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TESTS OF NEUTRAL GAUGE BOSON SELF-COUPPLINGS
WITH $e^+e^- \rightarrow \gamma Z$

F.M. Renard ^{*)}
CERN - Geneva

A B S T R A C T

We show how the reaction $e^+e^- \rightarrow \gamma Z$ provides a test of the $ZZ\gamma$ and $\gamma_V Z\gamma$ couplings. We calculate the third order QFD contribution (triangular diagrams). We show how anomalous terms (for example if the Z is a bound state) would reflect in this reaction and more explicitly in the photon spectrum of $e^+e^- \rightarrow \gamma + \text{lepton pairs}$.

^{*)} On leave of absence from U.S.T.L., Montpellier, France.

1. INTRODUCTION

One of the tasks of the future e^+e^- colliding beams will be to test the structure of electroweak interactions¹⁾. In the gauge theory framework these tests will concern the couplings of gauge bosons among themselves and of gauge bosons with fermions. Couplings of gauge bosons among themselves depend only upon the structure of the gauge group (and not upon the model assignment for leptons and quarks). Possible experimental tests of couplings involving charged gauge bosons (γW^+W^- , ZW^+W^- , $\gamma\gamma W^+W^-$, ...) have already been studied²⁾⁻⁴⁾; they require very high energies in order to produce at least a W^+W^- pair [i.e., $\sqrt{s} \gtrsim 200$ GeV, the highest energy for the LEP project¹⁾].

However, at lower energies a first interesting test concerns the couplings among (real or virtual) neutral gauge bosons (ZZZ , $ZZ\gamma$, $Z\gamma\gamma$, ...). Such couplings are a priori allowed by general invariance principles of electroweak interactions. For example, C invariance of electromagnetic interactions allows two independent couplings (electric dipole and magnetic quadrupole) for a real photon with two first class ($CP = +1$) spin one electroweak currents ($ZZ\gamma$ or $\gamma_V Z\gamma$). In standard QFD these couplings do not exist at lowest order (e). High order diagrams (triangular diagrams with internal fermion loops) give very small contributions which will be estimated below. But if QFD is only an effective low energy theory (for example, if fermions and gauge bosons are bound states of more fundamental constituents) such couplings may exist; their magnitude would be related to a characteristic scale Λ (for example, the size Λ^{-1} of the bound states).

The first simple reaction where this test can be carried out is $e^+e^- \rightarrow \gamma Z$, where γ is a detected hard photon. It requires moderate high energies ($\sqrt{s} \gtrsim M_Z$) (the same kind of tests could be done with $e^+e^- \rightarrow ZZ$ at higher energies). This reaction receives a standard contribution from electron exchange diagrams (Fig. 1); $ZZ\gamma$ and $Z\gamma\gamma$ couplings will allow an additional contribution to $e^+e^- \rightarrow \gamma Z$ (Fig. 2). Obviously radiative corrections (where additional undetected soft photons are emitted) will affect these basic terms; but as we give analytic expressions for the basic amplitudes these radiative corrections can be computed by standard methods for the specific detector to be used.

In Section 2 of this paper we define the general invariant forms for $ZZ\gamma$ and $Z\gamma\gamma$ couplings (with real and virtual photons) and relate them to multipole transitions. We calculate the contributions coming from triangular diagrams with internal lepton and quark loops (Fig. 3).

In Section 3 we calculate the additional contribution $e^+e^- \rightarrow (\gamma_V, Z) \rightarrow \gamma Z$ due to such couplings, we give the general form of the differential cross-section with arbitrary $e^\pm$ polarizations and we compare this contribution to the standard one (electron exchanges). We also discuss the observability of the effects.

In Section 4 we study the measurability of this reaction through the process $e^+e^- \rightarrow \gamma Z \rightarrow L^+L^-$, ($L^\pm = \mu^\pm, \dots$). There is a background contribution to $e^+e^- \rightarrow \gamma L^+L^-$ where the final L^+L^- pair does not form a Z which comes from $e^+e^- \rightarrow L^+L^-$ with a photon radiated from the $L^\pm$ lines. In order to take into account all interference effects we calculate the helicity amplitudes of the various processes. We compute the complete photon spectrum and we show how the Z peak appears in this spectrum and how it is sensitive to $ZZ\gamma$ and $Z\gamma\gamma$ couplings. The third order QFD contribution is surely unobservable. We give illustrations of observable effects due to hypothetical electric dipole ($d \approx 1/\Lambda$) and magnetic quadrupole ($K \approx 1/\Lambda^2$) transitions with $\Lambda = 50$ GeV.

A conclusion (Section 5) summarizes the results.

2. $ZZ\gamma$ AND $\gamma_V Z\gamma$ COUPLINGS

2.a General forms

We consider only the couplings of the spin one part of the weak neutral current (Z) or of the e.m. current (γ_V) with real Z and γ . The spin zero part of the current does not contribute in e^+e^- annihilation.

There are four independent coupling forms which are Lorentz invariant and gauge invariant with respect to the real photon:

$$\begin{aligned} I_1 &= i e \cdot k \varepsilon^{\mu\nu\rho\sigma} q_\mu k_\nu \varepsilon'_\rho e'_\sigma & I_2 &= i e' \cdot q \varepsilon^{\mu\nu\rho\sigma} q_\mu k_\nu \varepsilon_\rho e_\sigma \\ I_3 &= \varepsilon \cdot e e' \cdot k - \varepsilon \cdot e' e \cdot k & I_4 &= e \cdot k (\varepsilon \cdot e' k \cdot p - \varepsilon \cdot p e' \cdot k) \end{aligned}$$

$(e, q), (e', p)$ and (ε, k) are respectively the polarization and momentum four-vectors of the intermediate Z or γ_V , of the real Z and of the real γ ; $s \equiv q^2$.

These forms are combinations of electric dipole, magnetic quadrupole, magnetic dipole and electric quadrupole transitions.

