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STUDY OF THE (μ^-, e^+) REACTION MEDIATED BY MAJORANA NEUTRINOS

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A B S T R A C T

The exotic lepton violating (μ^-, e^+) reaction has been studied in a gauge theory model which assumes the existence of a Majorana neutral lepton. In addition to the usual mechanism involving only two nucleons, the reaction mechanisms which consider a virtual Δ^{++} present in the nucleus or pions in flight between the interacting nucleons have also been included. The total (μ^-, e^+) branching ratio was computed as a function of the various parameters of the theory. It was found to be very sensitively dependent on the mass m_σ of the Majorana particle and it becomes very small for $m_\sigma > 10 \text{ GeV}/c^2$. The dependence of the branching ratio on the other parameters of the theory is also discussed.

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1. INTRODUCTION

The concept of lepton conservation¹⁻³⁾ has recently been revived in the context of modern gauge theories, in which there exists a similarity between strangeness non-conservation and lepton non-conservation. Since the non-conservation of strangeness is an experimentally established fact, it is concluded that lepton conservation cannot be exact. Most of the gauge⁴⁾ theories, including the standard one of Weinberg-Salam⁵⁾, accommodate lepton non-conservation, and they differ only in the level at which such non-conservation shows up or in the mechanism by which it is produced. Lepton conservation is also not expected to hold in grand unification theories⁶⁾.

Many lepton-violating processes are possible. The interesting ones among them are those occurring in nuclei such as

$$\mu^- + A(Z) \rightarrow A^*(Z) + e^- \quad [\text{muon conversion}^{2)}] \quad (1)$$

$$A(Z) \rightarrow A(Z+2) + e^- + e^- \quad [\text{neutrinoless}^{7-8)} \text{ double } \beta\text{-decay}] \quad (2)$$

$$\mu^- + A(Z) \rightarrow A(Z-1) + e^+ \quad [\text{muon-positron conversion}^{3)}] \quad (3)$$

In the usual additive assignment of lepton numbers¹⁰⁾, process (1) does not conserve the family (muon) quantum number, but it does conserve the total lepton number. Processes (2) and (3), which violate the total lepton number, are more exotic^{*)}.

In the present work we will focus on reaction (3); this can be measured simultaneously with reaction (1), which is at present under experimental investigation²⁾. Reaction (3) has been studied by Kamal and Ng⁹⁾ in a model whereby it is mediated by a Majorana neutrino^{**)}, and by Vergados¹¹⁾ in a model in which it is mediated by Higgs particles. In the present work we will use exactly the same gauge theory model as that discussed in Ref. 9. We will, however, relax some of the assumptions made in this earlier work. Thus:

*) Process (3) conserves total lepton number in the Konopinski-Mahmoud scheme in which e^- , ν_e , μ^+ , ν_μ have the same lepton quantum number ($= +1$). (See, for example, Ref. 10.)

**) A Majorana neutrino differs from a Dirac neutrino in that it is its own anti-particle. More information is given elsewhere¹¹⁾.

- i) we will include the contribution of the axial current since we know that it is important in other similar processes, e.g. reaction (2);
- ii) we will take into consideration the reaction mechanism which includes particles other than nucleons; namely, we will consider the double charge exchange on a virtual Δ^{++} and also on a π^+ in flight between the two nucleons. This mechanism has been previously considered by Shuster and Rho¹²⁾, who pointed out the necessity of its inclusion. The reason why, in our model, it is necessary to incorporate these mechanisms is that the Majorana neutrino, which sets the range of the process, is now believed to be rather massive (if it exists), making this process a very short-ranged one. Therefore, the amplitude for the mechanism which involves only two nucleons is expected to be heavily suppressed by the short-range correlations.

The present paper will be organized as follows. After the introduction, we give the essentials of the relevant gauge theory model in Section 2. In Section 3 we construct the two-body operator for the mechanism involving only nucleons and give an expression for the branching ratio. In Sections 4 and 5 we build the effective operators corresponding to the elementary processes $\mu^- + \Delta^{++} \rightarrow n + e^+$ and $\mu^- + \pi^+ \rightarrow \pi^- + e^+$. A method of computing the nuclear matrix elements entering in total (μ^-, e^+) capture rates will be given in Section 6. In Section 7 we present a discussion of our results, and in Section 8 we summarize our conclusions.

2. GAUGE THEORY MODELS

The model that we will use has been discussed in Ref. 9. We will briefly mention its basic features here in order to make the paper as self-sufficient as possible and to establish the notation.

In order to accommodate reactions (2) and (3), the standard model⁵⁾ must be extended to include at least one more lepton pair put in a left-handed doublet of weak isospin, i.e. the left-handed leptons are

$$\begin{pmatrix} \nu_1 \\ e^- \end{pmatrix}_L, \begin{pmatrix} \nu_2 \\ \mu^- \end{pmatrix}_L, \begin{pmatrix} \nu_3 \\ \tau^- \end{pmatrix}_L, \begin{pmatrix} \nu_4 \\ N^- \end{pmatrix}_L \quad (4)$$

where $L = \frac{1}{2}(1-\gamma_5)$ and ν_i , $i = 1, 2, 3, 4$, are the eigenstates¹³⁾ of weak interactions which are linear combinations of the well-known neutrinos ν_e , ν_μ , ν_τ and the new Majorana neutrino N_0 . For our purposes it is adequate to consider the Cabibbo-like mixing angle¹⁴⁾ between N_0 and the muonic and electronic neutrinos. Thus we may write

$$\begin{aligned} (\nu_1)_L &\approx (\nu_e)_L + \beta (N_0)_L \\ (\nu_2)_L &\approx (\nu_\mu)_L + \rho (N_0)_L \end{aligned} \quad (5)$$

The mixing angles β and γ will be treated as free parameters. The $\mu^- \rightarrow e^+$ reaction proceeds via the above mixings, and the amplitude is proportional to $(\beta\gamma)$. Because of the Majorana nature¹⁵⁾ of the N_0 , there can be a mass term in the Lagrangian of the form

$$M_\sigma \bar{N}^\sigma N^\sigma \quad (6)$$

which violates the lepton number by two units. Thus the interaction Lagrangian takes the form

$$\mathcal{L} = f (\beta \bar{e}_L \gamma^\lambda N_L^\sigma + \gamma \bar{\mu}_L \gamma^\lambda N_L^\sigma) W_\lambda + M_\sigma \bar{N}_0 N_0 \quad (7)$$

where f is the usual gauge-coupling constant $f^2/m_W^2 = 4\sqrt{2}G_F$ and W_λ is the W-boson field. Right-handed leptons do not contribute in this charge-changing process since they are put in weak isosinglets.

With the Lagrangian density (7), one can write down the appropriate Feynman diagrams in perturbation theory. The process (3) must involve two nucleons since it is forbidden on a single nucleon. One possibility is that only nucleons are involved, as is represented in Fig. 1. However, if the mass of the Majorana neutrino is large, the double charge exchange is more likely to occur on a virtual Δ^{++} or on a positive π^+ in flight between the two nucleons. The corresponding diagrams will be presented later.

