Ref.TH.3180-CERN

NUMERICAL CHECKS OF THE LATTICE DEFINITION INDEPENDENCE
OF TOPOLOGICAL CHARGE FLUCTUATIONS

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ABSTRACT

The stability of our previous results on the topological charge fluctuation in $SU(2)$ is checked against some variations of its lattice definition. The method consists of subtracting a partly computed perturbative tail to Monte Carlo data, whose high statistics are achieved by use of the icosahedral subgroup of $SU(2)$.

In a previous paper¹⁾ (referred to hereafter as I) we attempted a lattice calculation of the topological charge fluctuation:

$$A \equiv \int d^4x \langle 0 | Q(x) Q(0) | 0 \rangle \quad (1)$$

in the quarkless SU(2) gauge theory, where

$$Q(x) = g^2 / 64\pi^2 \epsilon_{\mu\nu\rho\sigma} F_{\mu\nu}^a(x) F_{\rho\sigma}^a(x)$$

is the so-called topological charge density.

A non-zero value of A would provide a solution of the $U_A(1)$ problem of QCD²⁾. Actually, for the agreement with the phenomenology of the pseudo-scalar nonet, one needs for colour SU(3):

$$A \underset{\text{phen.}}{\simeq} (180 \text{ MeV})^4 \quad (2)$$

In I, with a simple definition of $Q(x)$, and using Monte Carlo techniques for the full SU(2) group, we obtained:

$$A^{1/4} \simeq (0.11 \pm 0.02) K^{1/2} \quad (3)$$

where K is the (quarkless) string tension.

In this paper we want to show that different lattice definitions of $Q(x)$ and A lead to results in agreement with Eq. (3) (within errors), as required by the existence of a continuum limit of lattice QCD.

The definition of $Q(x)$ on the lattice used in I can be written as

$$Q_n^{(1)} = - \sum_{(\mu, \nu, \rho, \sigma) = \pm 1}^{\pm 4} \frac{\tilde{\epsilon}_{\mu\nu\rho\sigma}}{2^4 32 \pi^4} \cdot \text{Tr} (U_{n,\mu} U_{n+\mu,\nu} \cdot \quad (4)$$

$$\cdot U_{n+\mu+\nu,\rho} U_{n+\mu+\nu+\rho,\sigma} U_{n+\nu+\rho+\sigma,\mu}^{\dagger} U_{n+\rho+\sigma,\nu}^{\dagger} U_{n+\sigma,\rho}^{\dagger} U_{n,\sigma}^{\dagger})$$

with $1 = \tilde{\epsilon}_{1234} = -\tilde{\epsilon}_{2134} = -\epsilon_{1234}$ and $U_{n,\mu}$ the usual link variable. The link structure of Eq. (4) is shown in Fig. 1a.

In I we also mentioned another definition of $Q(x)$ that has the same naive continuum limit as $Q^{(1)}$. It reads^{*)} (Fig. 1b):

*) This is the definition called $Q^{(s)}$ in I.

$$Q_n^{(2)} = - \sum_{(\mu\nu\rho\sigma)=\pm 1}^{\pm 4} \frac{\tilde{\epsilon}_{\mu\nu\rho\sigma}}{2^4 32 \pi^4} \text{Tr} (U_{n,\mu\nu} U_{n,\rho\sigma}) \quad (5)$$

where $U_{n,\mu\nu}$ is the usual plaquette variable constructed out of four links:

$$U_{n,\mu\nu} = U_{n,\mu} U_{n+\mu,\nu} U_{n+\nu,\mu}^+ U_{n,\nu}^+ \quad (6)$$

In terms of the quantities (5) and (6), we can construct the following three lattice definitions of A :

$$A_{11} = \frac{1}{a^4} \sum_n \langle 0 | Q_n^{(1)} Q_0^{(1)} | 0 \rangle \quad (7a)$$

$$A_{22} = \frac{1}{a^4} \sum_n \langle 0 | Q_n^{(2)} Q_0^{(2)} | 0 \rangle \quad (7b)$$

$$A_{12} = \frac{1}{a^4} \sum_n \langle 0 | Q_n^{(1)} Q_0^{(2)} | 0 \rangle \quad (7c)$$

The results of I summarized by Eq. (3) refer to A_{11} .

Here we shall present the results for A_{11} , A_{22} and A_{12} as obtained by Monte Carlo simulations on a 4^4 lattice with periodic boundary conditions. Our computer programme is based on that of Ref. 3), which uses the 120 element (icosahedron) subgroup of $SU(2)$. For each value of $\beta = 4/g^2$ we made 20,000 to 40,000 sweeps. We used hot and cold starts and checked that equilibrium was reached after a small number of sweeps. Consequently we have always ignored the first 500 sweeps.