If electroweak currents are first class only (i.e., $CP = +1$, 1^{--} and 1^{++} currents), charge conjugation invariance for the couplings of the real photon forbids $(1^{--}1^{--}\gamma)$ and $(1^{++}1^{++}\gamma)$ terms which correspond to magnetic dipole and electric quadrupole transitions described by I_3 and I_4 . We are left

with $(1^{--}1^{++}\gamma)$ or $(1^{++}1^{--}\gamma)$ terms which correspond to electric dipole and magnetic quadrupole transitions described by I_1 and I_2 . Notice that at the limit of two real photons $(\gamma Z \gamma)$ Bose statistics also forbid I_1 and I_2 (Yang's theorem).

Let us write the general form of $1^{--} \rightarrow 1^{++}\gamma$ (or $1^{++} \rightarrow 1^{--}\gamma$) transitions in the e^+e^- c.m. frame:

$$R_{fi} = f_1 I_1 + f_2 I_2 = -i f_1 \sqrt{s} \vec{e} \cdot \vec{k} \epsilon_{ijk} k^i \epsilon^j e^k + i f_2 \frac{s}{p^0} \vec{e} \cdot \vec{k} \epsilon_{ijk} k^i \epsilon^j e^k,$$

where f_i are (eventually s dependent) coupling constants. The decay probability can be written in the canonical form:

$$\Gamma = \frac{k}{4\pi s} \left[\frac{1}{3} |E_1|^2 + \frac{1}{5} |M_2|^2 \right] = \frac{k^5}{42\pi} \left[|f_1|^2 + \frac{s}{M_Z^2} |f_2|^2 \right],$$

with the multipole transitions:

$$E_1 = k^2 \sqrt{\frac{s}{2}} \left(\frac{\sqrt{s}}{M_Z} f_2 + f_1 \right) \quad M_2 = k^2 \sqrt{\frac{5s}{6}} \left(\frac{\sqrt{s}}{M_Z} f_2 - f_1 \right).$$

In the static limit $|\vec{k}| \equiv k \rightarrow 0$ (i.e., for Z or γ_V of mass M_Z) one gets the electric dipole and the magnetic quadrupole moments of the Z (or of the γ_V - Z transition) defined by $E_1 \rightarrow -2\sqrt{s}|\vec{k}|ed$ and $M_2 \rightarrow \sqrt{s/3}|\vec{k}|^2 eK$, such that

$$ed = -\frac{k}{2\sqrt{2}} \left(f_1 + \frac{\sqrt{s}}{p^0} f_2 \right) \quad eK = \sqrt{\frac{s}{2}} \left(-f_1 + \frac{\sqrt{s}}{p^0} f_2 \right).$$

In a fermion-antifermion bound state model (a case similar to $A_1 \rightarrow \rho\gamma$ in the non-relativistic quark model) we have:

$$d \propto R \quad K \propto \sqrt{s} \mu R$$

where R is the average distance between constituents and μ their magnetic moment. If Λ is a typical mass scale of the binding interactions, we have $d \propto \Lambda^{-1}$ and $K \propto \Lambda^{-2}$.

2.b QFD contribution at third order

In standard QFD these couplings f_1 and f_2 (or the electric dipole and the magnetic quadrupole transitions) receive a contribution at third order (e^3) from the triangular diagrams of Fig. 3 where the fermion line (f) is a charged lepton or a quark.

We take the standard couplings $e(a_f - b_f \gamma^5)$ for the Z or γ_V current and $e'(a'_f - b'_f \gamma^5)$ for the final Z given in Table 1, and get the amplitude:

$$R_{fi} = - \frac{8e^3 Q_f}{(2\pi)^4} \int d_4 r \frac{T}{D}$$

with

$$\begin{aligned} D &= (r^2 - m^2)((r-k)^2 - m^2)((r-q)^2 - m^2) \\ T &= m^2(a_f b'_f - b_f a'_f)[e' k \epsilon e] + (a_f b'_f + b_f a'_f) \{ -m^2([q e' \epsilon e] - [r e' \epsilon e]) \\ &\quad + r^2[(q+k-r)e' \epsilon e] - q \cdot k [e' \epsilon r e] + e' \cdot q [k \epsilon r e] + \epsilon \cdot r [q e' (k-2r)e] \\ &\quad - \epsilon \cdot e [q e' k r] + e \cdot r [(q-2r)e' k \epsilon] + e' \cdot k [q \epsilon r e] \} , \end{aligned}$$

r^μ , m and Q_f are the running momentum, the mass and the charge (in units of e) of the internal fermion f ; $[abcd]$ stands for $\epsilon^{\mu\nu\rho\sigma} a_\mu b_\nu c_\rho d_\sigma$.

This amplitude is projected on the invariant forms I_1 and I_2 . The classical treatment of the Feynman integrals and the anomalies is given in Appendix A.

The result is:

$$\begin{aligned} f_1 &= \frac{8e^3 Q_f}{(2\pi)^4 q \cdot k} \left\{ 2a'_f b_f m^2 J_0 + (a_f b'_f + a'_f b_f) [(2q \cdot k - s) K_0 - s(Q_0 - 2Q_1) - \pi^2] \right\} \\ f_2 &= \frac{8e^3 Q_f}{(2\pi)^4 s q \cdot k} \left\{ 2a'_f b_f (s - 2q \cdot k) m^2 J_0 + (a_f b'_f + a'_f b_f) [(s - 2q \cdot k)(3q \cdot k - s) K_0 \right. \\ &\quad \left. - 2q \cdot k Q_1 - \pi^2) - s(q \cdot k (K_0 - Q_0) + (s - q \cdot k)(Q_0 - 2Q_1))] \right\} , \end{aligned}$$

where J_0 , K_0 , Q_0 , Q_1 are convergent integrals given in Appendix A. The terms $-\pi^2$ are related to the anomalies⁵⁾ [we assume that the spectrum of leptons and quarks is such that these anomalies cancel⁶⁾].