3. THE EFFECTIVE TRANSITION OPERATOR FOR THE NUCLEONIC DIAGRAM

If the (μ^-, e^+) reaction proceeds via a mechanism involving only nucleons, as indicated in Fig. 1, the invariant amplitude associated with reaction (3) takes the form⁹⁾

$$\begin{aligned}
 m = & f^4 \frac{\beta\gamma}{16} \int d^4x \int d^4y \int \frac{d^4k}{(2\pi)^4} \frac{F((p_\mu - k)^2)}{(p_\mu - k)^2 - m_\omega^2} \frac{F((p_e - k)^2)}{(p_e - k)^2 - m_\omega^2} \frac{1}{k^2 - m_\sigma^2} \\
 & e^{ip_e y} e^{-iE_\mu y_0} e^{-ik(x-y)} \phi_\mu(\vec{y}) \\
 & \bar{u}^c(p_e) \gamma^\mu (1-\gamma_5) (\not{k} + m_\sigma) m_\sigma (1-\gamma_5) (\not{k} + m_\sigma) \gamma^\nu (1-\gamma_5) u(p_\mu) \\
 & \sum_n e^{-i(E_i - E_n) y_0} e^{-i(E_n - E_f) x_0} \langle f | J_\mu(\vec{x}) | n \rangle \langle n | J_\nu(\vec{y}) | i \rangle
 \end{aligned} \quad (8)$$

where $|i\rangle$, $|f\rangle$, and $|n\rangle$ refer to the initial, final, and intermediate nuclear states with energies E_i , E_f , and E_n , respectively; p_e and p_μ are the positron and muon four-momenta, m_σ is the mass of the Majorana neutrino, and $\phi_\mu(\vec{y})$ the muon wave function; J_μ and J_ν are the usual hadronic currents which are familiar from the ordinary β -decay. The constants β , γ , and f were defined in the previous section. In evaluating the integrals of Eq. (8), the following approximations were made:

- i) all external momenta appearing in the propagators were neglected⁹⁾, which is reasonable since they are of order m_μ , which is much smaller than m_W or m_σ ; the external momenta were also neglected in the nucleon form factors;
- ii) closure over the intermediate hadronic states was employed, i.e. we write

$$\begin{aligned}
 & \sum_n e^{-i(E_i - E_n) y_0} e^{-i(E_n - E_f) x_0} \langle f | J_\mu(\vec{x}) | n \rangle \langle n | J_\nu(\vec{y}) | i \rangle \\
 & \approx e^{-i(E_i - \langle E_n \rangle) y_0} e^{-i(\langle E_n \rangle - E_f) x_0} \langle f | J_\mu(\vec{x}) J_\nu(\vec{y}) | i \rangle
 \end{aligned}$$

with some appropriate mean energy $\langle E_n \rangle$ of the intermediate states, which in the end will not affect the final results in this reaction mechanism.

With the above approximations after collecting all the γ -matrices together and performing the x_0 , y_0 , k_0 , and $\vec{k}$ integrations with standard contour integral techniques, we find that to leading order the amplitude takes the form

$$\begin{aligned} \mathcal{M} = & \frac{f^4}{16} (\beta_\gamma) \frac{m_\sigma^2}{M^4} \int d^3 \vec{x} \int d^3 y \ e^{-i \vec{p}_0 \cdot \vec{x}} \ \phi_\mu(\vec{y}) \\ & \left\{ \tilde{F}_1(r) \ \bar{u}^c(p_0) \ \gamma^\mu \gamma^0 \gamma^\nu \ (1-\gamma_5) \ u(p_\mu) \ \frac{\Delta}{m} \right. \\ & \left. + \tilde{F}_2(r) \ \bar{u}^c(p_0) \ \gamma^\mu \ \frac{\vec{p} \cdot \hat{r}}{i} \ \gamma^\nu \ (1-\gamma_5) \ u(p_\mu) \right\} \\ & \langle \ell | \mathcal{J}_\mu(\vec{x}) \mathcal{J}_\mu(\vec{y}) | i \rangle \end{aligned} \quad (9)$$

where $r = |\vec{x} - \vec{y}|$ and $\Delta = E_i + m_\mu - \langle E_n \rangle$.

The functions $\tilde{F}_1(r)$ and $\tilde{F}_2(r)$ depend, of course, on the shape of the nucleon form factors. If we set $F(q^2) \approx 1$, we obtain

$$\begin{aligned} \tilde{F}_1(r) &= \frac{1}{(m_\omega^2 - m_\sigma^2)^2} \left\{ \frac{\Delta}{m_\sigma} \left(1 + \frac{4 m_\sigma^2}{m_\sigma^2 - m_\omega^2} \frac{1}{m_\sigma r} \right) e^{-m_\sigma r} + m_\omega \leftrightarrow m_\sigma \right\} \\ \tilde{F}_2(r) &= \frac{1}{(m_\omega^2 - m_\sigma^2)^2} \left\{ \left(1 + \frac{4(1 + m_\sigma r)}{(m_\sigma^2 - m_\omega^2) r^2} \right) e^{-m_\sigma r} + m_\omega \leftrightarrow m_\sigma \right\} \end{aligned}$$

The following limiting cases are of interest:

$$\tilde{F}_1(r) = \frac{1}{M^4} \frac{\Delta}{m} e^{-mr} \quad \tilde{F}_2(r) = \frac{1}{M^4} e^{-mr} \quad m \ll M$$

with $m = \min(m_\sigma, m_\omega)$ and $M = \max(m_\sigma, m_\omega)$. We note that since $\Delta = E_i + m_\mu - \langle E_n \rangle \approx m_\mu$, and m is expected to be greater than $0.5 \text{ GeV}/c^2$, the time component is negligible in front of the space component. With our normalization, the hadronic current takes the form:

$$\mathcal{J}_\mu(\vec{x}) = \frac{1}{2} \sum_i \delta(\vec{x} - \vec{r}_i) \mathcal{L}_\mu(i) \quad (10)$$

where $\vec{r}_i$ is the coordinate of the i^{th} nucleon, and $\mathcal{L}_\mu(i)$ for non-relativistic nucleons is given by:

$$\mathcal{L}_\mu(i) = \begin{cases} f_v \tau_-(i) & \mu = 0 \\ -f_A \tau_-(i) \vec{\sigma}(i) & \mu \neq 0 \end{cases} \quad (11)$$

