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ON THE POSSIBLE EXISTENCE OF HIGGS FIELDS
WITH WEAK ISOSPIN LARGER THAN $\frac{1}{2}$

J. Iliopoulos
Laboratoire de Physique Théorique *)
Ecole Normale Supérieure
75231 Paris Cedex 05

D.V. Nanopoulos
CERN - Geneva

and

T. N. Tomaras +)
California Institute of Technology
Pasadena - CA 91125

A B S T R A C T

Higgs fields with high values of weak isospin are thought to be unacceptable because they may spoil the successful predictions of the standard model. In this paper we show that this is not necessarily true. We study a specific example of a theory containing an $I = \frac{1}{2}$ as well as an $I = 1$ scalar field and we find that there exist finite regions in the coupling constant space for which the symmetry breaking is still dominated by the vacuum expectation value of the isodoublet Higgs field.

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1. - INTRODUCTION

One of the great successes of the standard $SU(2) \times U(1)$ model of electromagnetic and weak interactions ^{1),2)} is the special form of the neutral current which implies that a single parameter, the angle θ_w , describes both the overall strength of the neutral current and the amount of parity violation ²⁾. This is sometimes expressed as a relation among the vector boson masses

$$M_W^2 = M_Z^2 \cos^2 \theta_w \quad (1)$$

Equation (1) is a direct consequence of the fact that the symmetry breaking in the standard model is obtained through the non-vanishing vacuum expectation value of a Higgs field with $I = \frac{1}{2}$ ^{2),3)}. Therefore, in all attempts to construct grand unified theories one tries to make sure that, after the first breaking at $M \sim 10^{14}$ GeV, all scalar fields which remain light have weak isospin equal to $\frac{1}{2}$. We want to argue in this paper that this may be an unnecessary restriction.

Our approach is based on the idea that the low energy properties of a quantum field theory are governed by the infra-red attractive fixed points of the renormalization group equations and these properties may be quite different from those of the high energy theory ⁴⁾. This idea has been used in an ambitious attempt to derive the observed symmetries of low-energy physics, such as Lorentz invariance ⁵⁾ as well as invariance under internal symmetries, global or local ⁶⁾, starting from more or less chaotic theories at extremely high energies. A more pragmatic programme is to try to obtain relations among the measured parameters of the theory, such as masses and mixing angles, from the infra-red attractive fixed points of the underlying theory ⁷⁾.

In this paper we address the following question : let us suppose that, at energies $M \sim 10^{12}-10^{13}$ GeV, one is left with a certain number of massless scalar fields belonging to different representations of the weak $SU(2)$. We shall assume that the Higgs fields are originally massless and spontaneous symmetry breaking occurs through the Coleman and Weinberg mechanism ⁸⁾. When the effective potential develops a minimum away from the origin in classical field space (for a realistic theory this must happen at $M \sim 10^2$ GeV), the $SU(2) \times U(1)$ symmetry will be spontaneously broken. For one scalar field ϕ , this happens when λ , the ϕ^4 coupling constant, becomes of order g^4 , where g is the gauge coupling constant ⁸⁾. If we have several scalar fields ϕ_i we can have several quartic coupling constants but the previous result remains essentially true ⁹⁾ : spontaneous symmetry breaking occurs when a certain linear combination

of these coupling constants vanishes to order g^2 . The direction of the breaking depends on this linear combination. We shall show that, for the $SU(2) \times U(1)$ theory, there exists a finite region in the coupling constant space, such that the direction of the symmetry breaking is determined by the non-vanishing vacuum expectation value of an isodoublet Higgs scalar, although the theory contains scalar fields with higher values of weak isospin.

