writing this way, we have factored out $F_i(\phi_i)$ which is invariant under $U(1)$ PQ symmetry. This means, that, in general F_i will involve terms of type $\Pi_{i=0,1,2} \phi_i^{a_i}$ for various a_i , such that

$$\sum_{i=0,1,2} a_i X_i = 0 \quad (17)$$

However, from Eq. (9), it follows that, each such term in F_i will lead to the following linear combinations of ξ_i s to appear:

$$\prod_i \phi_i^{a_i} = \left(\prod_i (v_i + \eta_i) e^{i a_i \chi_i} \right) e^{i \sum_i (a_i \xi_i / v_i)} \quad (18)$$

It is now clear from Eqs (10) and (17) that, all linear combinations of ξ_i - fields present in F_i are orthogonal to the axion. Thus, F_i does not involve the axion field at all (as long as we ignore the derivative couplings). Thus, the effect of higher weak interactions is simply to modify the over-all $\bar{\psi}_L \psi_R \phi_i$ coupling. Thus, the same chiral rotation that diagonalizes the fermion mass matrices will also diagonalize the axion coupling, which will then remain pseudoscalar. This proves our result. It is then obvious that, once there appears a non-zero θ_{eff} in the theory, the $\arg \det m$ term will be non-zero leading to an explicit breaking of PQ symmetry. Thus barring accidental cancellations, we will expect the induced scalar coupling of the superlight axion to be bounded above by $f_\pi \theta_{\text{eff}} / M_U$. If $\theta_{\text{eff}} \lesssim 10^{-9}$, the strength of this coupling $\lesssim 10^{-25}$.

4. From the discussion of the previous section, we see that the axion coupling to light fermions is suppressed by a heavy mass M_U . In this section, we prove that, its coupling to light Higgs and light bosons are also similarly suppressed. The $SU(3)_C \times SU(2)_L \times U(1)$ singlet nature of the field ϕ_0 and Eq. (11) make it clear that, coupling to $SU(2) \times U(1)$ gauge bosons are suppressed by $1/M_U$. Let us therefore concentrate on the coupling to light Higgs bosons. It is clear that we have to analyze the Higgs potential $V(\phi_i)$. But again as in the previous

section [cf. the discussion of $F_i(\phi_i)$], the $U(1)_{PQ}$ invariance of $V(\phi_i)$ leads to the result that the Goldstone boson field will not appear in $V(\phi_i)$. Thus, no unsuppressed coupling of axion to Higgs bosons will appear. Again, here, the inclusion of instanton effects will break the $U(1)_{PQ}$ symmetry by interactions whose strengths $\sim \theta_{eff}$. Thus, any possible coupling of axion to light Higgs will be of order $\theta_{eff} (\lesssim 10^{-9})$.

5. Before concluding, we wish to point out that the minimal way of implementing the PQ mechanism in $SU(5)$ runs into a problem with cosmological baryon generation [see also Ref. 7)].

To study this question we make the standard assumption that the vector boson decays do not contribute to the baryon generation; rather it is the colour triplet Higgs bosons coupling to fermions that are responsible for baryon production. The essence of our observation is that, for the minimal $SU(5)$ model under consideration, the PQ symmetry restricts the Higgs fermion coupling in such a way that in lowest orders of perturbation theory the $\Delta B \neq 0$ decay widths of Higgs bosons, Γ , satisfy the relation $\Gamma = \bar{\Gamma}$ for each channel, inspite of the presence of CP violation.

Let us consider for example the T_2 boson decay. The only lowest order diagram that could give rise to baryon asymmetry is shown in the figure; all other a priori possible graphs are prohibited by the $U(1)_{PQ}$ symmetry. This diagram is, however, CP conserving and therefore yields $n_B = n_{\bar{B}}$. Thus, any non-zero n_B/n_γ must come only in higher orders and will presumably be small. It therefore appears that the model has to be extended by including at least one more 5 dimensional Higgs multiplet transforming as H_1 or H_2 under the PQ symmetry, in order to understand the presently observed magnitude of the baryon to photon ratio.

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FIGURE CAPTION

The lowest non-trivial diagram that could have contributed to $n_B - n_{\bar{B}}$. Z stands for the complex mixing between the Higgs triplets T_1 and T_2 .

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