Using standard identities of the Dirac matrices¹⁶⁾, after some algebra we obtain:

$$m = G \langle f | \Omega | i \rangle [\langle \phi_\mu^2 \rangle]^{1/2} \quad (12)$$

where

$$G = \frac{1}{2} \beta \nu (G_F m_p)^2 \quad (13)$$

$$\Omega = l_0 \Omega_0 + \vec{\Omega} \cdot \vec{l}$$

$$\Omega_0 = \sum_{i+j} e^{-i\vec{p}_0 \cdot \vec{r}_j} F(r) \tau_-(i) \tau_-(j) \omega_{ij}^0 \quad (14a)$$

$$\vec{\Omega} = \sum_{i+j} e^{-i\vec{p}_0 \cdot \vec{r}_j} F(r) \tau_-(i) \tau_-(j) \vec{\omega}_{ij} \quad (14b)$$

$$l^\lambda = \bar{u}^c(p_e) \gamma^\lambda (1 - \gamma_5) u(p_\mu) \quad (15)$$

with

$$\omega_{ij}^0 = -\frac{A}{m} (f_V^2 + f_A^2 \vec{\sigma}_i \cdot \vec{\sigma}_j) - f_A^2 (\vec{\sigma}_i \times \vec{\sigma}_j) \cdot \frac{\hat{r}}{i} + f_V f_A (\vec{\sigma}_i + \vec{\sigma}_j) \cdot \frac{\hat{r}}{i} \quad (16a)$$

$$\begin{aligned} \vec{\omega}_{ij} = & \frac{A}{m} [f_V f_A (\vec{\sigma}_i + \vec{\sigma}_j) + f_A^2 i \vec{\sigma}_i \times \vec{\sigma}_j] - f_A^2 (\vec{\sigma}_i - \vec{\sigma}_j) \times \hat{r} \\ & + (f_V^2 + f_A^2 \vec{\sigma}_i \cdot \vec{\sigma}_j) \frac{\hat{r}}{i} + f_A^2 \left(\frac{\vec{\sigma}_i \cdot \hat{r}}{i} \cdot \vec{\sigma}_j + \vec{\sigma}_i \cdot \frac{\vec{\sigma}_j \cdot \hat{r}}{i} \right) \end{aligned} \quad (16b)$$

and

$$F(r) \approx \left(\frac{m_\sigma}{m_p} \right)^2 \left(\frac{m_\omega}{M} \right)^4 e^{-m|\vec{r}_i - \vec{r}_j|} \quad (17)$$

m_p = proton mass.

The muon ls wave function can safely be taken out of the integral since it is approximately constant inside the nucleus. Its distortions do not affect the branching ratio. After summing over all final lepton spin states and averaging over the initial ones, we get:

$$|m(i \rightarrow f)|^2 = G^2 \langle \phi_\mu^2 \rangle |M.E|^2 \quad (18)$$

with $|M.E|^2$, the nuclear matrix element, given by

$$|M.E|^2 = |\langle f | Q_0 | i \rangle|^2 + |\langle f | \vec{Q} | i \rangle|^2 \quad (19)$$

Since

$$\langle \phi_\mu^2 \rangle \approx \frac{1}{\pi} \left(\frac{\alpha m_\mu c^2}{\hbar c} \right)^3 Z_{eff}^4 / Z$$

the transition width from an initial state $|i\rangle$ to a final nuclear state $|f\rangle$ takes the form

$$\Gamma(i \rightarrow f) = \frac{1}{(2\pi)^2} E_e p_e (4\pi) |m(i \rightarrow f)|^2$$

or

$$\Gamma(i \rightarrow f) = (\beta\gamma)^2 \frac{1}{4\pi^2} (G_F m_p^2)^4 \frac{E_e p_e c}{(m_p c^2)^2} \left(\frac{m_\mu}{m_p} \right)^2 Z_{eff}^4 / Z \quad (20)$$

$$m_\mu c^2 |M.E|^2$$

The ordinary total muon capture rate is given by¹⁷⁾

$$\Gamma(\mu^-, \nu_\mu) = \frac{\alpha^3}{2\pi} (G_F m_p^2)^2 m_\mu c^2 Z_{eff}^4 [f_V^2 + 3f_A^2 + f_P^2 - 2f_P f_A] f(A, Z) \quad (21)$$

where f_V , f_A , f_P are the vector, axial vector, and pseudoscalar form factors entering muon decay, and

$$f_V^2 + 3f_A^2 + f_P^2 - 2f_P f_A \approx 5.87$$

The function $f(A, Z)$ is a correction due to the two-nucleon correlation given approximately¹⁸⁾ by

$$f(A, Z) \approx 1 - .030 \frac{A}{Z} + .25 \left(\frac{A}{2Z} - 1 \right) + 3.24 \left(\frac{Z}{2A} - \frac{1}{2} - \left| \frac{1}{8Z} - \frac{1}{4A} \right| \right)$$

which is 0.16 for $A = 2Z$.

Thus the branching ratio of the (μ^-, e^+) conversion to a specific final state takes the form

$$R = \frac{1}{2} (\beta_g)^2 (G_F m_p^2)^2 \frac{E_e p_e c}{(m_\mu c^2)^2} \frac{|M_E|^2}{Z [f_V^2 + 3f_A^2 + f_p^2 - 2f_p f_A] f(Z, A)} \quad (22)$$

Before we proceed with the evaluation of the nuclear matrix elements, a task that will be accomplished in Section 5, we note that the radial function $F(r)$ is an exponential function of m and it will be greatly damped for large values of m . In such situations one may not be able to neglect the dependence of the nucleon form factor on the momentum transfer. Assuming that the form factor is approximately described by a dipole shape, i.e. is given by

$$F(q^2) = \frac{1}{(1 - q^2/m_A^2)^2} \quad m_A = .85 \text{ GeV}/c^2$$

we find that the radial function $F(r)$ in Eqs. (14) is given by

$$F(r) = \frac{1}{(1 - m_\sigma^2/m_A^2)^2} \left\{ \frac{8}{(1 - m_\sigma^2/m_A^2)^3} \frac{(x_A + 1) e^{-x_A} - (x_\sigma + 1) e^{-x_\sigma}}{x_A^2} + \frac{3 e^{-x_A} + e^{-x_\sigma}}{(1 - m_\sigma^2/m_A^2)^2} + \frac{1}{2} \frac{x_A e^{-x_A}}{1 - m_\sigma^2/m_A^2} + \frac{1}{24} x_A^4 Y_1(x_A) \right\} \quad (23)$$

where

$$x_\sigma = m_\sigma r \quad x_A = m_A r \quad Y_1(x) = \left(\frac{1}{x^2} + \frac{1}{x} \right) \frac{e^{-x}}{x}$$

Thus the range of the transition operator is now determined by the "size" of the nucleon and not by the mass of the Majorana particle. The effect of the form factor will be to reduce the branching ratio for small values of m_σ but enhance it for large values.

4. THE EFFECTIVE TRANSITION OPERATOR INVOLVING Δ^{++}

There are two possible mechanisms involving Δ^{++} ; they correspond to the elementary processes

$$\mu^- + \Delta^{++} \rightarrow e^+ + n \quad \text{and} \quad \mu^- + \Delta^{++} \rightarrow e^+ + \Delta^0$$

The second one, which is of higher order in the probability of the presence of a Δ , has been neglected. In order to construct the effective transition operator for the first one, two problems have to be tackled:

- i) To compute the elementary amplitude associated with $\mu^- + \Delta^{++} \rightarrow e^+ + n$ which is shown diagrammatically in Fig. 2. The evaluation of this amplitude can be done in two ways. Firstly, the Δ^{++} and the neutron are treated as being composed of quarks. For this we apply the same formalism as described for the nucleons in the previous section, the only modification being that i and j now label quarks rather than nucleons. Secondly, we treat the hadrons involved as elementary particles, and we use phenomenological expressions for the relevant vertices. In this case we have to evaluate convergent loop integrals instead of the Fourier transform of Eq. (8).
- ii) We have to know the probability of finding a Δ^{++} in the initial nuclear state. In a first approach we take this probability from the literature^{19,20}). In a second approach we have assumed that the creation of Δ^{++} arises from the exchange of a π^+ as shown in Fig. 3, and we have explicitly calculated the contribution of this diagram.