The data for $\pi^+ 2^{18} a^4 A$ are presented in Figs. 2a, b and c for the three quantities defined respectively in Eqs. 7a, b and c, with their statistical errors as a function of β . Note the large differences obtained in the data with the three definitions, which forced us to use different scales.

The data of Fig. 2a agree well with those obtained in I, showing that the restriction to the icosahedral subgroup of $SU(2)$ is an excellent approximation in the region of β considered (we stay faraway from the phase transition point of this finite group, $\beta_c \sim 6$).

The strong coupling expansion for A_{11} has already been given in I [Eqs. (4.4) and (4.11)] and we report it here as a function of β for completeness:

$$\pi^4 2^{18} a^4 A_{11} = 6144 \left(1 - \beta^2/4 + \frac{23}{3 \cdot 2^7} \beta^4 - \frac{139}{9 \cdot 2^{10}} \beta^6 + O(\beta^8) \right) \quad (8)$$

For A_{22} and A_{12} one finds respectively

$$\pi^4 2^{18} a^4 A_{22} = 2304 (1 + O(\beta^4)) \quad (9)$$

$$\pi^4 2^{18} a^4 A_{12} = 72 \beta^4 + O(\beta^6) \quad (10)$$

This behaviour is in agreement with the shape of our Monte Carlo data for small β (see Figs. 2a, b and c).

In order to extract the physical value of A we proceed as in I by first subtracting from each set of data the appropriate perturbative tail which we write as:

$$\pi^4 2^{18} a^4 A^{\text{Pert.}} = C_3/\beta^3 + C_4/\beta^4 \quad (11)$$

The coefficients C_3 have been computed, giving for a 4^4 lattice $C_3 = 14321.2$, 6414 and 4690 for A_{11} , A_{22} and A_{12} , respectively. The computation of C_4 in (11) is very complicated and we have not attempted it. As in I, we have determined it by a fit of the data in the weak coupling region $2.8 < \beta < 4.5$. We obtain $C_4 = 4987, 4897, -884$ for A_{11} , A_{22} and A_{12} , respectively. The perturbative tails, as defined in Eq. (11), are also shown in Figs. 2a, b and c on top of the data.

In Fig. 3 we plot together, on a logarithmic scale, the differences between our Monte Carlo data for $\pi^4 2^{18} a^4 A$ and the perturbative tail [Eq. (11)] for the three definitions of A . The three sets of data agree well with the (two-loop) renormalization group behaviour shown by the solid lines^{*)}. Comparing these results with those on the string tension $K^{(4)}$, we get

$$A^{1/4} K^{-1/2} = \begin{cases} 0.13 \pm 0.02 & \text{for } A_{11} \\ 0.14 \pm 0.02 & \text{for } A_{22} \\ 0.11 \pm 0.02 & \text{for } A_{12} \end{cases} \quad (12)$$

in reasonable agreement with each other and with the results for A_{11} for the full $SU(2)$ group.

^{*)} The results are practically unchanged if the one-loop renormalization group behaviour is used instead.

If we use the string model value for the string tension $K^{1/2} = 1/\sqrt{2\pi\alpha'} = 420 \text{ MeV}$ and an average value from Eq. (12), we get

$$A^{1/4} = (55 \pm 10) \text{ MeV} \quad (13)$$

Possible reasons for the smallness of this number as compared with the phenomenological value (2) [absence of quark loops, going from $SU(2)$ to $SU(3)$, current algebra extrapolation,...] have already been discussed in I.

Although the results we have presented here are quite encouraging, two weak points should be mentioned.

The first one is the subtraction of the perturbative tail which is a delicate one due to higher order uncertainties. Recently, however, Lüscher⁵⁾ constructed a topological charge density for the $SU(2)$ gauge theory on a lattice which is free of perturbative corrections. It would be very interesting to use his definition in a Monte Carlo computation.

The other problem has to do with the small size of our lattice. We are exploring the possibility of enlarging the lattice from 4^4 to 8^4 points.

ACKNOWLEDGMENTS

We thank C. Lang and C. Rebbi for kindly providing us with their Monte Carlo programme for the icosahedral subgroup of $SU(2)$. This allowed us to drastically improve our statistics.

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FIGURE CAPTIONS

- Fig. 1 : a) Link structure of the definition of Q_n given in Eq. (4);
b) same for definition given in Eq. (5).
- Fig. 2 : a) Monte Carlo data, strong coupling expansion and perturbative tail for A_{11} ;
b) same for A_{22} ;
c) same for A_{12} .
- Fig. 3 : Subtracted data in the region $2.1 \leq \beta \leq 2.4$ for the three definitions A_{11} , A_{22} and A_{12} .

(a)

(b)

FIG. 1

FIG. 2a

FIG. 2b

FIG. 2c

FIG. 3