We can now compute the electric dipole and the magnetic quadrupole moments related to f_1 and f_2 . For $m^2 \ll s$ we get analytically:

$$d \simeq \frac{\alpha}{\pi\sqrt{2s}} Q_f (a_f b'_f + a'_f b_f) (2 - \log \frac{s}{m^2})$$

$$K \simeq -\frac{\alpha}{\pi\sqrt{2s}} \cdot \frac{\sqrt{s}}{\sqrt{s}} Q_f (a_f b'_f + a'_f b_f) \log \frac{s}{m^2}$$

for each fermion loop. For a complete family (e, u, d with three coloured quarks) the mass independent terms cancel like the anomaly and we are left with logarithms of mass ratios.

For $ZZ\gamma$ transitions:

$$\frac{\pi\sqrt{2s}}{\alpha} d = \frac{\pi\sqrt{2s}}{\alpha} \cdot \frac{\sqrt{s}}{\sqrt{s}} K = \frac{1}{\sin^2 \theta_W} \left(\log \frac{m_u^2}{m_e m_d} + \frac{4}{3} \sin^2 \theta_W \log \frac{m_e^3 m_d}{m_u^4} \right),$$

for $\gamma_V Z \gamma$ transitions:

$$\frac{\pi\sqrt{2s}}{\alpha} d = \frac{\pi\sqrt{2s}}{\alpha} \cdot \frac{\sqrt{s}}{\sqrt{s}} K = \frac{1}{3 \sin^2 \theta_W} \log \frac{m_u^4}{m_e^3 m_d}.$$

An explicit numerical calculation with the six leptons and six quarks spectrum confirms this magnitude. For $\sqrt{s} \gtrsim M_Z$ one gets

$$|d| \simeq 10^{-5} \text{ GeV}^{-1} \quad \text{and} \quad |K| \simeq 10^{-6} \text{ GeV}^{-2}.$$

3. CONTRIBUTIONS TO $e^+e^- \rightarrow \gamma Z$

In standard QFD the lowest order diagrams contributing to this process are those of Fig. 1. They have already been calculated⁷⁾ and lead to the cross-section:

$$\frac{d\sigma^{(1)}}{d\Omega} = \frac{\alpha^2 x}{s} \left\{ \left[\frac{2(s^2 + M_Z^4)}{(s - M_Z^2)^2 \sin^2 \theta} - 1 \right] [(1 - P_L P'_L)(a^2 + b^2) + 2(P'_L - P_L) \text{Re}ab^* + P'_T P'_T (a^2 - b^2) \cos 2\varphi] \right\},$$

where $P_L(P'_L)$, $P_T(P'_T)$ are the $e^-(e^+)$ degrees of polarization, θ, ϕ are the polar and azimuthal angles of the γ with respect to the e^- beam and a and b are the couplings of the Z to the electron (see Table 1); $x = 1 - (M_Z^2/s) = 2k/\sqrt{s}$.

Non-zero $ZZ\gamma$ and $\gamma_V Z\gamma$ couplings would allow an additional contribution (Fig. 2) whose cross-section is given by:

$$\begin{aligned} \frac{d\sigma^{(2)}}{d\Omega} = \frac{\alpha x^5 s^3}{256\pi} \sum_{\alpha, \beta} \left\{ \right. & [(A^\alpha A^{\beta*} + B^\alpha B^{\beta*})(1 - P_L P'_L) + (A^\alpha B^{\beta*} + B^\alpha A^{\beta*})(P_L - P'_L)] \cdot \\ & \cdot [f_1^{\alpha Z} f_1^{\beta Z*} \sin^2 \theta + f_2^{\alpha Z} f_2^{\beta Z*} \frac{s}{2M_Z^2} (1 + \cos^2 \theta)] \\ & - P_T P'_T [(A^\alpha A^{\beta*} - B^\alpha B^{\beta*}) \cos 2\phi + i(A^\alpha B^{\beta*} - B^\alpha A^{\beta*}) \sin 2\phi] \cdot \\ & \cdot [f_1^{\alpha Z} f_1^{\beta Z*} - f_2^{\alpha Z} f_2^{\beta Z*} \frac{s}{2M_Z^2}] \sin^2 \theta \left. \right\}. \end{aligned}$$

$f_i^{\alpha Z}$ are the couplings defined in Section 2 for $ZZ\gamma$ or $\gamma_V Z\gamma$ transitions; α, β run over Z or γ_V intermediate currents,

$$A^\gamma = -\frac{1}{s}, \quad B^\gamma = 0, \quad A^Z = \frac{a}{D_Z(s)}, \quad B^Z = \frac{b}{D_Z(s)}$$

and

$$D_Z(s) = s - M_Z^2 + i M_Z \Gamma_Z.$$

This new contribution differs from the first one by:

- its angular distribution; $\sin^2 \theta$ and $(1 + \cos^2 \theta)$ terms instead of the dominant $1/\sin^2 \theta$ one in $d\sigma^{(1)}/d\Omega$;
- its azimuthal distribution (when $e^\pm$ are transversally polarized) with a new $\sin 2\phi$ term;
- its transverse polarization dependence associated with a $\sin^2 \theta$ term instead of a constant term;
- its longitudinal polarization dependence associated with a different energy dependence; in particular the polarization asymmetry

$$A_L = \frac{\sigma(+ -) - \sigma(- +)}{\sigma(+ -) + \sigma(- +)}$$

is now given by

$$A_L^{(2)} = \frac{\sum_{\alpha, \beta} (A^\alpha B^{\beta*} + B^\alpha A^{\beta*}) [f_1^{\alpha Z} f_1^{\beta Z*} \sin^2 \theta + f_2^{\alpha Z} f_2^{\beta Z*} \frac{s}{2M_Z^2} (1 + \cos^2 \theta)]}{\sum_{\alpha, \beta} (A^\alpha A^{\beta*} + B^\alpha B^{\beta*}) [f_1^{\alpha Z} f_1^{\beta Z*} \sin^2 \theta + f_2^{\alpha Z} f_2^{\beta Z*} \frac{s}{2M_Z^2} (1 + \cos^2 \theta)]}$$

instead of

$$A_L^{(1)} = \frac{2 \operatorname{Re} a b^*}{a^2 + b^2}$$

for the first contribution.