In evaluating the basic amplitude $m_1(\mu^- + \Delta^{++} \rightarrow e^+ + n)$ in the quark model, we made the following two approximations:

- i) the quarks are in S-states;
- ii) the spin-isospin wave functions of the Δ^{++} and n are adequately described by a pure SU(4) symmetric state.

Then only the parts of the effective operator which change the spin of the system and which are symmetric under the exchange of i and j contribute. Thus after invoking the long wavelength approximation, which is expected to hold well even for $p_e \approx 100$ MeV since the radii of the relevant hadrons are small (≤ 0.6 fm), we obtain

$$\Omega \approx -\frac{\delta_1}{2} \frac{p_e}{m_p} \tau^1 \cdot \hat{p}_e l_0 + \delta_2 \frac{\Delta}{m_p} \vec{l} \cdot \tau^1 - \frac{1}{2} \delta_1 [\tau^2 \otimes \hat{p}_e] \cdot \vec{l} \quad (24a)$$

where

$$\delta_1 \approx \frac{1}{3} \langle m_p | r | F(r) \rangle \quad \text{and} \quad \delta_2 = \frac{m_p}{m} \langle F(r) \rangle \quad (24b)$$

$$T_q^1 = \sum_{i \neq j} \tau_-(i) \tau_-(j) (\vec{\sigma}_i + \vec{\sigma}_j)_q \quad (24c)$$

$$T_q^2 = \sum_{i \neq j} \tau_-(i) \tau_-(j) (\vec{\sigma}_i \times \vec{\sigma}_j)_q^2 \quad (24d)$$

The quantities δ_1 and δ_2 depend on the radius R_0 of the quark density distribution [see Eqs. (90a) and (90b) of Section 6]. For reasonably massive Majorana neutrinos, they are approximately given by

$$\delta_1 = \delta_2 = \left(\frac{m_\sigma}{m_p} \right) \frac{3}{4\rho^3} \quad \rho = \frac{m_\sigma c^2 R_0}{\hbar c} \quad (25)$$

With the above operator, the amplitude can be cast into the form

$$M = G g_\Delta (m_\sigma, m_\omega, R_0) \bar{\ell} \cdot \langle \chi | \vec{S}^+ | \psi \rangle$$

with

$$g_\Delta (m_\sigma, m_\omega, R_0) = 4 \left[\frac{7}{4} \delta_1^2 \left(\frac{p_\sigma}{m_p} \right)^2 + \delta_2^2 \left(\frac{2.5 m_\mu}{m_p} \right)^2 \right]^{1/2} \quad (26)$$

In the above expression, $|\chi\rangle$ represents the nucleon two-component spinor, and $|\psi\rangle$ the (non-relativistic) four-component spinor associated with a $J = 3/2$ particle. The operator S^+ converts a Δ^{++} into a neutron and is normalized as follows:

$$\begin{aligned} \langle \frac{1}{2} m_p | \vec{q} \cdot \vec{S}^+ | \frac{3}{2} m_i \rangle &= \langle \frac{3}{2} m_i | \vec{q} \cdot \vec{S} | \frac{1}{2} m_f \rangle^* \\ &= \langle \frac{1}{2} m_p | 1 m_i - m_f | \frac{3}{2} m_i \rangle \vec{q}_{m_i - m_f} \end{aligned} \quad (27)$$

In the elementary particle (EP) treatment the basic amplitude is constructed from a phenomenological expression of the $\Delta^{++} p W^-$ and $p n W^-$ vertices. The second vertex is the usual one appearing in V-A theory of weak interactions and was given earlier.

The $\Delta^{++} p W^-$ vertex has been given by Schneiner and Hippel²¹⁾ and takes the form

$$\frac{1}{2} \bar{\psi}_\alpha \left\{ \left[\left(G_3^V \not{p}_\lambda + G_4^V \frac{(p_\Delta)_\lambda}{m_p} \right) \not{p}_3 + G_3^A \not{p}_\lambda + G_4^A \frac{(p_\Delta)_\lambda}{m_p} \right] \left(\frac{q^2 \omega^\alpha - q^\alpha \omega^2}{m_p} \right) + C_5^V \omega^\alpha \not{p}_3 + C_5^A \omega^\alpha \right\} u(p) \quad (28)$$

where $\bar{\psi}_\alpha$ is the Schwinger-Rarita symbol representing a relativistic spin $3/2$ particle with four-momentum $(p_\Delta)_\lambda$, and q is the four-momentum transfer $q = p_\Delta - p_p$. The form factors C_3^V and C_4^V are determined from electroproduction and photoproduction data, while the other form factors have been obtained from certain theoretical models. The values of these form factors at $q^2 = 0$ are presented in Table 1. For the momentum dependence of these form factors the dipole approximation was employed:

$$\frac{\Gamma(q^2)}{\Gamma(0)} = \frac{1}{(1 - q^2/m_A^2)^2}, \quad m_A = 0.85 \text{ GeV}/c^2 \quad (29)$$

Then the amplitude associated with Fig. 2 takes the form

$$m_1 = \frac{p^4}{16} (\beta p) \not{p}_\sigma^2 \int \frac{d^4 k}{(2\pi)^4} \frac{1}{Q^2 - m_p^2} \frac{1}{(k^2 - m_\sigma^2)^2} \frac{k_\alpha}{(k_1^2 - m_\omega^2)^2} \frac{m_A^8}{(q^2 - m_A^2)^4} \\ \bar{u}^c(p_e) \not{p}^\mu \not{p}^\alpha \not{p}^\nu (1 - \gamma_5) u(p_p) \\ \bar{\psi}(n) \not{p}_\mu (\not{p}_\nu - \not{p}_A \gamma_5) (\not{p} + m_p) T_{\nu\lambda} \psi^\lambda(p_\Delta) \\ Q = p_n + p_e - K \quad (30)$$

$T_{\nu\lambda}$ is obtained by casting Eq. (28) in the form

$$\bar{\psi}(p_n) T_{\nu\lambda} \psi^\lambda(p_\Delta) \quad (31)$$

We note that the dipole form factor was used for the pnW^- vertex as well. We also mention that the external momenta were neglected in the W and N_0 propagators.

