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QCD RADIATIVE CORRECTIONS TO HYPERFINE SPLITTING IN QUARKONIUM

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ABSTRACT

We calculate the leading radiative corrections in QCD to $\Delta E(1^{--}-0^{--})$ in quarkonium. We also present a summary of present knowledge of quarkonium rates together with numerical estimates for charmonium and bottomium.

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A lowest order calculation of the ratio

$$r = \frac{\Delta E(^3S_1 - ^1S_0)}{\Gamma(^3S_1 \rightarrow e^+e^-)} \quad (1)$$

between the quarkonium¹⁾ hyperfine splitting and its leptonic width is easily done in quarkonium models and QCD. One gluon exchange gives in fact

$$r = \frac{1}{e_Q^2} \frac{2}{9} \frac{M^2(Q\bar{Q})}{m_Q^2} \frac{\alpha_s}{\alpha^2} \frac{1}{1 - \frac{16}{3\pi} \alpha_s} \quad (2)$$

where a partial correction to the denominator Γ is given²⁾.

In this note we shall improve on the result (2) by consistently including QCD leading radiative corrections to the hyperfine splitting. We will then present an over-all summary of the present knowledge of quarkonium rates with numerical estimates for charmonium and bottomium. We shall take advantage for that purpose of the recent calculation of Mackenzie and Lepage³⁾ of the corrections to the 1^{--} gluonic width.

The general framework for the calculation is provided by the Bethe-Salpeter description of the bound state. It is natural to split the various corrections to ΔE into pieces proportional to C_F and C_A , in the usual notation. The C_F part is identical to the analogous correction in QED for positronium. The C_A part requires an explicit calculation. The diagrams which contribute to such a term in the kernel are those of Figs. 1a - 1d. In addition, one has the contribution of Fig. 1e.

Contrary to the QED-like C_F part, the kernel of the C_A part develops an infra-red behaviour in the mass-shell limit and for zero transferred momentum. Such a feature is of course a common one in bound state problems: it appears at higher orders in QED and has been found in non-leading orders in QCD. As an estimate of the effect, a cut-off of the size of a typical confinement radius $R = (400 \text{ MeV})^{-1}$ can be introduced. We report here on the various contributions coming from the individual singular diagrams within such a logarithmic approximation (the C_F part is finite and as such is not considered):

$$\frac{\alpha_s}{\pi} \frac{5}{12} C_A \ln \frac{q^2}{m_Q^2} \quad (3a)$$

$$\frac{\alpha_s}{\pi} \frac{1}{2} C_A \ln \frac{q^2}{m_Q^2} \quad (3b)$$

$$\frac{\alpha_s}{\pi} C_A \left(-\frac{1}{8} \ln \frac{q^2}{m_Q^2} - \frac{1}{2} \ln \frac{m_Q^2}{\lambda^2} \right) \quad (3c)$$

$$\frac{\alpha_s}{\pi} \frac{1}{2} C_A \ln \frac{m_Q^2}{\lambda^2} \quad (3d)$$

$$\frac{\alpha_s}{\pi} \frac{1}{6} \ln \frac{q^2}{m_Q^2} \quad (3e)$$

(for each light flavour)

where q^2 is the invariant momentum transfer and λ is an infra-red regularizing photon mass which disappears in the sum.

We note that the coefficient of the over-all infra-red term is expected to be renormalization scheme dependent. In place of Eq. (2) we are thus led to

$$r = \frac{1}{e^2} \frac{2}{9} \frac{M^2(q\bar{q})}{m_Q^2} \frac{\alpha_s}{\alpha^2} \frac{1 + \frac{\alpha_s}{\pi} \left(\frac{1}{4} - \frac{f-1}{3} \right) \ln(Rm_Q)}{1 - \frac{16}{3} \frac{\alpha_s}{\pi}} \quad (4)$$

where $(f-1)$ is the total number of light quarks contributing to the fermion loop ($f-1=3$ for the charm, etc.).

We now proceed to the summary of the present knowledge of radiatively corrected rates for quarkonium systems. Our sources are Refs. 2) - 4) and the result in Eq. (4). Whenever it is needed to be specified, the $\overline{MS}$ subtraction scheme is used. The relevant mass scale for all quarkonium systems is taken to be the corresponding constituent quark mass, so that, for instance, for charmonium α_s means $\alpha_s(m_c)$. In Table 1 we give the theoretical expressions for the various radiative corrections.

For comparing with data and for predictions we adopt the following procedure: from the ratio of the gluonic $[\Gamma(1^{--} \rightarrow 3g) = 27 \pm 7 \text{ keV}]$ to leptonic rate $[\Gamma(1^{--} \rightarrow \mu^+ \mu^-) = 1.16 \pm 0.15 \text{ keV}]$ for Y we infer

$$\alpha_s(m_b) = 0.156^{+0.012}_{-0.016}$$

and from the renormalization group we calculate

$$\alpha_s(m_c) = 0.210 \pm 0.028 \quad .$$

With the above values we obtain the figures in Table 2. For the corresponding gluonic ($44 \pm 6 \text{ keV}$) to leptonic ($4.8 \pm 0.6 \text{ keV}$) rate for ψ the agreement is good. The ratio of the hyperfine splitting over the leptonic width for the ψ remains on the contrary bigger than expected on a theoretical basis. In general, for the bottomium system the size of the corrections is acceptable, justifying the use of perturbation theory, whereas the same cannot be said for charmonium. An experimental effort to reduce the errors for existing measurements and to provide for the lacking data would be extremely welcome.