- the general energy - or x - dependence of the cross-section (which in addition can be modified by form factors in the f_i couplings).

We can compare the orders of magnitude of both contributions. We consider cross-sections integrated in a typical angular acceptance for the photon ($30^\circ \leq \theta \leq 150^\circ$):

$$\frac{\sigma^{(2)}}{\sigma^{(1)}} \simeq \frac{x^4 s^2}{256 \pi \alpha} \cdot (|f_i|^2 \text{ like terms})$$

or

$$\frac{\sigma^{(2)}}{\sigma^{(1)}} \simeq \frac{x^2 s}{8} \cdot (|d|^2 \text{ or } (x^2 s / 80) |K|^2 \text{ like terms})$$

This means that $\sigma^{(2)}/\sigma^{(1)}$ will be larger than 1% (a kind of lowest level of observability) for $\sqrt{s} \gtrsim M_Z$ if

$$|d| \gtrsim 2 \times 10^{-3} \text{ GeV}^{-1} \quad \text{or} \quad |K| \gtrsim 2 \times 10^{-4} \text{ GeV}^{-2}$$

(i.e., a scale $\Lambda \lesssim 500 \text{ GeV}$). This also shows that the third order QFD contribution calculated in Section 2b ($|d| \approx 10^{-5} \text{ GeV}^{-1}$, $|K| \approx 10^{-6} \text{ GeV}^{-2}$) will not be observable.

So any measurement showing a departure from the lowest order QFD contribution $d\sigma^{(1)}/d\Omega$ will be a signal for an anomalous effect. As an illustration we have computed $d\sigma^{(1)}/d\Omega$ and $d\sigma^{(2)}/d\Omega$ at $\sqrt{s} = 100, 140$ and 200 GeV, taking $|d| = 1/50 \text{ GeV}^{-1}$ and $|K| = 1/2500 \text{ GeV}^{-2}$ without form factor (i.e., constant in energy). We get the integrated cross-sections given in Table 2. In Fig. 4 we show the angular distributions at $\sqrt{s} = 140$ GeV.

4. CONTRIBUTIONS TO THE CROSS-SECTION OF $e^+e^- \rightarrow \gamma L^+L^-$

A good way of detecting $e^+e^- \rightarrow \gamma Z$ is via the lepton pair channel $e^+e^- \rightarrow \gamma L^+L^-$ (for example $L^\pm = \mu^\pm$) (Fig. 5a). There is, however, a background contribution where the lepton pair does not form a Z . This is the process of Fig. 5b where the γ is radiated by the final $L^\pm$ lines.

We now compute the complete cross-section due to the three processes of Fig. 5 a,b,c, taking into account the interference terms. We decompose the amplitudes on helicity states $e^-(\lambda), e^+(-\lambda), \gamma(\rho), L^\pm(\lambda_{L\pm})$. The photon spectrum in $e^+e^- \rightarrow \gamma L^+L^-$ is given by:

$$\frac{d\sigma}{dx d\Omega} = \frac{\alpha^2 x}{\pi} \left\{ (1 - P_L P'_L) R_1 + (P'_L - P_L) R_2 + P_L P'_L (R_3 \cos 2\phi + R_4 \sin 2\phi) \right\}$$

with

$$x = \frac{2k}{\sqrt{s}} = 1 - \frac{W^2}{s}, \quad W^2 = (p_{L^+} + p_{L^-})^2, \quad 0 \leq x \leq 1 - \frac{4m_L^2}{s}.$$

We define:

$$R_{(\frac{3}{2})} \equiv \frac{1}{4} (R_{--} \pm R_{++}), \quad R_{(\frac{3}{4})} \equiv \frac{1}{2} \frac{\text{Re}}{\text{Im}} (R_{-+})$$

with

$$R_{\lambda\lambda'} \equiv \frac{1}{2} \int d\rho_L \sum_{\rho, \lambda_{L\pm}} F_{\lambda\rho\lambda_L+\lambda_{L-}} F_{\lambda'\rho\lambda_L+\lambda_{L-}}^*$$

$F_{\lambda\rho\lambda_L+\lambda_{L-}}$ being the complete helicity amplitudes, and

$$d\rho_L = \frac{m_L^2}{4\pi^2} \cdot \frac{p_L^*}{W} d\Omega_L$$

the lepton pair phase space in its centre-of-mass.

In Appendix B we collect the expressions of the helicity amplitudes:

$E_\alpha(\lambda, \rho, \eta)$ for the electron exchange processes (involved in Fig. 5a; η is the helicity of γ_V or Z),

$A_\alpha(\lambda, \tau)$ for the e^+e^- annihilation vertex (involved in Fig. 5b, c; τ is the helicity of γ_V or Z),

$C_{\alpha\alpha'}(\tau, \rho, \eta)$ for the central $ZZ\gamma$ or $\gamma_V Z\gamma$ vertices (involved in Fig. 5c with $\alpha = \gamma, Z$ and $\alpha' = Z$).

We also define (see Appendix B):

- the quantity $X_{\alpha\alpha'\beta\beta'}(\tau, \tau') \equiv \sum_{\rho, \eta} C_{\alpha\alpha'}(\tau, \rho, \eta) C_{\beta\beta'}^*(\tau', \rho, \eta)$ for the square of diagram 5c;

- the structure functions

$$\pi_{\mu\nu}^{\alpha\beta} = p_\mu p_\nu \mathcal{V}_1^{\alpha\beta} + (p_\mu p_\nu - W^2 g_{\mu\nu}) \mathcal{V}_2^{\alpha\beta}$$

for the fermion loop $\gamma_V, Z \rightarrow L^+ L^- \rightarrow \gamma_V, Z$ appearing in the square of diagrams (5a and 5c);

- the inclusive structure functions of the photon

$$G_i^{\alpha\beta}(s, x) \quad i=1,6$$

appearing in the square of diagram 5b $[(\gamma_V, Z) + \gamma \rightarrow L^+ L^- \rightarrow \gamma + (\gamma_V, Z)]$;

- the mixed structure functions $I_{\alpha\beta}(\tau, \rho, \tau')$ for the interference terms $(\gamma_V, Z) + \gamma \rightarrow L^+ L^- \rightarrow (\gamma_V, Z)$.