The above loop integral can be evaluated using standard Feynman parameter techniques as described by 't Hooft and Veltman²²⁾. The exact answer is very

complicated. It is approximately given by the following expression:

$$I_L = \frac{1}{M^4} \mathcal{S}_L, \quad \mathcal{S}_L = \frac{1}{32\pi} \mathcal{S}(\mu, \nu) \quad (32)$$

with

$$\mathcal{S}(\mu, \nu = \infty) = \frac{1}{(1-\mu)^3} \left[-4(1-\mu) - 2(1+\mu) \ln \mu \right] \quad (33a)$$

$$\begin{aligned} \mathcal{S}(\mu, \nu) = & \frac{1}{(1-\mu)^2} \left\{ \frac{1}{(\frac{\mu}{\nu}-1)^4} \left[\frac{(\frac{\mu}{\nu})^2 - 8(\frac{\mu}{\nu}) - 17}{3} + 2 \frac{3(\frac{\mu}{\nu}) + 1}{\frac{\mu}{\nu}-1} \ln \left(\frac{\mu}{\nu} \right) \right] \right. \\ & + \frac{\nu^2}{(1-\nu)^4} \left[\frac{1-8\nu-17\nu^2}{3} - \frac{2\nu^2(\nu+1)}{1-\nu} \ln \nu \right] \Big\} \\ & + \frac{2}{3} \frac{1}{(1-\mu)^3} \left\{ \frac{\nu}{(\frac{\mu}{\nu}-1)^3} \left[\left(\frac{\mu}{\nu} \right)^2 - 5 \left(\frac{\mu}{\nu} \right) - 2 + \frac{6 \frac{\mu}{\nu}}{\frac{\mu}{\nu}-1} \ln \frac{\mu}{\nu} \right] \right. \\ & \left. \left. - \frac{\nu^2}{(1-\nu)^3} \left[1-5\nu-2\nu^2 - 6 \frac{\nu^2}{1-\nu} \ln \nu \right] \right\} \right. \quad (33b) \end{aligned}$$

where

$$\mu = \frac{m^2}{M^2} \quad \nu = \frac{m_A^2}{M^2} \quad m = \min(m_\sigma, m_\omega), \quad M = \max(m_\sigma, m_\omega)$$

Then the required amplitude takes the form

$$m_1 = G \mathcal{S}_L \left(\frac{m_\sigma}{m_p} \right)^2 \left(\frac{m_\omega}{M} \right)^4 \bar{u}^c(p_e) \gamma^\lambda \gamma^\alpha \gamma^\nu (1-\gamma_5) u(p_\mu) K_{\lambda\alpha\nu} \quad (34)$$

$$\begin{aligned} K_{\lambda\alpha\nu} = & \bar{u}(p_n) L_\lambda \left[\frac{2\not{p} + m_p}{m_p} (S_\alpha \psi_\nu - S_\nu \psi_\alpha) \right. \\ & + \not{p}_\alpha \left((p+p_\mu)_\beta S^\beta \psi_\nu - \frac{(p+p_\mu)_\beta}{m_p} S_\nu \psi^\beta \right. \\ & \left. \left. + \frac{p_\alpha}{m_p} (\not{p}_\beta S^\beta \psi_\nu - \not{p}_\beta S_\nu \psi^\beta) \right] \right. \quad (35a) \end{aligned}$$

with

$$\begin{aligned} p &= p_e + p_n \\ L_\lambda &= \not{p}_\lambda (\not{p}_\nu - \not{p}_\alpha \gamma_5) + \frac{(\not{p}_\lambda)_\nu}{m_p} (G_4^A + G_4^V \not{p}_5) \\ S_\nu &= \not{p}_\nu (G_3^A + G_3^V \not{p}_5) + \frac{(\not{p}_\lambda)_\nu}{m_p} (G_4^A + G_4^V \not{p}_5) \end{aligned} \quad (35b)$$

It should be remarked at this point that in the diagram of Fig. 2 the proton is not the only intermediate hadronic state, but the N^{*+} should also be included. In the earlier quark treatment, since we applied closure, the contribution of N^{*+} was automatically included. In the EP treatment the inclusion of N^{*+} is quite tedious and it was omitted. Rough quark model calculations indicate that it adds coherently to the proton contribution with approximately the same amplitude.

We made the further approximation of neglecting in Eq. (34) all terms which depend on the momenta of the external particles. Then we obtain the following expression:

$$m_1 = G g_\Delta(m_\sigma, m_\omega, m_A) \vec{\ell} \cdot \langle \chi | \vec{S}^+ | \psi \rangle \quad (36)$$

where

$$g_\Delta = \xi_1 \xi_2 \left(\frac{m_\sigma}{m_p} \right)^2 \left(\frac{m_\omega}{M} \right)^4 \quad (37)$$

The quantity ξ_1 , which is a function of the various form factors appearing in Table 1, is given in the same table. For our numerical calculations, the average square value of $\xi_1 = 12.5$ was employed.

Once the amplitude m_1 is known, the effective transition operator can be written down since the amplitude $pp \rightarrow \Delta^{++}n$ indicated in Fig. 3b has already been calculated⁽²¹⁾. If this amplitude has already been incorporated in the usual nuclear calculation to yield the Δ -component of the nuclear wave function, the required effective operator takes the form

$$\Omega = e^{-i\vec{p}_e \cdot \vec{r}} \vec{\ell} \cdot \vec{S}^+ \quad (38)$$

where $\vec{r}$ is the coordinate of the Δ^{++} . The total amplitude now takes the form

$$m = G g_\Delta \langle f | \Omega | i \rangle \quad (39)$$

where g_Δ is given by Eqs. (26) and (37). It may be useful to write $\vec{S}^+$ in a more familiar form

$$\vec{S}_q^+ = \frac{2}{\sqrt{3}} \left[\alpha_{1/2}^+ + (\alpha_{3/2}) \right]_q^1 \quad (40)$$

where $t(a_{3/2}, m)$ is the tensor operator associated with the destruction Δ -operator $a_{3/2, m}$ and $a_{1/2}^+$ is the neutron creation operator.

The effective operator can also be constructed by explicitly calculating the whole diagram (3a). Then it is expressed solely in terms of the nucleon coordinates. The amplitude associated with Fig. 3b is approximately²³⁾ given by

$$m_2 = \frac{\chi_{n\Delta}}{m_n^2} \frac{1}{q^2 - m_n^2} \langle x'_1 | \vec{\sigma}_1 \cdot \vec{q} | x_1 \rangle \langle \psi | \vec{S} \cdot \vec{q} | x_2 \rangle \quad (41)$$

with

$$\chi_{n\Delta} \approx 4\pi \frac{2\sqrt{2}}{\sqrt{3}} p_{nnn}^* p_{nnn} \approx 3.4 (4\pi) p_{nnn}^2 \quad (42)$$

Thus the full amplitude associated with Fig. 3 becomes

$$m = G g_\Delta \frac{\chi_{n\Delta}}{m_n^2} \sum_{\Delta} \frac{\langle x'_1 x'_2 | \vec{S} \cdot \vec{S}^+ | x'_1 \Delta \rangle}{\epsilon_\Delta + \epsilon'_1 - \epsilon_1 - \epsilon_2} \frac{\langle x'_1 \Delta | \vec{S} \cdot \vec{q} \vec{\sigma}_1 \cdot \vec{q} | x_1 x_2 \rangle}{q^2 - m_n^2} \quad (43)$$

