Ref.TH.3173-CERN

m_b/m_t : A SENSITIVE FLAVOUR-METER

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A B S T R A C T

We recalculate the complete two-loop correction to the m_b/m_t ratio, allowing for the possibility of a large t-quark Yukawa coupling constant, comparable to g_{QCD} . We find that our previous stringent limit on the number of flavours becomes even more stringent; i.e. only six flavours are allowed. As a by-product we find that the mass of the t-quark has to be smaller than 155 GeV.

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1. One of the most striking successes of grand unified theories (GUTs) is the correlation between quark and lepton masses. A well-known example is the ratio m_b/m_τ , which is equal to one^{*)} at superhigh energies and gets its experimental value (~ 2.7), at present energies, through renormalization effects which are different for quarks and leptons^{1,2)}. There is a very good reason for most of the discussion of the quark-lepton mass relations to centre around m_b/m_τ : the values of m_b and m_τ are large enough so that the results obtained from calculating renormalization corrections to the tree level Yukawa couplings are affected very little by non-perturbative effects, while at the same time the fact that these Yukawa couplings are comfortably smaller than the gauge coupling (strong or electroweak) simplifies considerably the study of the appropriate renormalization group equations and avoids some of the usual pitfalls of perturbation theory.

A few years ago we calculated²⁾ the two-loop corrections to the matrix elements of the fermion mass operator in a general gauge theory and applied them to the m_b/m_τ ratio in GUTs. To our satisfaction we found that a previous limit¹⁾ on the allowed number of flavours (six or at most eight) coming from the agreement between theory and experiment on the m_b/m_τ ratio, became even more stringent: *no more than six flavours are allowed* when these two-loop corrections are taken into account²⁾.

In our previous calculation we tacitly assumed that the t-quark Yukawa coupling was much smaller than the gauge couplings. This assumption has recently been criticized^{3,4)}, especially by Hill⁴⁾, who raised the point that the inclusion of a large t-quark Yukawa coupling -- comparable, say, to the strong gauge coupling -- may undo our stringent limit on the number of flavours and thus render possible the existence of more than six flavours.

In this letter we add to our previous two-loop calculation of the ratio m_b/m_τ , by allowing for the possibility of a large t-quark Yukawa coupling. We find that our previous limit becomes even more stringent; that is, that indeed only six

*) J. Ellis suggests that anyone who does not accept this assumption has no need to read any more of this paper.

flavours are allowed and, moreover, the mass of the t -quark must be smaller than 155 GeV, if the excellent agreement between theory and experiment for m_b/m_t is to be retained.

Before discussing the calculation, we make a few comments on its possible impact.

2. As is well known, GUTs in their present form do not shed much light on the problem of flavour origin. Therefore any *independent* information on the number of flavours is most welcome. The only other piece of information on the number of flavours, that we are aware of (apart from m_b/m_t), is of cosmological origin. Surprisingly enough, in the standard Big-Bang model, three or at most four "normal" two-component neutrinos are allowed⁵⁾ if the successful evaluation (25% by mass) of the ^4He abundance in the universe is to remain valid. We find this result encouraging and it supplements our assertion about the maximum number of allowed flavours. Indeed, taken at face value, the ratio m_b/m_t may put severe constraints on model building. For example, if the desert between 100 GeV and 10^{15} GeV becomes polluted, then there is a tendency for m_b/m_t to increase beyond tolerable bounds. More specifically, the recently proposed supersymmetric grand unified models⁶⁾, which attempt to resolve the notorious gauge hierarchy problem (at least in their simplest form), increase⁷⁾ m_b/m_t by between 50% and 100% relative to their "standard" value, which is certainly unacceptable.

From a more practical point of view, the existence of a fourth family may have the unfortunate experimental consequence of drastically increasing⁸⁾ the proton lifetime^{*}). It should be mentioned that a fourth family with suitable masses could save¹⁰⁾ the "naïve" GUTs¹¹⁾ in their effort to explain the baryon-antibaryon asymmetry of the universe.