So we get the contributions to the photon spectrum:

- the square of diagram 5a:

$$R_{\lambda\lambda'}^{aa} = \frac{1}{2} \sum_{\substack{\alpha\beta \\ \tau\rho}} \frac{E_\alpha(\lambda, \rho, \tau) E_\beta^*(\lambda', \rho, \tau)}{D_\alpha(W) D_\beta^*(W)} W^2 e^2 \mathcal{V}_2^{\alpha\beta}$$

α and β run over γ or Z , $D(W)$ are the corresponding propagators

$$D_\gamma(W) = W^2, \quad D_Z(W) = W^2 - M_Z^2 + iM_Z \Gamma_Z$$

- the square of diagram 5b:

$$R_{\lambda\lambda'}^{bb} = \sum_{\alpha\beta} \left\{ \sum_{\tau} A_\alpha(\lambda, \tau) A_\beta^*(\lambda', \tau) G_2^{\alpha\beta} + \sum_{\tau\tau'} A_\alpha(\lambda, \tau) A_\beta^*(\lambda', \tau') \frac{\tau\tau' \sin^2 \theta}{2} G_3^{\alpha\beta} \right\} \frac{e^2}{D_\alpha(s) D_\beta^*(s)}$$

- the interference term of 5a and 5b:

$$R_{\lambda\lambda'}^{ab} = \sum_{\substack{\alpha\beta \\ \tau\tau'}} \frac{E_\alpha(\lambda, \tau) A_\beta^*(\lambda', \tau')}{D_\alpha(W) D_\beta^*(s)} e^2 I_{\alpha\beta}(\tau, \tau') + \text{crossed term}.$$

- the square of diagram 5c:

$$R_{\lambda\lambda'}^{cc} = \frac{1}{2} \sum_{\substack{\alpha\alpha'\beta\beta' \\ \tau\tau'}} \frac{A_\alpha(\lambda, \tau) A_{\beta'}^*(\lambda', \tau')}{D_\alpha(s) D_{\beta'}^*(s)} \cdot \frac{W^2 e^2 \mathcal{V}_2^{\alpha'\beta'} \chi_{\alpha\alpha'\beta\beta'}(\tau, \tau')}{D_{\alpha'}(W) D_{\beta'}^*(W)}.$$

- the interference term of 5a and 5c:

$$R_{\lambda\lambda'}^{ac} = \frac{1}{2} \sum_{\substack{\alpha\beta\beta' \\ \tau\tau'}} \frac{E_\alpha(\lambda, \tau) A_\beta^*(\lambda', \tau') C_{\beta\beta'}^*(\tau', \tau)}{D_\alpha(W) D_\beta^*(s) D_{\beta'}^*(W)} W^2 e^2 \mathcal{V}_2^{\alpha'\beta'} + \text{crossed term}.$$

- the interference term of 5b and 5c:

$$R_{\lambda\lambda'}^{bc} = \sum_{\substack{\alpha\beta\beta' \\ \tau\tau' b p}} \frac{A_\alpha(\lambda, \tau) A_{\beta'}^*(\lambda', \tau') C_{\beta\beta'}^*(\tau', p, b)}{D_\alpha(s) D_{\beta'}^*(s) D_{\beta'}^*(W)} e^2 I_{\beta'\alpha}^*(b, \tau) + \text{crossed term}.$$

Using the same parameters as in Section 3 ($d = 1/50 \text{ GeV}^{-1}$ and $K = 1/2500 \text{ GeV}^{-2}$) we illustrate in Fig. 6 (for $\sqrt{s} = 140 \text{ GeV}$) how the photon spectrum $d\sigma/dx$ (integrated in the range $30^\circ \leq \theta \leq 150^\circ$) reflects the Z peak. In Fig. 7 we show how this peak is sensitive to the value of the scale Λ which controls the magnitude of the $ZZ\gamma$ and $\gamma_V Z\gamma$ couplings (we integrate the photon spectrum $d\sigma/dx$ over the Z peak). This cross-section now differs from the one given in Table 2 by the occurrence of the background contribution of the interferences with the intermediate γ_V contributions and of the lepton pair branching ratio.

5. CONCLUSIONS

We have shown how the reaction $e^+e^- \rightarrow \gamma Z$ provides interesting tests of the gauge nature of the neutral bosons γ and Z . The standard contribution to this reaction is due to the electron exchange diagrams and is large enough to be well observable in high energy e^+e^- colliding beams just above the Z mass ($\sqrt{s} \gtrsim M_Z$) (see Fig. 4 and Table 2). An additional contribution can come from e^+e^- annihilation through a virtual photon or a Z followed by $\gamma_V Z \gamma$ or $ZZ\gamma$ vertices. These couplings correspond to electric dipole (d) and magnetic quadrupole (K) transitions. The third order standard QFD contributions (with internal lepton or quark loops) gives

$$|d| \propto \frac{\alpha}{\pi} \cdot \frac{1}{\sqrt{2s}} \quad \text{and} \quad |K| \propto \frac{\alpha}{\pi} \cdot \frac{1}{\sqrt{2s}} \cdot \sqrt{\frac{s}{s_0}}$$

which is surely too weak to be observable. However, if QFD is only an effective low energy theory (for example, if the gauge bosons are fermion-antifermion bound states) one can expect $d \approx 1/\Lambda$ and $K \approx 1/\Lambda^2$ (or μ/Λ) where Λ is some mass scale related to the binding interactions. We have shown that observable effects can appear for $\sqrt{s} \gtrsim M_Z$ if $\Lambda < 500$ GeV. These effects consist of modifications of the angular distributions of the γZ final state, of the energy dependence and of the polarization dependence of the cross-section of $e^+e^- \rightarrow \gamma Z$. This reaction can be measured by the process $e^+e^- \rightarrow \gamma Z \rightarrow L^+L^-$ (for example, $L^\pm = \mu^\pm$); there is a background due to $e^+e^- \rightarrow L^+L^-$ followed by the radiation of a γ from the final $L^\pm$ lines. We have calculated the complete cross-section including interference effects and showed how the Z peak emerges well from this background and is sensitive to sizeable values of d or K for the $ZZ\gamma$ and $\gamma_V Z \gamma$ couplings.

