Ref.TH.3163-CERN
LAPP-TH-47

ON PSEUDO-GOLDSTONE BOSON MASSES FROM
BROKEN GAUGE INTERACTIONS

P. Binétruy
CERN - Geneva
and
LAPP - Annecy-Le-Vieux

S. Chadha^{*)} and P. Sikivie^{**)}
CERN - Geneva

A B S T R A C T

We investigate the effects of extended technicolour (ETC) interactions on the masses of pseudo-Goldstone bosons in one technifamily models. We find that the P^0 and P^3 cannot acquire any masses from ETC, while the $P^\pm$ only acquire contributions of order $g_W g_{\text{ETC}} (\lesssim 5 \text{ GeV})$. This removes a major source of uncertainty in the masses of these particles.

---

^{*)} Address after Oct. 1, 1981: Rutherford Appleton Laboratory, Chilton, Didcot, England.

^{**)} Present address: Department of Physics, Univ. of Florida, Gainesville, Florida 32611, U.S.A.

1. An important problem in theories in which symmetries are broken dynamically, such as the recently popular technicolour theories¹⁾, is the so-called "subgroup alignment" or "vacuum orientation" problem²⁾. In such theories, which are devoid of both elementary scalar fields and fermion bare mass terms, the fermion interactions are assumed to be invariant under a gauge group G^{TC} as well as a global flavour symmetry group G^{TF} . At a scale $\Lambda^{TC} \sim 1 \text{ TeV}$ the global group G^{TF} is assumed to be broken spontaneously to a subgroup H^{TF} through the formation of fermion bilinear condensates. This breakdown gives rise to a manifold of degenerate vacua. Generally, however, there exist in the theory various perturbations which are not G^{TF} invariant; these will then lift the energy degeneracy among the vacua. Determining the true vacuum of the theory is what is referred to as the vacuum orientation problem. The correct choice of vacuum has evident physical implications. We outline below two criteria which are helpful in determining this choice.

The G^{TF} violating perturbations responsible for selecting the true vacuum may arise in several ways. One possibility is that a certain subgroup G_W of G^{TF} is weakly gauged so as to introduce the electroweak interactions. The G_W forces then explicitly break G^{TF} . In this case Peskin and Preskill³⁾ have shown that the vacuum condensate orients itself so that the overlap of the unbroken subgroup H^{TF} and G_W is maximized. More precisely, the two subgroups H^{TF} and G_W align themselves so as to minimize the trace of the $(\text{mass})^2$ matrix of the G_W vector bosons.

In this note we are more interested in the consequences of another possibility for the G^{TF} violating perturbation, viz., that G^{TC} is itself part of a larger gauge group G^{ETC} which breaks down to the former at a scale $\mu_{ETC} > \Lambda^{TC}$. This is the so-called extended technicolour (ETC) mechanism^{4),5)}. The perturbation is now provided by the effective four fermion interactions generated by the massive gauge bosons in G^{ETC}/G^{TC} . It should be noted that at some scale larger than Λ^{TC} , where the ETC gauge coupling constant g_{ETC} is small, these interactions may be reliably calculated using perturbation theory. They may then be renormalized down to low energy. In all the examples studied by us⁶⁾ we found that those technifermions condense which have the effect of giving the usual (i.e., technisinglet) fermions the most mass in the sense of maximizing the quantity $\text{tr}(m_f^\dagger m_f)$ where m_f is the fermion mass matrix. Moreover, these preferred condensates, which are technisinglets so as not to break G^{TC} , are in general linear combinations of elements from several different ETC multiplets. Further details concerning vacuum alignment by ETC interactions may be found in Ref. 6).

As emphasized above the physical consequences of a theory depend critically on which vacuum is dynamically preferred. In particular the technifermion condensates which form must reproduce the correct pattern of electroweak symmetry breaking. In the class of models to be discussed below we shall assume that this physical requirement is satisfied.