The paper is organized as follows : in Section 2 we present the model which contains the usual complex $I = \frac{1}{2}$ scalar field $\phi(x)$ and in addition, a real one $\Phi(x)$, with $I = 1$. We compute the one-loop β functions and the effective potential. In Section 3 we integrate numerically the renormalization group equations and we determine the direction of the spontaneous symmetry breaking. We show that this direction may still be dominated by the vacuum expectation value of $\phi(x)$. This result is natural, in the technical sense of the word, i.e., it holds true for a finite range in the initial values of the coupling constants.

2. - THE MODEL

Our model is the standard $SU(2) \times U(1)$ theory of weak and electromagnetic interactions with an additional $I = 1$ real scalar field. The normalizations and conventions are given in the following Lagrangian density :

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4} T_2 (A_{\mu\nu} A^{\mu\nu}) - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \sum_{i=1}^{n/2} (\bar{\psi}_{iL} i \not{\partial} \psi_{iL} + \bar{\psi}_{iR} i \not{\partial} \psi_{iR}) \\ & + (D_\mu \phi)^\dagger (D^\mu \phi) + \frac{1}{2} T_2 (D_\mu \Phi) (D^\mu \Phi) \\ & - \frac{1}{2} (\phi^\dagger \phi)^2 - \frac{\lambda}{4} [T_2 (\Phi^2)]^2 - \frac{1}{2} \phi^\dagger \Phi^2 \phi \end{aligned} \quad (2)$$

where

$$\begin{aligned} A_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - \frac{i}{\sqrt{2}} g [A_\mu, A_\nu] ; \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \\ D_\mu \phi &= \partial_\mu \phi - \frac{i}{\sqrt{2}} g A_\mu \phi - \frac{i}{2} g' B_\mu \phi \\ D_\mu \Phi &= \partial_\mu \Phi + \frac{i}{\sqrt{2}} g [\Phi, A_\mu] ; \quad A_\mu = \frac{1}{\sqrt{2}} \tau^i A_\mu^i ; \quad \Phi = \frac{1}{\sqrt{2}} \tau^i \Phi^i \\ D_\mu \psi_{iL} &= \partial_\mu \psi_{iL} - \frac{i}{\sqrt{2}} g A_\mu \psi_{iL} - i g' B_\mu Y_{iL} \psi_{iL} \end{aligned}$$

$$D_H \Psi_{iR} = \partial_H \Psi_{iR} - i g' B_H Y_{iR} \Psi_{iR}$$

The $U(1)$ generator is defined by $Y = T_3 - Q$ and $f/2 = 3$ is the number of fermion families.

The effective potential of the scalar fields in the tree approximation is given by :

$$V_{tree} = \frac{\lambda}{4} \phi^4 + \frac{h}{4} \Phi^4 + \frac{f}{2} \phi^2 \Phi^2 \quad (3)$$

where $\phi^2 = \phi^\dagger \phi$ and $\Phi^2 = \Phi^\dagger \Phi$. For $\lambda, h, f > 0$ it is positive definite and vanishes only for $\phi = \Phi = 0$. When one-loop corrections are included we add terms proportional to g^4, g'^4 and $g^2 g'^2$ [see Eq. (12) below]. In this case the minimum of the potential will correspond to vacuum expectation values

$$\langle \phi \rangle = v, \quad \langle \Phi \rangle = V, \quad V^2 = \rho v^2 \quad (4)$$

The vector boson masses are now equal to :

$$M_W^2 = \frac{1}{2} g^2 (2V^2 + v^2), \quad M_Z^2 = \frac{1}{2} (g^2 + g'^2) v^2 \quad (5a)$$

which modify (1) to :

$$M_W^2 = M_Z^2 \cos^2 \theta_W (2\rho + 1) \quad (5b)$$

where θ_W is still given by $\cos^2 \theta_W = g^2 / (g^2 + g'^2)$. The success of Eq. (1) implies that, at low energies, ρ must be much smaller than one, in other words, the minimum of the potential must be such that $v^2 \gg V^2$.