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APPENDIX A - FEYNMAN INTEGRALS FOR THE TRIANGULAR CONTRIBUTIONS

The amplitude corresponding to the triangle diagram of Fig. 3 is:

$$R_{fi} = \frac{ie^3 Q_f}{(2\pi)^4} \int d_4 q_1 \frac{\text{Tr} \{ (\not{q}_3 + m) \not{\epsilon}' (a_f' - b_f' \gamma^5) (\not{q}_2 + m) \not{\epsilon} (\not{q}_1 + m) \not{\epsilon} (a_f - b_f \gamma^5) \}}{(q_1^2 - m^2)(q_2^2 - m^2)(q_3^2 - m^2)}$$

with $q_1 = r$, $q_3 = r - q$, $q_2 = r - k$. Expanding the trace and adding the crossed diagram we get the expression given in Section 3.

We now define the following integrals putting

$$D \equiv (r^2 - m^2)((r - k)^2 - m^2)((r - q)^2 - m^2):$$

$$J_0 = i \int \frac{d_4 r}{D}$$

$$i \int \frac{d_4 r}{D} r^\mu \equiv K_0 k^\mu + Q_0 q^\mu$$

with $Q_0 = \frac{i}{q \cdot k} \int d_4 r \frac{r \cdot k}{D}$ $K_0 = -i \frac{s}{(q \cdot k)^2} \int d_4 r \frac{r \cdot k}{D} + \frac{i}{q \cdot k} \int d_4 r \frac{r \cdot q}{D}$.

$$i \int \frac{d_4 r}{D} r^\mu r^\nu \equiv Q_2 q^\mu q^\nu + K_2 k^\mu k^\nu + Q_1 (q^\mu k^\nu + q^\nu k^\mu) + G_0 g^{\mu\nu}$$

with $Q_2 = \frac{i}{(q \cdot k)^2} \int d_4 r \frac{(r \cdot k)^2}{D}$ $Q_1 = \frac{i}{2(q \cdot k)^3} \left\{ 4k \cdot q \int d_4 r \frac{r \cdot k \cdot q}{D} - 3s \int d_4 r \frac{(r \cdot k)^2}{D} - (k \cdot q)^2 \int d_4 r \frac{r^2}{D} \right\}$

$$K_2 = \frac{1}{(k \cdot q)^2} \left\{ i \int d_4 r \frac{(r \cdot q)^2}{D} - s^2 Q_2 - 2s q \cdot k Q_1 - s G_0 \right\}$$

$$H_0 = i \int d_4 r \frac{r^2}{D} \quad G_0 = -\frac{i}{2(q \cdot k)^2} \left\{ 2q \cdot k \int d_4 r \frac{r \cdot k \cdot q}{D} - s \int d_4 r \frac{(r \cdot k)^2}{D} - (q \cdot k)^2 \int d_4 r \frac{r^2}{D} \right\}.$$

$$i \int \frac{d_4 r}{D} r^2 r^\mu \equiv K_1 k^\mu + Q_1 q^\mu$$

with

$$Q_1 = \frac{i}{q \cdot k} \int d_4 r \frac{r^2 r \cdot k}{D} \quad K_1 = -i \frac{s}{(q \cdot k)^2} \int d_4 r \frac{r^2 r \cdot k}{D} + \frac{i}{q \cdot k} \int d_4 r \frac{r^2 r \cdot q}{D}$$

$J_0, K_0, Q_0, Q_2, K_2, Q_1$ are convergent, H_0, G_0 are logarithmically divergent and K_1, Q_1 are linearly divergent. We also define

$$Q'_1 = Q_1 + m^2 Q_0 \quad K'_1 = K_1 + m^2 K_0$$

and notice that

$$H_0 = Q_2 s + 2 Q_1 q \cdot k + 4 G_0.$$

The amplitude is then given by:

$$\begin{aligned} R_{fi} = i \frac{8e^3 Q_f}{(2\pi)^4} \left\{ m^2 (a_f b'_f - b_f a'_f) I_0 [e' k \varepsilon e] + (a_f b'_f + b_f a'_f) ([e' k \varepsilon e] \cdot \right. \\ \cdot (K'_1 - J_0 + K_0 q \cdot k + 2 G_0) + e \cdot k [q k \varepsilon e'] (K_0 - 2 Q_1) \\ \left. - e' \cdot q [q k \varepsilon e] (K_0 - Q_0) + e \cdot q [q e' k \varepsilon] (Q_0 - 2 Q_2) + [q e' \varepsilon e] \cdot \right. \\ \left. \cdot (J_0 - m^2 I_0 - Q'_1 - Q_0 q \cdot k - 2 G_0) + [q e' k e] \varepsilon \cdot q (Q_0 - 2 Q_1) \right\}. \end{aligned}$$

For the identification of the anomaly we compute the amplitude of the same triangular diagram with the current (e^μ, q^μ) replaced by a pseudoscalar (q^μ) with the coupling $eb_f \gamma^5$ to the fermion line. We get:

$$\bar{R}_{fi} = i \frac{e^3 b_f Q_f}{(2\pi)^4} \int d_4 q_1 \frac{\text{Tr} \left\{ (q_3 + m) \not{\varepsilon}' (a'_f - b'_f \gamma^5) (q_2 + m) \not{\varepsilon} (q_1 + m) \gamma^5 \right\}}{(q_1^2 - m^2)(q_2^2 - m^2)(q_3^2 - m^2)}.$$

Expanding the trace and adding the crossed diagram, we get, with the same notations as above,

$$\bar{R}_{fi} = i \frac{8e^3 Q_f b_f a'_f m}{(2\pi)^4} \varepsilon^{\mu\nu\sigma} q_\mu k_\nu \varepsilon_\sigma e'_\sigma J_0.$$

We now take the $(1^{++}1^{--}1^{--})$ part of the R_{fi} amplitude (i.e., $b'_f \equiv 0, a_f \equiv 0$) and replace e^μ by q^μ . We get

$$\frac{R_{fi}(e^k \Rightarrow q^k)}{b_f a'_f} = 2m \frac{\bar{R}_{fi}}{b_f a'_f} + A \left(i \frac{8e^3 Q_f}{(2\pi)^4} \varepsilon^{\mu\nu\rho\sigma} q_\mu k_\nu \varepsilon_\rho e'_\sigma \right)$$

with the anomaly given by $A = Q'_1 - K'_1$ in terms of the linearly divergent integrals.