2. We now wish to consider the effects of the ETC interactions on the masses of the (pseudo-) Goldstone bosons (PGBs) accompanying the spontaneous breakdown $G^{TF} \rightarrow H^{TF}$. This will be done within the context of the one technifamily models⁷⁾. These models assume the presence of just one family of technifermions composed of four weak doublets $[(U_c, D_c), (N, E)]$, where the techniquarks U_c and D_c are colour triplets while the technileptons N and E are colour singlets. We assume that at the scale Λ^{TC} the following condensates develop:

$$\langle \bar{U}_c U_c \rangle = \langle \bar{D}_c D_c \rangle = \langle \bar{N} N \rangle = \langle \bar{E} E \rangle \simeq (\Lambda^{TC})^3. \quad (1)$$

Since there are eight species of technifermions the global chiral symmetry group of the TC sector is either $G^{TF} = SU_L(8) \times SU_R(8) \times U_V(1)$ or $G^{TF} = SU(16)$ depending on whether the technifermions and their antiparticles transform according to inequivalent or equivalent representations of G^{TC} respectively. In the first case the chiral symmetry is spontaneously broken to the diagonal subgroup $H^{TF} = SU_V(8) \times U_V(1)$, while in the second case $H^{TF} = O(16)$ ^{*)}. The Goldstone bosons accompanying this spontaneous breakdown are 63 in the first case and 135 in the second; evidently the 63 Goldstone bosons of the first case are included as a subset of the 135 Goldstone bosons of the second. Of particular interest is the subset of Goldstone bosons labelled $\{P^0, P^a, P_8^0, P_8^a\}$ where P^0 and P^a are colour singlets, P_8^0 and P_8^a are colour octets, and $a = 1, 2, 3$ is an isotriplet index. These correspond to the axial generators:

$$\begin{aligned} P^0: & \quad \gamma_5 \frac{1}{2\sqrt{2}} \quad 1_2 \otimes \Lambda_{15}, & P^a: & \quad \gamma_5 \frac{1}{2\sqrt{2}} \quad \tau_a \otimes \Lambda_{15}, \\ P_8^0: & \quad \gamma_5 \frac{1}{2\sqrt{2}} \quad 1_2 \otimes \Lambda_\alpha, & P_8^a: & \quad \gamma_5 \frac{1}{2\sqrt{2}} \quad \tau_a \otimes \Lambda_\alpha, \end{aligned} \quad (2)$$

($\alpha = 1, 2, 3$; $\alpha = 1, \dots, 8$)

where the Λ_r are the usual 4×4 Hermitean matrices which generate $SU(4)$ normalized according to $\text{tr}(\Lambda_r \Lambda_s) = 2\delta_{rs}$, τ_a are the Pauli matrices and 1_2 denotes the 2×2 unit matrix.

^{*)} The possibility $H^{TF} = Sp(16)$ in the second case is in conflict with phenomenology.

Now, the following facts are known concerning the masses of these particles from previous work. The colour octet particles P_8^0 and P_8^a acquire masses of order $g_s \Lambda^{TC}$ from gluon exchange (g_s is the colour coupling constant); these masses are estimated to be about 250 GeV^{7),3)}. The situation with the colour singlet particles is somewhat more interesting. The neutral singlets P^0 and P^3 do not acquire any mass at all from electronuclear interactions and hence remain strictly massless at this level. On the other hand, the charged singlet bosons $P^\pm$ only get higher order mass contributions of $O(g_w g'_w)$ where g_w and g'_w are the $SU_L(2)$ and hypercharge gauge coupling constants respectively. The masses of the $P^\pm$ from electroweak forces are estimated to be between 5 - 8 GeV in the case of complex TC and between 8 - 14 GeV for real TC^{3),9),10)}. Despite this information, however, the implications of these numbers for phenomenology are obscure since all these particles could in principle acquire substantial (~ 50 GeV) mass contributions from the ETC interactions¹¹⁾. We therefore consider this problem next.