We hope that we have provided enough motivations to justify this calculation of the full two-loop corrections to the ratio m_b/m_t .

3. At the tree diagram level we find the contribution to the anomalous dimension of the bottom quark mass from the exchange of scalar particles which couple the bottom quark to the top quark to be

*) For a different point of view, see Ref. 9.

$$\gamma_0^H = -\frac{3}{2} \frac{g_t^2}{16\pi^2} \Theta(q^2 - (m_t + m_W)^2) \quad (1)$$

This is in agreement with previous calculations^{3,4}). The Θ function indicates that if the effective bottom-quark mass is measured at q^2 below $(m_t + m_W)^2$ (m_t is the top-quark mass, m_W is the mass of the W-boson which is equal to the mass of the charged scalar particles in the Feynman gauge), this contribution is suppressed by virtue of the Appelquist-Carazzone theorem¹²).

At the two-loop level there are two types of diagrams:

- i) those in which one scalar particle and one gluon are exchanged;
- ii) those in which two scalar particles are exchanged.

In both of these there are contributions which are suppressed for $q^2 < (m_t + m_W)^2$ and those which are not. In order to separate out these contributions, it was necessary to separate out diagrams which involved overlapping ultraviolet divergences into pieces for which the divergences are nested. The contribution from each piece is suppressed by a two-loop extension of the Appelquist-Carazzone theorem if a top-quark and scalar particle appear in the outer loop, irrespective of whether or not they also appear in the inner loop. An example of a diagram which is suppressed for $q^2 < (m_t + m_W)^2$ is shown in Fig. 1a, whereas the contribution from Fig. 1b continues all the way down to m_b . The diagram shown in Fig. 1c has to be separated into several parts, some of which are suppressed and some of which are not. The result of our calculation is a contribution

$$\gamma_{(1)}^{HG} = \frac{g_t^2}{16\pi^2} \frac{\alpha_s}{4\pi} C_F \left\{ \frac{21}{4} - \frac{141}{4} [\Theta(q^2 - (m_t + m_W)^2)] \right\} \quad (2)$$

with $C_F = 4/3$, for the contribution from diagrams involving the exchange of one scalar particle and one gluon, and

$$\begin{aligned} \gamma_{(1)}^{HH} = - \left(\frac{g_t^2}{16\pi^2} \right)^2 & \left\{ \frac{1}{2} \Theta(q^2 - (m_b + m_W)^2) + \frac{1}{2} \Theta(q^2 - (m_b + m_H)^2) - \right. \\ & \left. - \frac{61}{4} \Theta(q^2 - (m_t + m_W)^2) \right\} \quad (3) \end{aligned}$$

where m_H is the mass of the physical Higgs particle. Note that the first two terms are not suppressed by the top-quark, but are suppressed by the scalar particles in the diagram. Unlike the calculation of Ref. 2, these anomalous dimensions contain all the subleading contributions, i.e. the coefficient function has been chosen to be equal to 1.

In order to work consistently to fourth order in the coupling constants, it is necessary to take into account the running of the Yukawa coupling constant g_t , instead of keeping it constant at its threshold value $g_t(Q^2 = m_t^2)$ as was done in Ref. 4 (that being consistent with using the one-loop anomalous dimensions)*). The set of coupled renormalization group equations governing the q^2 dependence of g_t are given in Ref. 4. (It should be noted that the effect of g_t on the running of the strong coupling constant vanishes to this order as the result of a Ward identity.) These have been integrated numerically to find m_b as a function of m_t up to two loops. Using $\Lambda_{\overline{MS}} = 0.1$ GeV, the results are shown in Fig. 2. They are in very good agreement with the graph of the one-loop contributions presented by Mahaček and Vaughn³⁾. The effect of the two-loop corrections in this case is therefore very small. This is the result of a partial cancellation between the contributions which are suppressed for $q^2 < (m_t + m_W)^2$ and those which are not (these two contributions have the opposite sign).