In the general spirit of our approach we assume that, at $M \sim 10^{13}$ GeV, all coupling constants are small but of the same order of magnitude : $\lambda \sim h \sim f \sim g^2 \sim g'^2$. From this point we must follow the evolution of these coupling constants as the energy scale goes down. Therefore, the next step is to compute the corresponding one-loop β functions. We find :

$$16\pi^2 \beta_\lambda = 6\lambda^2 + 6f^2 - 9\lambda g^2 - 3\lambda g'^2 + \frac{9}{2} g^4 + \frac{3}{2} g'^4 \quad (6)$$

$$16\pi^2 \beta_h = 22h^2 + 24g^2 - 24hg^2 + 12g^4 \quad (7)$$

$$16\pi^2 \beta_f = 4f^2 + 10hf + 3\lambda f - \frac{33}{2}fg^2 + 6g^4 - \frac{3}{2}fg'^2 \quad (8)$$

$$16\pi^2 \beta_g = -\frac{17}{6}g^3 \quad (9)$$

$$16\pi^2 \beta_{g'} = \frac{123}{18}g'^3 \quad (10)$$

In order to obtain $v \gg V$ we must have $\lambda \ll g^2$ ($\lambda \sim g^4$) while h remains of order g^2 and $f \geq 0$. Furthermore this must yield a physically acceptable theory, which means that the following conditions must be fulfilled : we start at $M \sim 10^{13}$ GeV with $V(\phi, \Phi) > 0$ and $\langle \phi \rangle = \langle \Phi \rangle = 0$. We follow the evolution of $\lambda(\mu)$, $h(\mu)$ and $f(\mu)$. At a certain scale μ_0 spontaneous symmetry breaking occurs. We want μ_0 to be of order 10^2 GeV, in other words, no breaking should occur before this scale. At this point we have $V(\phi, \Phi; \mu_0) = 0$ along a whole line in classical field space. The direction of this line determines the ratio ρ which must be much smaller than one. Finally, no coupling constant should be allowed to become large, of order unity, in the entire range between 10^{13} GeV and 10^2 GeV. In the next section we show, by integrating numerically Eqs. (6) to (10), that such an acceptable solution can indeed be found for a finite range in the initial values of the coupling constants.

Before closing this section, let us rephrase this argument in a different, but equivalent, way : let us compute the one-loop corrections to the effective potential. Using the renormalization conditions

$$\left. \frac{\partial^2 V}{\partial \phi^2} \right|_{\phi=\Phi=0} = 0 = \left. \frac{\partial^2 V}{\partial \Phi^2} \right|_{\phi=\Phi=0} \quad (11a)$$

$$\left. \frac{\partial^4 V}{\partial \Phi^4} \right|_{\phi=\Phi=\mu} = 6h \quad \text{e.t.c.} \quad (11b)$$

we obtain :

$$\begin{aligned}
 V = & \frac{1}{4} \phi^4 + \frac{h}{4} \Phi^4 + \frac{f}{2} \phi^2 \Phi^2 \\
 & - \frac{1}{4!} \left[\frac{145}{72\eta^2} g^4 + \frac{25}{16\eta^2} (g^2 + g'^2)^2 \right] \phi^4 \\
 & - \frac{1}{4!} \frac{193}{18\eta^2} g^4 \Phi^4 - \frac{1}{2} \frac{47}{72\eta^2} g^4 \phi^2 \Phi^2 \\
 & + \frac{g^4}{8\eta^2} \left[\Phi^2 + \frac{1}{2} \phi^2 \right]^2 \ln \frac{\Phi^2 + \frac{1}{2} \phi^2}{\frac{3}{2} \mu^2} + \frac{1}{64\eta^2} (g^2 + g'^2)^2 \phi^4 \ln \frac{\phi^2}{\mu^2} \\
 & + O(\lambda^2, h^2, f^2, \dots)
 \end{aligned} \tag{12}$$

The conditions for a minimum are :

$$\frac{\partial V}{\partial \phi} = 0 = \frac{\partial V}{\partial \Phi} \tag{13}$$

If we choose $v^2 = \mu^2$ and $V^2 = \rho v^2$, we can easily see that the conditions (13) can be satisfied with $\rho \ll 1$ only if $\lambda \ll g^2, h$.