After the completion of our hyperfine calculation we received a preprint⁵⁾ by Buchmüller, Ng and Tye who also calculate the hyperfine splitting corrections. Our result for the leading logarithm agrees with their expression, which disagrees with previous results in the literature. In Ref. 5) the corresponding finite parts are also evaluated, which however does not significantly change the numerical results.

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Table 1
First order radiative corrections *)

	charmonium	bottomium
$\frac{\Gamma(1^{--} \rightarrow \mu^+ \mu^-)}{[\Gamma(1^{--} \rightarrow \mu^+ \mu^-)]^0}$	$1 - 5.33 \frac{\alpha_s}{\pi}$	$1 - 5.33 \frac{\alpha_s}{\pi}$
$\frac{\Gamma(1^{--} \rightarrow 3g)}{[\Gamma(1^{--} \rightarrow 3g)]^0}$	$1 - (3.8 \pm 0.5) \frac{\alpha_s}{\pi}$	$1 - (4.2 \pm 0.5) \frac{\alpha_s}{\pi}$
$\Delta E / \Delta E^{(0)}$	$1 - 0.75 \frac{\alpha_s}{\pi} \ln(Rm_c)$	$1 - 1.08 \frac{\alpha_s}{\pi} \ln(Rm_c)$
$\frac{\Gamma(0^{-+} \rightarrow 2g)}{[\Gamma(0^{-+} \rightarrow 2g)]^0}$	$1 - 5.84 \frac{\alpha_s}{\pi}$	$1 + 5.41 \frac{\alpha_s}{\pi}$
$\frac{\Gamma(0^{-+} \rightarrow 2\gamma)}{[\Gamma(0^{-+} \rightarrow 2\gamma)]^0}$	$1 - 3.38 \frac{\alpha_s}{\pi}$	$1 - 3.38 \frac{\alpha_s}{\pi}$
$B(0^{-+}) / B(0^{++})$	$1 + 0.9 \frac{\alpha_s}{\pi}$	$1 + 2.1 \frac{\alpha_s}{\pi}$
$B(2^{++}) / B(0^{++})$	$1 + 6.5 \frac{\alpha_s}{\pi}$	$1 + 4.0 \frac{\alpha_s}{\pi}$
$\frac{\Gamma(0^{++} \rightarrow 2g)}{\gamma(2^{++} \rightarrow 2g)}$	$\frac{15}{4} (1 + 12 \frac{\alpha_s}{\pi})$	$\frac{15}{4} (1 + 9.5 \frac{\alpha_s}{\pi})$
$\frac{\Gamma(1^{++} \rightarrow gq\bar{q} + 3g)}{\Gamma(1^{++} \rightarrow 2g)}$	$\frac{\alpha_s}{\pi} [-\frac{4}{9} \ln(Rm_c) - 0.23]$	$\frac{\alpha_s}{\pi} [-\frac{16}{27} \ln(Rm_c) - 0.30]$

*) $B(J^{PC}) = \Gamma(J^{PC} \rightarrow \gamma\gamma) / \Gamma(J^{PC} \rightarrow \text{"hadrons"})$

Table 2
Numerical estimates

	charmonium			bottomium		
	prediction	rad corr	exp	prediction	rad corr	input
$\frac{\Gamma(1^{--} \rightarrow 3g)}{\Gamma(1^{--} \rightarrow \mu^+ \mu^-)}$	$15.5^{+7.7}_{-5.7}$	16%	9.2 ± 2.4	23.3 ± 6.7	8%	input
$\Delta E / \Gamma(1^{--} \rightarrow \mu^+ \mu^-)$	$(11.4^{+2.4}_{-2.1}) \times 10^3$	45%	$(24.8 \pm 5) \times 10^3$	$(27.6^{+2.6}_{-3.3}) \times 10^3$	18%	
$\frac{\Gamma(0^{--} \rightarrow 2g)}{\Gamma(1^{--} \rightarrow 3g)}$	173^{+11}_{-7}	86%	300 ± 150	200^{+12}_{-8}	60%	
$\frac{\Gamma(0^{--} \rightarrow \gamma\gamma)}{\Gamma(1^{--} \rightarrow \mu^+ \mu^-)}$	$1.60^{+0.07}_{-0.05}$	20%		0.377 ± 0.066	13%	
$B(0^{--}) / B(0^{++})$	1.06 ± 0.01	6%		1.10 ± 0.01	10%	
$B(2^{++}) / B(0^{++})$	1.43 ± 0.06	43%		1.20 ± 0.02	20%	
$\frac{\Gamma(0^{++} \rightarrow 2g)}{\Gamma(2^{++} \rightarrow 2g)}$	6.8 ± 0.4	80%		5.5 ± 0.16	47%	
$\frac{\Gamma(1^{++} \rightarrow g\bar{q}q + 3g)}{\Gamma(0^{++} \rightarrow 2g)}$	$(2.48^{+0.34}_{-0.33})\%$	—		$(5.83^{+0.45}_{-0.60})\%$	—	

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FIGURE CAPTION : Graphs contributing to the C_A part of hyperfine structure.

(a)

(b)

(c)

(d)

(e)

FIG.1

(4)

(5)