We note that these effects tend to increase the magnitude of m_b , thus strengthening our previous limit of six flavours. This calculation also sets an upper limit on the value of m_t . If m_b increases by 10% from its value for small m_t , and $\Lambda_{\overline{MS}}$ is 0.1 GeV, it would give us the maximum acceptable value consistent with the observed value of the T mass. Thus unless $\Lambda_{\overline{MS}}$ is even lower than 0.1 GeV, the top-quark mass should not exceed 155 GeV **). We therefore have a

*) This differs from the case of a pure QCD calculation where α_s has to be calculated up to two loops in order to calculate all subleading logarithms, because g_t is flat for $q^2 < m_t^2$ owing to the Appelquist-Carazzone theorem, which is not the case for α_s .

**) Binétruy and Schücker¹³⁾ have shown that by using a definition of m_b , which takes into account threshold effects, they get a value for m_b which is 12% higher than ours, so that using their definition the upper limit on m_t would be even lower.

more stringent constraint than Hill⁴⁾, who observed that if m_t exceeds 230 GeV, perturbation breaks down because the Yukawa coupling constant g_t becomes unacceptably large below the unification mass.

It has also been suggested⁴⁾ that in a grand unified model in which there was a broken "family" group, i.e. there were Higgs particles which mediated transitions between fermions of different families, a fourth family could be tolerated if the effect of these Higgs particles were to suppress the value of m_b . Any quantitative calculation of the effects of such a scheme is of course highly model-dependent. However, we have looked at one simple case in which we assume that there is one uncharged Higgs particle that mediates transitions between the third and fourth families, with Yukawa coupling g_H . We also assume that the fermions in the fourth family are all very heavy with a mass around M .

If the family group commutes with the subgroup of all interactions within a family, then the effect of the Higgs on m_b will be exactly the same as its effect on m_τ , leaving the ratio m_b/m_τ unchanged. This is true at the one-loop level. However, in practice m_b and m_τ will receive different corrections, since the Yukawa coupling to the b-quark has QCD corrections, whereas the Yukawa coupling to the τ -lepton does not. As explained above, if we are taking these effects into account we should also look at the two-loop diagrams involving the exchange of a Higgs particle and a gluon. As before, we find that some of the contributions are suppressed if $q^2 < M^2$ and some are not. We obtain a correction to the ratio m_b/m_τ ,

$$\Delta\left(\frac{m_b}{m_\tau}\right) = \exp \left\{ -\frac{M}{m_b} \left[\int_{M^2}^{4M_x^2} \frac{dq^2}{q^2} \frac{g_H^2}{16\pi^2} \left(\left(\frac{\ln \frac{q^2}{\Lambda^2}}{\ln \frac{M^2}{\Lambda^2}} \right)^{-\frac{24}{19}} - 1 + \frac{91}{19 \ln \left(\frac{q^2}{\Lambda^2} \right)} \right) + \int_{4m_b^2}^{M^2} \frac{dq^2}{q^2} \frac{g_H^2}{16\pi^2} \frac{1}{7 \ln \left(\frac{q^2}{\Lambda^2} \right)} \right] \right\} \quad (4)$$

where g_H is the value of the Yukawa coupling constant at the threshold of the fourth family $q^2 \approx M^2$. This has an effect of further increasing the ratio m_b/m_τ

so that this particular model is excluded. In other models it may be possible that the sign of the exponent is different; however, to obtain an exact cancellation between this effect and the 20% increase in m_b due to the presence of an extra family of fermions would require very special values for g_H and M .

In conclusion it seems that the m_b/m_τ ratio is a very sensitive flavour-metre indeed, and that six flavours is still the favourite number.

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Figure captions

- Fig. 1 : a) Example of a diagram which is Appelquist-Carazzone suppressed.
b) Example of a diagram which is not Appelquist-Carazzone suppressed.
c) Example of a diagram with overlapping divergences, part of which
is Appelquist-Carazzone suppressed.
- Fig. 2 : Plot of m_b against m_t with $\Lambda_{\overline{MS}}$ taken to be 0.1 GeV (for five
flavours).

FIG. 1

FIG. 2