Neglecting the momentum dependence of the energies of the intermediate states, it is straightforward to sum over the intermediate states and transform into coordinate space to obtain

$$m = G g_\Delta \xi_\Delta^{\text{exch}} \vec{\ell} \cdot \langle p | \vec{\Omega}_\Delta^{\text{exch}} | i \rangle \quad (44)$$

where

$$\xi_\Delta^{\text{exch.}} = \chi_{n\Delta} \frac{m_n}{3(m_n^* m_p)} \approx 0.6 p_{nnn}^2 \quad (45)$$

and

$$\vec{\Omega}_\Delta^{\text{exch}} = \sum_{i \neq j} e^{-i \vec{p} \cdot \vec{r}_i} \tau_{-}(i) \tau_{-}(j) \left\{ 2 \left[\hat{r} (\vec{\sigma}_i \cdot \hat{r}) x_n^2 Y_2(x_n) - \vec{\sigma}_i Y_1(x_n) \right] \right. \\ \left. + i (\vec{\sigma}_i \times \hat{r}) \vec{\sigma}_j \cdot \hat{r} x_n^2 Y_2(x_n) - i (\vec{\sigma}_i \times \vec{\sigma}_j) Y_1(x_n) \right\} \quad (46)$$

with

$$x_{ij} = \frac{m_{\pi} c^2 r}{\hbar c}, \quad \vec{r} = \vec{r}_i - \vec{r}_j$$

$$Y_2(x) = \left(\frac{3}{x^4} + \frac{3}{x^3} + \frac{1}{x^2} \right) Y_0(x) \quad (47a)$$

$$Y_1(x) = \left(\frac{1}{x^2} + \frac{1}{x} \right) Y_0(x) \quad (47b)$$

$$Y_0(x) = \frac{e^{-x}}{x} \quad (47c)$$

The relevant nuclear matrix element is

$$|ME|^2 = g_A^2 (S_A^{\text{exch}})^2 \left| \langle p | \hat{\vec{Q}}_{\text{exch}}^A | i \rangle \right|^2 \quad (48)$$

5. THE EFFECTIVE TRANSITION OPERATOR FOR THE (π^+, π^-) MECHANISM

The effective transition operator associated with this reaction mechanism can be constructed along lines similar to those given in the previous section. The relevant diagram is now depicted in Fig. 4.

In the non-relativistic quark treatment, those components of the operator of Eq. (17) contribute which do not change the spin, i.e.

$$\omega_{ij}^* \approx -\frac{\Delta}{m} (1 + \vec{\sigma}_i \cdot \vec{\sigma}_j) \quad \text{and} \quad \vec{\omega}_{ij} = \left(1 - \frac{1}{3} \vec{\sigma}_i \cdot \vec{\sigma}_j\right) \frac{\hat{r}}{r}$$

Proceeding in a fashion analogous to that in the previous section, we obtain

$$m_i = G g_{\pi} (m_{\sigma}, m_{\omega}, R_0) \left[L_0 - \vec{L} \cdot \hat{\vec{p}}_e \right] 2m_{\pi} \quad (49)$$

with

$$g_{\pi} = \left[\frac{1}{4} \left(\frac{p_0}{m_p} \right)^2 \delta_1^2 + \left(\frac{m_{\mu}}{m_p} \right)^2 \delta_2^2 \right]^{1/2} \quad (50)$$

where δ_1 and δ_2 are given by Eq. (24), and R_0 is the radius of the pion (radius of quark confinement).

In the EP treatment we proceed as in the (Δ^{++}, n) case. Now we need the vertices $\pi^+ \pi^0 W^\lambda$ and $\pi^0 \pi^- W^\lambda$, which have a structure determined by the CVC hypothesis. They take the form²⁴⁾

$$\frac{\sqrt{2}}{2} (2q_1 + p_\pi - k)_\lambda w^\lambda \quad \text{and} \quad \frac{\sqrt{2}}{2} (2q_2 + p_\pi - k)_\nu W^\nu$$

where W^λ is the charged W-boson field. Each vertex is multiplied by a form factor $F(q^2)$, which was assumed to be of the same form as before and is given by Eq. (29).

Thus the required amplitude takes the form

$$m_1 = \frac{p^4}{16} m_\sigma^2 2f_v^2 \int \frac{d^4 k}{(2\pi)^4} \frac{(p_\mu + 2q_1 - k)_\lambda K_\alpha (p_\pi + 2q_2 - k)_\nu}{(k^2 - m_\sigma^2)^2 (k^2 - m_\omega^2) ((p - k)^2 - m_\eta^2)} \frac{m_A^8}{(q^2 - m_A^2)^4} \quad (51)$$

$$\bar{u}^c(p_\pi) \gamma^\lambda \gamma^\alpha \gamma^\nu (1 - \gamma_5) u(p_\mu)$$

where

$$p = q_2 + p_\pi = q_1 + p_\mu$$

After performing the loop integration, we obtain an equation similar to Eq. (34) with $k_{\mu\alpha\nu}$ in the present case being

$$k_{\mu\alpha\nu} = f_v^2 \left[(p_\mu + 2q_1)_\lambda g_{\alpha\nu} + (p_\pi + 2q_2)_\lambda g_{\alpha\nu} + \frac{1}{3} (p_\lambda g_{\alpha\nu} + p_\nu g_{\alpha\lambda} + p_\alpha g_{\lambda\nu}) \right] \quad (52)$$

Thus the required amplitude takes the form

$$m_1 = 6 g_\pi(m_\sigma, m_\omega, m_A) \left[2 (\ell_0 - \vec{\ell} \cdot \hat{p}_\pi) + 4 \frac{\vec{\ell} \cdot \vec{q}_2}{m_\mu} \right] m_\eta \quad (53)$$

where

$$g_\pi(m_\sigma, m_\omega, m_A) = 5 \frac{m_\mu}{m_\eta} f_v^2 \left(\frac{m_\sigma}{m_\rho} \right)^2 \left(\frac{m_\omega}{M} \right)^4 \xi_L \quad (54)$$

with ξ_L given by Eq. (32).

In order to make the calculations that follow tractable, and to retain the similarity between the above amplitude and that obtained in the quark model, we will make the approximation of omitting the momentum-dependent term $\vec{\ell} \cdot \vec{q}_2$ and write

$$m_1 \approx G g_n (m_e, m_\omega, m_A) (L_0 - \vec{\ell} \cdot \hat{p}_e) 2 m_n \quad (55)$$

Then, following a procedure analogous to that described in the previous section, we find that the full amplitude associated with Fig. 5 yields, in coordinate space, the following expression for the nuclear matrix element:

$$|ME|^2 = 2 (g_n \xi_n^{\text{exch}})^2 |\langle f | \Omega_n^{\text{exch}} | i \rangle|^2 \quad (56)$$

where

$$\Omega_n^{\text{exch}} = \sum_{i \neq j} \tau_{-}(i) \tau_{-}(j) \frac{(\vec{\nabla}_i \cdot \vec{\sigma}_i)(\vec{\nabla}_j \cdot \vec{\sigma}_j)}{m_\pi^2} e^{-i\vec{p}_e \cdot \vec{r}_i} I(\vec{p}_e, \vec{r}, p_e r, m_\pi r) \quad (57)$$

with

$$\xi_n^{\text{exch}} = f_{nnn}^2$$

and

$$I(\vec{p}_e, \vec{r}, p_e r, m_\pi r) = \frac{2}{\pi^2} \int \frac{d^3 q}{(q^2 + m_\pi^2) [(q + \vec{p}_e)^2 + m_\pi^2]} e^{-i\vec{q} \cdot \vec{r}}$$

The above operator is still somewhat complicated. Thus we make the approximation of retaining only the lowest non-vanishing terms in a $p_e |m_\pi$ expansion of the above integral I. Thus we find

$$\Omega_n^{\text{exch}} \approx \sum_{i \neq j} e^{-i\vec{p}_e \cdot \vec{R}} \tau_{-}(i) \tau_{-}(j) [-\vec{\sigma}_i \cdot \vec{\sigma}_j Y_0(x_\pi) + \vec{\sigma}_i \cdot \hat{r} \vec{\sigma}_j \cdot \hat{r} Y_2(x_\pi)] \quad (58)$$

with $\vec{R} = \frac{1}{2}(\vec{r}_i + \vec{r}_j)$, and the functions $Y_1(x)$ and $Y_0(x)$ given by Eqs. (47b) and (47c).