These integrals are fixed by the requirement of gauge invariance for the two vector currents (we impose a vanishing contribution when replacing e^μ by k^μ or $e^{\mu\prime}$ by p^μ):

$$Q'_1 = -m^2 J_0 + Q_2 S + 2 G_0$$

$$K'_1 = m^2 J_0 + 4q.k Q_1 + 2 G_0 + (Q_2 - Q_0) S + (s - 2q.k) K_0 .$$

The anomaly is now convergent and given by:

$$A = s Q_0 - 4q.k Q_1 - 2m^2 J_0 + (2q.k - s) K_0 .$$

The evaluation of the convergent integrals is done using Feynman parametrization. We obtain

$$J_0 = \frac{\pi^2}{2q.k} \int_0^1 dy \frac{L}{y} \quad Q_0 = 2 Q_2 = \frac{\pi^2}{2q.k} \int_0^1 dy L$$

$$Q_1 = \frac{\pi^2}{2q.k} \left[\frac{1}{2} - \int_0^1 dy \frac{m^2 - sy(1-y)}{2q.k y} L \right]$$

$$K_0 = \frac{\pi^2}{2q.k} \left[-1 + \int_0^1 dy \left(\frac{1}{y} - \frac{m^2 - sy(1-y)}{2k.q y^2} L \right) \right]$$

with

$$L \equiv \log \left| 1 + \frac{2q.k y(1-y)}{m^2 - sy(1-y)} \right| + i\pi \{ \theta(y-y_1) - \theta(y-y_2) + \theta(y-y_3) - \theta(y-y_4) \}$$

$$y_{(1)} = \frac{1}{2} \left(1 \mp \sqrt{1 - \frac{4m^2}{s}} \right) \quad y_{(2)} = \frac{1}{2} \left(1 \mp \sqrt{1 - \frac{4m^2}{s-2q.k}} \right) .$$

We also check that $sQ_0 - 4q \cdot kQ_1 - 2m^2J_0 = -\pi^2$, which is the value of the anomaly in the case of two final real photons (when $p^2 = s - 2q \cdot k = 0$).

With these gauge invariance constraints and the expression of the anomaly we get the final and convergent expression of the amplitude R_{fi} given in Section 3 in terms of f_1 and f_2 .

APPENDIX B - HELICITY AMPLITUDES AND STRUCTURE FUNCTIONS FOR $e^+e^- \rightarrow \gamma L^+L^-$

Helicity amplitudes:

For the electron exchange process (in Fig. 5a)

$$e^+(-\lambda) + e^-(\lambda) \longrightarrow \gamma(p) + (\gamma_\nu(b) \text{ or } Z(b)) :$$

$$E_\alpha(\lambda, p, b) = \frac{a^\alpha - 2\lambda b^\alpha}{k} \left\{ \frac{W^2(2\lambda \cos \theta - b)}{2\sqrt{s} \sin \theta} \delta_{p,b} - \frac{\lambda \sqrt{s}}{\sin \theta} (2\lambda p + \cos \theta) \delta_{p,-b} - \frac{2\lambda p W}{\sqrt{2}} \delta_{b,0} \right\}$$

(α runs over γ_ν and Z). For the e^+e^- annihilation vertex (in Fig. 5b,c):

$$e^+(-\lambda) + e^-(\lambda) \longrightarrow \gamma_\nu(\tau) \text{ or } Z(\tau) :$$

$$A_\alpha(\lambda, \tau) = \frac{\lambda \sqrt{s}}{e} (a^\alpha - 2\lambda b^\alpha) \frac{\tau + 2\lambda}{\sqrt{2}} (1 - \delta_{\tau,0}) .$$

For the central $ZZ\gamma$ or $\gamma_\nu Z\gamma$ vertices (in Fig. 5c)

$$\text{with } \alpha = \gamma_\nu(\tau) \text{ or } Z(\tau) \text{ and } \alpha' = \gamma_\nu(b) \text{ or } Z(b) :$$

$$C_{\alpha\alpha'}(\tau, p, b) = \left\{ -\sqrt{s} f_1^{\alpha\alpha'} \frac{k^2 \sin \theta}{2\sqrt{2}} \tau(p+b)(1 - \delta_{b,0}) + f_2^{\alpha\alpha'} \frac{s k^2}{2W} (\tau \cos \theta + p) \delta_{b,0} \right\} (1 - \delta_{\tau,0}) .$$

Structure functions:

For the square of diagram 5c:

$$\sum_{\alpha\alpha'/\beta\beta'} X(\tau, \tau') = s f_1^{\alpha\alpha'} f_1^{\beta\beta'} \frac{k^4 \sin^2 \theta}{2\sqrt{2}} \tau \tau' + f_2^{\alpha\alpha'} f_2^{\beta\beta'} \frac{s^2 k^4}{W^2} (\delta_{\tau,\tau'} - \frac{\tau \tau'}{2} \sin^2 \theta) .$$

For the fermion loop appearing in the square of diagrams 5a and 5b:

$$\Pi_{\mu\nu}^{\alpha\beta} = p_\mu p_\nu \mathcal{V}_1^{\alpha\beta} + (p_\mu p_\nu - W^2 g_{\mu\nu}) \mathcal{V}_2^{\alpha\beta}$$