3. - THE SYMMETRY BREAKING PATTERNS

The renormalization group equations (6) to (10) have been integrated numerically for different sets of initial values of the coupling constants. Taking initial values at random we end up, in most cases, with unacceptable results. Either the symmetry breaking happens at unwanted scales (too large or too small with respect to 10^2 GeV), or in the few cases when it does occur at $\mu_0 \sim 10^2$ GeV, this happens in the direction of the higher isospin scalar multiplet ($I=1$ in our case), thus yielding $\rho > 1$. This was to be expected ¹⁰⁾ because the higher isospin fields tend to have larger β functions for their quartic couplings, therefore these coupling constants tend to decrease faster with decreasing energy scale and become first of order g^4 . However, this argument is heuristic for a system of coupled differential equations and a more refined analysis is needed. The numerical method we used is the following.

We first start from a point corresponding to $\lambda \approx 0$, $f \geq 0$, $h \sim g^2 \sim g'^2$. We take it to be the final point at $M^2 \sim 10^2$ GeV. We then integrate the equations upwards up to an energy scale $M \sim 10^{13}$ GeV. We check that all coupling constants

remain small in the entire range of scales. If the low-energy point is appropriately chosen (we found several such choices), we end up at $M \sim 10^{13}$ GeV at a point where $\lambda = \lambda_0$, $f = f_0$, $h = h_0$, $g^2 = g_0^2$ and $g'^2 = g_0'^2$ with $\lambda_0 \sim f_0 \sim h_0 \sim g_0^2 \sim g_0'^2$. Obviously this point could have been chosen as an initial high energy point and it would have yielded an acceptable low-energy theory with $\rho \ll 1$. We now start moving in the vicinity of this point by changing $\lambda \rightarrow \lambda_0 + \delta\lambda$ etc. Each time we integrate the equations down to $M \sim 10^2$ GeV and check whether the resulting theory is still acceptable in the sense explained in the previous section. After several steps the final point ceases to be acceptable, i.e., either ρ becomes larger than a given ϵ (we put $\epsilon = 0.02$ in the programme), or one of the other conditions is violated. This way we obtain the boundaries of a finite region in the space of high energy scale coupling constants inside which all points give acceptable low-energy theories. We have found many such regions and we have, by no means, conducted an exhaustive study. Some examples are given below.

The initial values of g and g' at 10^{13} GeV are taken to be $g \sim 0.58$ and $g' \sim 0.41$. They correspond to the grand unification values with our conventions and they do not change significantly since we do not allow ρ to exceed the value of 0.02. The other coupling constants λ , h and f may vary inside limited regions. For example, such an allowed region is bounded by the values : $0.27 < \lambda < 0.28$; $0.4 < h < 0.425$ and $0.32 < f < 0.33$. The actual boundary lies inside this three-dimensional rectangle but its precise shape is not important and can only be given numerically. As a second example we give the rectangle $0.23 < \lambda < 0.24$; $0.26 < h < 0.29$; $0.205 < f < 0.22$. The size of these regions should not come as a surprise. To begin with, the requirement that the symmetry breaking occurs at a scale $\sim 10^2$ GeV severely restricts the admissible values of the parameters in the effective potential. In our search we have allowed this scale to vary by at most one order of magnitude. A similar condition on the standard model, with only one isodoublet Higgs scalar, yields an allowed region for λ with a width of the same order as the ones found here. The over-all size of our regions reflects this freedom in the scale of symmetry breaking as well as the condition $\rho < 0.02$.

We conclude that a gauge theory may contain higher isospin scalar fields and still exhibit the successful relation (1) which is characteristic of the symmetry breaking through an isodoublet Higgs field.

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