Let us define ETC quite generally to denote any "horizontal" gauge interactions which unify each of the six $G_w = SU^C(3) \times SU_L(2) \times U_Y(1)$ multiplets (U_L, D_L) , U_R , D_R , (N_L, E_L) , N_R and E_R with an unspecified set of quarks and leptons. Such interactions are in contradistinction to "vertical" gauge interactions which unify G_w multiplets amongst themselves. Thus, for example, Pati-Salam unification of quarks and leptons, where lepton number is treated as the fourth colour, is an interaction of the vertical type¹²⁾. The six ETC multiplets obtained in this way can, a priori, all transform as different representations of G^{ETC} . Then the minimal global flavour symmetry group of the ETC interactions is given by

$$G^{ETC} = SU_L^{(Q)}(6) \times SU_L^{(L)}(2) \times SU_R^{(U)}(3) \times SU_R^{(D)}(3) \\ \times U_L^{(0,15)}(1) \times U_R^{(0,15)}(1) \times U_V^{(0,0)}(1) \\ \times U_R^{(3,0)}(1) \times U_R^{(3,15)}(1). \quad (3)$$

Here the superscripts (Q) and (L) refer to techniquarks and technileptons respectively, and $U_{L(R)}^{(a,r)}(1)$ is generated by

$$Q_{L(R)}^{(a,z)} = \frac{1}{2\sqrt{2}} \int (d\vec{x}) \psi^\dagger \tau_a \otimes \Lambda_z \frac{1 \pm \gamma_5}{2} \psi, \quad (4)$$

where ψ denotes the column of ETC multiplets, $\tau_0 = 1_2$ and $\Lambda_0 = (1/\sqrt{2})1_4$. Note that any generator of G^{ETC} may be written as the sum of a generator acting on the technifermions (F) and a generator acting on the ordinary fermions (f):

$$Q^{ETF} = Q^{ETF}(F) + Q^{ETF}(f). \quad (5)$$

However, in evaluating the PGB mass matrix from Dashen's formula

$$m_{xy}^2 = -\frac{1}{F_\pi^2} \langle 0 | [Q_A^{(x)}, [Q_A^{(y)}, \delta \mathcal{H}_{ETC}]] | 0 \rangle, \quad (6)$$

the second term in Eq. (4) may be omitted since $Q^{ETF}(f)|0\rangle = 0$. In Eq. (6) F_π is the PGB decay constant^{*}, $Q_A^{(x)}$ is the axial charge corresponding to the PGB denoted by x , and $\delta \mathcal{H}_{ETC}$ is the G^{TF} violating Hamiltonian density.

The process of technifermion condensate formation, Eq. (1), now breaks the group G^{ETF} down to its largest vector subgroup. This unbroken subgroup is

$$H^{ETF} = SU_V^{(U)}(3) \times SU_V^{(D)}(3) \times U_V^{(0,15)}(1) \\ \times U_V^{(0,0)}(1) \times U_V^{(3,0)}(1) \times U_V^{(3,15)}(1). \quad (7)$$

There are thus 39 generators of G^{ETF} which are broken by the technifermion condensates. This implies that 39 PGBs cannot acquire any masses from the ETC interactions alone. These are the 36 PGBs P^0, P^a, P_8^0, P_8^a , plus the three technipions which are removed from the spectrum by the dynamical Higgs mechanism. That this is so follows from Dashen's formula Eq. (6). Explicitly, the mass submatrix for these PGBs is given by ($\mu = 0,1,2,3; \alpha, \beta = 0,1,\dots,8,15$)

$$m_{(\mu,\alpha),(\nu,\beta)}^2 = -\frac{1}{F_\pi^2} \langle 0 | [Q_A^{(\mu,\alpha)}, [Q_A^{(\nu,\beta)}, \delta \mathcal{H}_{ETC}]] | 0 \rangle \\ = +\frac{1}{F_\pi^2} \langle 0 | [Q_A^{(\mu,\alpha)}, [Q_V^{(\nu,\beta)}, \delta \mathcal{H}_{ETC}]] | 0 \rangle \quad (8)$$

where we have used the fact that $Q_L^{(\nu,\beta)}$ generates a symmetry of $\delta \mathcal{H}_{ETC}$. Using further the Jacobi identity this becomes