For $m_L/W \ll 1$ we need only

$$\mathcal{V}_2^{\alpha\beta} = \frac{1}{6\pi} (a_L^\beta a_L^{\alpha*} + b_L^\beta b_L^{\alpha*}) .$$

The structure functions of the photon are defined⁷⁾ by:

$$\frac{1}{2} \sum_p \int d^4 p_L J_\mu^\alpha J_\nu^{\beta*} = \frac{q_\mu q_\nu}{s} G_1^{\alpha\beta} + \left(\frac{q_\mu q_\nu}{s} - g_{\mu\nu} \right) G_2^{\alpha\beta} + \frac{\tilde{k}_\mu \tilde{k}_\nu}{k^2} G_3^{\alpha\beta} - i \frac{\varepsilon_{\mu\nu\alpha\beta} k^a q^b}{k\sqrt{s}} G_4^{\alpha\beta} + \frac{\tilde{k}_\mu q_\nu + q_\mu \tilde{k}_\nu}{k\sqrt{s}} G_5^{\alpha\beta} + i \frac{k_\mu q_\nu - k_\nu q_\mu}{k\sqrt{s}} G_6^{\alpha\beta}.$$

J_μ^α is the intermediate (γ_V or Z) current; $\tilde{k}_\mu = k_\mu - (|\vec{k}|/\sqrt{s})q_\mu$. We need only G_2 and G_3 (the other terms give vanishing contributions in our case). For $m_L/\sqrt{s} \ll 1$ we have:

$$G_2^{\alpha\beta} = 2\alpha Q_L^2 (a_L^\beta a_L^{\alpha*} + b_L^\beta a_L^{\alpha*}) \frac{1}{x^2} \left(-(2-x)^2 + [1+(1-x)^2] \log \frac{s(1-x)}{m_L^2} \right)$$

$$G_2^{\alpha\beta} + G_3^{\alpha\beta} = 8\alpha Q_L^2 (a_L^\beta a_L^{\alpha*} + b_L^\beta a_L^{\alpha*}) \frac{1-x}{x^2}, \quad \alpha = \frac{2k}{\sqrt{s}}.$$

The mixed structure functions for the interference terms (5a x 5b) or (5c x 5b) are taken from Ref. 7):

$$I_{\alpha\beta}(\tau, p, \tau') = - \frac{e Q_L}{2\pi} (b_L^\beta a_L^{\alpha*} + b_L^{\alpha*} a_L^\beta) \frac{1-x}{x^2} \left\{ -k \left(1 + \frac{k}{\sqrt{s}} \right) \cdot \right.$$

$$\left. \cdot \left(\frac{\tau+p}{2} \right) \frac{\sin \theta}{\sqrt{2}} \tau' (1 - \delta_{\tau,0}) - \frac{k\sqrt{s}}{2W} \left(1 - \frac{k\sqrt{s}}{W^2} \right) (\tau' \cos \theta + p) \delta_{\tau,0} \right\}.$$

Table 1

Standard γ and Z couplings
 for the photon: $a = Q$, $b = 0$
 for the Z:

	ν	e	u	d
$a \times 2\sin^2\theta_W$	1	$-1 + 4\sin^2\theta_W$	$1 - \frac{8}{3}\sin^2\theta_W$	$-1 + \frac{4}{3}\sin^2\theta_W$
$b \times 2\sin^2\theta_W$	1	-1	1	-1

Table 2

Integrated cross-sections $\sigma(e^+e^- \rightarrow \gamma Z)$

$\sqrt{s}$ (GeV)	100	140	200
$\sigma^{(1)}$ (el. exchange) (nb)	0.20	0.22×10^{-1}	0.64×10^{-2}
$\sigma^{(2)}$ (central term) $\Lambda = 50$ GeV (nb)	0.13×10^{-2}	0.75×10^{-2}	0.16×10^{-1}

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FIGURE CAPTIONS

- Fig. 1 : Standard QFD contribution (electron exchanges) to $e^+e^- \rightarrow \gamma Z$.
- Fig. 2 : $e^+e^- \rightarrow \gamma Z$ via $ZZ\gamma$ and $\gamma_V Z\gamma$ couplings.
- Fig. 3 : Triangular diagrams for $ZZ\gamma$ and $\gamma_V Z\gamma$ couplings.
- Fig. 4 : Angular distribution $d\sigma/d\cos\theta$ ($e^+e^- \rightarrow \gamma Z$) at $\sqrt{s} = 140$ GeV:
 ----- standard contribution (electron exchanges)
 additional contribution with $d = 1/50 \text{ GeV}^{-1}$, $K = 1/2500 \text{ GeV}^{-2}$.
- Fig. 5 : Diagrams for $e^+e^- \rightarrow \gamma L^+ L^-$:
 (a) standard contribution $e^+e^- \rightarrow \gamma + \underbrace{\gamma_V, Z}_{\rightarrow L^+ L^-}$
 (b) background contribution with photon radiated by $L^\pm$ lines
 (c) additional contribution with $ZZ\gamma$ and $\gamma_V Z\gamma$ couplings.
- Fig. 6 : Photon spectrum $d\sigma/dx$ in $e^+e^- \rightarrow \gamma L^+ L^-$ at $\sqrt{s} = 140$ GeV:
 xxxxxx background contribution (diag. 5b)
 ----- standard contribution and background (diag. 5a + 5b)
 additional contribution (diag. 5c) with $d = 1/50 \text{ GeV}^{-1}$,
 $K = 1/2500 \text{ GeV}^{-2}$.
 ——— total
- Fig. 7 : Cross-section $\bar{\sigma} = \int dx (d\sigma/dx)(e^+e^- \rightarrow \gamma L^+ L^-)$ integrated over the Z peak (see text):
 ----- standard contribution and background
 total with $d = 1/\Lambda$ alone
 .-.-.- total with $K = 1/\Lambda^2$ alone
 ——— total with $d = 1/\Lambda$ and $K = 1/\Lambda^2$.

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG.6

FIG.7