6. CALCULATION OF THE TOTAL (μ^-, e^+) TRANSITION RATE

The transition rate for (μ^-, e^+) conversion to any final state can be computed once the matrix element $|\langle f | \Omega | i \rangle|^2$ is calculated. The operator Ω is given, for the cases considered in this work, by Eqs. (13) to (17), (38), (46), or (58).

Transitions to individual final states, in particular to the ground state, are of

great experimental interest since, by a judicious choice of the target nucleus, the background problem associated with pair production from radiative muon capture is minimized. Our work indicates, however, that such individual rates are very small; thus we will focus our attention on predicting total rates. Then, using closure, we find

$$\sum_f |\langle f | \Omega | i \rangle|^2 = \langle i | \Omega^\dagger \Omega | i \rangle \quad (59)$$

where $\Omega^\dagger$ is the Hermitian adjoint of Ω . The evaluation of the above matrix element is fairly straightforward if the Δ -component in the nuclear wave function is known. Indeed, then the nuclear wave function is given by

$$|JM\rangle = \frac{1}{\sqrt{1+P_\Delta}} \left\{ \psi_{JM}^0 + \sqrt{P_\Delta} \sum_{n, J_\Delta} c_{n, J_\Delta} [\psi_{Jn}^n \otimes \psi_{J_\Delta}] JM \right\} \quad (60)$$

where ψ_{JM}^0 is the usual shell-model wave function, ψ_{Jn}^n represents a basis for the (A-1, Z-2) nuclear system, ψ_{J_Δ} is the wave function of the Δ , and P_Δ is the probability of finding the Δ in the nucleus. Then

$$\begin{aligned} \langle f | \Omega^\dagger \Omega | i \rangle &= \\ &= \frac{P_\Delta}{1+P_\Delta} \sum_{n, J_\Delta} \frac{|c_{n, J_\Delta}|^2}{2J_\Delta+1} \sum_{m_\Delta} \langle \psi_{J_\Delta m_\Delta} | \vec{\ell} \cdot \vec{S} \vec{\ell} \cdot \vec{S}^\dagger | \psi_{J_\Delta m_\Delta} \rangle \\ &= \frac{1}{3} \frac{P_\Delta}{1+P_\Delta} \end{aligned} \quad (61)$$

Thus the final result is proportional to the probability (per nucleon) of finding the Δ in the nucleus and does not depend on the details of the structure of the Δ -component of the wave function. The probabilities P_Δ which have been quoted in the literature^{19,20)} are of the order of 2-4%.

For all the other operators considered in the present work, we write

$$\begin{aligned} \Omega^\dagger \Omega &= \\ &= \sum_{i \neq j} e^{i\vec{p}_e \cdot \vec{r}_i} \omega_{ij}^\dagger \sum_{\kappa \neq \ell} e^{-i\vec{p}_e \cdot \vec{r}_\kappa} \omega_{\kappa\ell} \\ &= \sum_{i \neq j} \omega_{ij}^\dagger \omega_{ij} + \sum_{\substack{i \neq j, \kappa \neq \ell \\ (i,j) \neq (\kappa,\ell)}} e^{-i\vec{p}_e \cdot (\vec{r}_\kappa - \vec{r}_i)} \omega_{ij}^\dagger \omega_{\kappa\ell} \end{aligned} \quad (62)$$

The first term is a two-body operator, which is independent of the positron momentum, while the second is a complicated four-body operator. At this point we neglect the second term and write

$$\langle i | \Omega^\dagger \Omega | i \rangle \approx \langle i | (\Omega^\dagger \Omega)_A | i \rangle \quad (63)$$

Since it is fairly complicated to use spin densities explicitly, we make the approximation of assuming that the two nucleons undergoing the transition are spin singlets. Then we obtain

$$\langle i | (\Omega^\dagger \Omega) | i \rangle = \sigma_N (f_V, f_A) \langle i | Q_N | i \rangle \quad \begin{array}{l} \text{(nucleons} \\ \text{only)} \end{array} \quad (64a)$$

$$\langle i | (\vec{Q}_\Delta^{\text{exch}})^\dagger (\vec{Q}_\Delta^{\text{exch}}) | i \rangle \approx 3 \langle i | Q_\Delta | i \rangle \quad (\Delta^{++}, \pi) \quad (64b)$$

$$\langle i | (Q_\eta^{\text{exch}})^\dagger (Q_\eta^{\text{exch}}) | i \rangle \approx \langle i | Q_\eta | i \rangle \quad (\pi^+, \pi^-) \quad (64c)$$

where

$$\sigma_N (f_V, f_A) = \left(\frac{A}{m} \right)^2 \left[(\rho_V^2 - 3\rho_A^2)^2 + 12\rho_A^4 \right] + (\rho_V^2 + \rho_A^2)^2 + 28\rho_A^4 \quad (65a)$$

$$Q_N = \sum_{i \neq j} \tau_+(i) \tau_-(i) \tau_+(j) \tau_-(j) \left| F(r_{ij}) \right|^2 \quad (65b)$$

$$Q_\Delta = \sum_{i \neq j} \tau_+(i) \tau_-(i) \tau_+(j) \tau_-(j) \left[Y_0(x_{ij}) \right]^2 \quad (66)$$

$$Q_\eta = \sum_{i \neq j} \tau_+(i) \tau_-(i) \tau_+(j) \tau_-(j) \left[3Y_0(x_{ij}) - x_{ij}^2 Y_1(x_{ij}) \right]^2 \quad (67)$$

The assumption of spin-singlet states is not expected to be crucial. For example, in the mechanism which involves only nucleons, we find that σ_N is only about 50% smaller if we make the assumption that the two-nucleon states are in spin triplets. From the structure of $\sigma_N (f_V, f_A)$ we notice that the matrix element is dominated by the axial current.