---

^{*}) F_π is normalized such that the corresponding constant in QCD ≈ 95 MeV.

$$m_{(\mu,\alpha),(\nu,\beta)}^2 = \frac{1}{F_\pi^2} \left[\langle 0 | [[Q_A^{(\mu,\alpha)}, Q_V^{(\nu,\beta)}], \delta \mathcal{M}_{ETC}] | 0 \rangle - \langle 0 | [[Q_A^{(\mu,\alpha)}, \delta \mathcal{M}_{ETC}], Q_V^{(\nu,\beta)}] | 0 \rangle \right]. \quad (9)$$

The first term in this expression is simply a sum of terms like

$$\langle 0 | [Q_A^{(p,\gamma)}, \delta \mathcal{M}_{ETC}] | 0 \rangle = - \langle 0 | [Q_V^{(p,\gamma)}, \delta \mathcal{M}_{ETC}] | 0 \rangle. \quad (10)$$

The mass matrix therefore vanishes since the vector generators annihilate the vacuum. We have thus proved that within the context of the one technifamily models the 36 PGBs P^0 , P^a , P_8^0 and P_8^a cannot acquire any mass from the ETC interactions alone.

3. There still remains the important question of whether some of these PGBs can acquire higher order mass contributions from a combination of the ETC and electro-nuclear interactions. This question is perhaps uninteresting for the octet PGBs since they have already acquired large leading order masses from coloured gluon exchange. We shall therefore consider only the P^0 , P^3 and $P^\pm$ bosons in detail.

It is not possible for the neutral PGBs P^0 and P^3 to acquire any masses even when the electronuclear interactions are combined with ETC. They thus remain completely massless to all orders. This is so simply because the generators $Q_A^{(0,15)}$ and $Q_R^{(3,15)}$ not only belong to G^{ETF} but are also symmetries of the electronuclear interactions. In the now familiar fashion, these invariances prevent the P^0 and P^3 from acquiring any mass at all.

Similar arguments may be employed to eliminate those possible contributions to the $P^\pm$ masses which are proportional to $g_S g_{ETC}$ or $g'_W g_{ETC}$. For if such contributions existed they would be unaltered as $g_W \rightarrow 0$. But in this limit the generators $Q_L^{(1,15)}$ and $Q_L^{(2,15)}$ are invariances of the remaining G^{TF} violating Hamiltonian, and these symmetries would prevent the $P^\pm$ from acquiring any mass.

The same conclusion does not extend to the $O(g_W g_{ETC})$ contributions to the $P^\pm$ masses. The symmetries which prevent the $P^\pm$ from acquiring masses from the weak $SU_L(2)$ gauge bosons alone are the right-handed $Q_R^{(1,15)}$ and $Q_R^{(2,15)}$. But these right-handed generators are not, in general, symmetries of the ETC interactions. By contrast, the symmetries which forbid the ETC interactions alone from contributing to the $P^\pm$ masses are left-handed: $Q_L^{(1,15)}$ and $Q_L^{(2,15)}$. It follows that when the weak $SU_L(2)$ and ETC interactions are considered in

conjunction there are no symmetries which will keep the $P^\pm$ massless. Thus the $P^\pm$ are expected to acquire higher order mass contributions of $O(g_W g_{\text{ETC}})$. We estimate these to be $O((\sqrt{\alpha_W/\pi}) \times 50) \sim 5 \text{ GeV}$ in order of magnitude. These contributions are to be added in quadrature to the 5 - 14 GeV masses that the $P^\pm$ obtain from the electroweak contributions.

As a final remark, we mention that massless P^0 and P^3 are, of course, phenomenologically unacceptable. It has therefore been suggested^{13),9)} that the global axial symmetries corresponding to the P^0 and P^3 should be broken by "vertically" unifying (techni-) quarks and (techni-) leptons into a single multiplet of $SU^{PS}(4)$ in the manner of Pati and Salam¹²⁾. The mass thus imparted to the P^0 and P^3 is $\sim 2 \text{ GeV}$. Even after such unification, however, these symmetries are not completely broken and the vacuum remains discretely degenerate. Details are contained in Ref. 6).

4. To summarize, we have found that interactions involving ETC do not contribute any masses to the P^0 and P^3 , and give at most higher order contributions of $O(g_W g_{\text{ETC}})$ ($\lesssim 5 \text{ GeV}$) to the masses of the $P^\pm$. A major source of uncertainty in the masses of these particles is now removed. We may thus conclude that the 5 - 14 GeV electroweak contributions to the $P^\pm$ masses are not substantially altered by further corrections involving ETC. Reference 14) provides an extensive discussion of PGB phenomenology. The absence of these particles in the predicted mass range would seem to be fairly conclusive evidence against the class of models considered here. It is therefore discouraging that the JADE collaboration has recently searched for the $P^\pm$ through the decay modes $\tau\nu_\tau$ and $c\bar{s}(\bar{c}s)$ in the mass range 5 - 14 GeV, and have not reported any evidence for them¹⁵⁾. Further experimental searches are clearly warranted.

We would like to take this opportunity to thank John Ellis for innumerable helpful discussions over the course of the last year, and for his very special, but unflagging, brand of encouragement and humour. We also thank Serge Rudaz for several useful suggestions concerning the manuscript.

REFERENCES

- 1) S. Weinberg, Phys. Rev. D19 (1979) 1277;
L. Susskind, Phys. Rev. D20 (1979) 2619.
- 2) R. Dashen, Phys. Rev. 183 (1969) 1245; D3 (1971) 1879;
S. Weinberg, Phys. Rev. D13 (1976) 974.
- 3) M.E. Peskin, Nucl. Phys. B175 (1980) 97;
J.P. Preskill, Nucl. Phys. B177 (1981) 21.
- 4) S. Dimopoulos and L. Susskind, Nucl. Phys. B155 (1979) 237.
- 5) E. Eichten and K. Lane, Phys. Lett. 90B (1980) 125.
- 6) P. Binétruy, S. Chadha and P. Sikivie, CERN preprint TH.3122 (1981).
- 7) S. Dimopoulos, Nucl. Phys. B168 (1980) 69.
- 8) S. Coleman and E. Witten, Phys. Rev. Lett. 45 (1980) 100.
- 9) S. Dimopoulos, G. Kane and S. Raby, Nucl. Phys. B182 (1981) 77.
- 10) S. Chadha and M.E. Peskin, Nucl. Phys. B187 (1981) 541.
- 11) K. Lane and M.E. Peskin, Proceedings of the XV Rencontre de Moriond, ed.
J. Tran Thanh Van (Editions Frontières, France 1980);
P. Sikivie, Varenna Lectures (1980), CERN preprint TH.2951 (1980).
- 12) J.C. Pati and A. Salam, Phys. Rev. D8 (1973) 1240.
- 13) S. Dimopoulos, S. Raby and P. Sikivie, Nucl. Phys. B176 (1980) 449.
- 14) J. Ellis, M.K. Gaillard, D.V. Nanopoulos and P. Sikivie, Nucl. Phys. B182
(1981) 529;
J. Ellis, D.V. Nanopoulos and P. Sikivie, Phys. Lett. 101B (1981) 387;
P. Sikivie, Moriond Lecture (1981), CERN preprint TH.3083 (1981);
A. Ali, Implications of Dynamical Symmetry Breaking for High Energy Experi-
ments, DESY preprint 81-032 (1981).
- 15) JADE collaboration, talk presented by J. Burger at the Bonn Conference (1981).