In order to proceed further, we need to know the two-body nuclear density, which can be written as follows:

$$\rho(\vec{r}_1, \vec{r}_2) = \rho(\vec{r}_1) \rho(\vec{r}_2) [1 - G(|\vec{r}_1 - \vec{r}_2|)] \quad (68)$$

where $\rho(\vec{r}_1)$ and $\rho(\vec{r}_2)$ are the one-body densities, and $G(\vec{r}_1, \vec{r}_2)$ is the two-nucleon correlation function. Since the range of the operators Ω_π and Ω_Δ is determined by the pion mass, we do not expect their results to be sensitively dependent on $G(r)$. On the other hand, the range of the operator Ω_N is determined by $m = \min(m_\sigma, m_W)$. Thus if, as expected, the mass of the Majorana neutrino (if it exists) is large^{25,26}, e.g. $m_\sigma > 9 \text{ GeV}/c^2$, the results are extremely sensitive to $G(r)$, which is not well known. This sensitivity can be adequately exhibited assuming the following simple correlation function:

$$G(r) = \begin{cases} 1 & r < r_c \\ 0 & r > r_c \end{cases} \quad (69)$$

where r_c is the hard-core radius ($\sim 0.4 \text{ fm}$). For illustrative purposes, we will exhibit our results as a function of r_c .

Furthermore, since we are interested in the gross behaviour of the total (μ^-, e^+) branching ratio as a function of A and Z , we will make the simplifying but quite adequate assumption that the one-body density is constant inside the nucleus. Then the matrix elements of the operators Ω given by Eqs. (65b), (66), and (67) take the form

$$\langle i | \Omega | i \rangle = Z(Z-1) \langle f(r) \rangle \quad (70)$$

where

$$\langle f(r) \rangle = \frac{3}{(2R_A)^3} \int_{r_c}^{2R_A} f(r) r^2 dr, \quad R_A = r_0 A^{1/3}, \quad r_0 = 1.1 \text{ fm} \quad (71)$$

where $f(r)$ is a radial function associated with each of the above operators. Equation (70) states that the total (μ^-, e^+) conversion rate is proportional to the number of proton pairs present in the initial nuclear state. Thus we obtain

$$\langle i | Q_N | i \rangle = \frac{Z(Z-1)}{A} \frac{6}{(4\eta_2)^3} \Phi_N(m_\sigma, m_\omega, r_c) \quad (72)$$

$$\langle i | Q_A | i \rangle = S(A, Z) \frac{12}{(4\eta_3)^3} e^{-\eta_1} \quad (73)$$

$$\langle i | Q_N | i \rangle = \frac{Z(Z-1)}{A} \frac{30}{(4\eta)^3} \left[\frac{1}{10} (\eta_1^2 - 6\eta_1) + 1 \right] e^{-\eta_1} \quad (74)$$

where

$$S(A, Z) \text{ is } \frac{Z(Z-1)}{A} \text{ or } \frac{1}{3} P_A \quad (75)$$

$$\Phi_N = \begin{cases} \left(\frac{m_\sigma}{m_p}\right) \left\{ \frac{1}{2} p^2 + p + 1 \right\} e^{-p} & m_\sigma < m_\omega \\ \left(\frac{m_\omega}{m_\sigma}\right)^4 \left(\frac{m_\omega}{m_p}\right) \left\{ \frac{1}{2} \tilde{p}^2 + \tilde{p} + 1 \right\} e^{-\tilde{p}} & \end{cases} \quad (76a)$$

$$p = 2r_c \frac{m_\sigma c^2}{\hbar c} \quad \tilde{p} = 2r_c \frac{m_\omega c^2}{\hbar c} \quad (77a)$$

$$\eta_2 = r_0 \frac{m_p c^2}{\hbar c} \quad \eta_3 = r_0 \frac{m_n c^2}{\hbar c} \quad \eta_1 = 2r_c \frac{m_n c^2}{\hbar c} \quad (77b)$$

Equation (76a) is drastically modified if a nucleon form factor of the type of Eq. (29) is included. The actual function Φ_N is very complicated, but it assumes a very simple form in the following situations:

$$\Phi_N = \begin{cases} \left(\frac{m_\sigma}{m_p}\right) \frac{1}{2} \left\{ (p-3)^2 + 1 \right\} e^{-p} & m_\sigma \ll m_A \\ \left(\frac{m_A}{m_\sigma}\right)^4 \left(\frac{m_A}{m_p}\right) P(p_A) e^{-p_A} & m_\sigma \gg m_A \end{cases} \quad (76b)$$

with $p_A = 2r_c (m_A c^2 / \hbar c)$ and

$$P(p_A) = \frac{1}{18432} \left[p_A^6 + 2p_A^5 - 34p_A^4 - 136p_A^3 + 360p_A^2 + 3074p_A + 5328 \right]$$

Thus the branching ratio can be cast in the form

$$R = \frac{1}{2} \left(G_F m_p^2 \right)^2 \frac{\langle E_e^2 \rangle}{(m_p c^2)^2} \frac{S(A,Z)}{Z P(Z,A)} \frac{\Phi(m_\sigma, m_\omega, r_c, R_0)}{p_\nu^2 + 3p_A^2 + p_p^2 - 2p_p p_A} \quad (78)$$

where $S(A,Z) = P_\Delta/Z$ in the case of the (Δ^{++}, n) mechanism in which the Δ -component of the wave function is known. In all other cases

$$S(A,Z) = k \frac{Z(Z-1)}{A}$$

with

$$k = \begin{cases} \frac{6}{(4\eta_2)^3} \sigma(p_\nu, p_A) \approx 5.0 \times 10^{-2} & \text{(nucleons only)} \quad (79a) \\ 3 \times \frac{6}{(4\eta_3)^3} (\xi_\Delta^{exch})^2 \approx 3.0 \times 10^{-3} & (\Delta^{++}, n) \quad (79b) \\ 2 \times \frac{30}{(4\eta_3)^3} (\xi_n^{exch})^2 \approx 1.2 \times 10^{-2} & (\pi^+, \pi^-) \quad (79c) \end{cases}$$

The function Φ is given by Eq. (76) in the mode involving only nucleons. In the other two cases we get

$$\phi = \phi_\Delta(m_\sigma, m_\omega, r_c, R_0) = e^{-\eta_1} \{g_\Delta(m_\sigma, m_\omega, R_0)\}^2 \quad (80a)$$

$$\phi = \phi_n(m_\sigma, m_\omega, r_c, R_0) = \left[\frac{1}{10} (\eta_1^2 - 6\eta_1 + 1) \right] e^{-\eta_1} [g_n(m_\sigma, m_\omega, R_0)]^2 \quad (80b)$$

with g_Δ and g_π given by Eqs. (26) or (37) and (50) or (54), respectively. In the quark-model treatment, we still have to compute the functions δ_1 and δ_2 . Assuming that the quark matter is distributed uniformly inside the bag with radius R_0 , we obtain

$$\delta_1 = \frac{3}{4} \frac{m_\sigma}{m_p} \left[1 - \left(\frac{4}{3} \eta^3 + 2\eta^2 + 2\eta + 1 \right) e^{-2\eta} \right] \frac{1}{\eta^3} \quad m_\sigma < m_\omega \quad (81a)$$

$$\delta_2 = \frac{3}{4} \frac{m_\sigma}{m_p} \left[1 - (2\eta^2 + 2\eta + 1) e^{-2\eta} \right] \frac{1}{\eta^3} \quad m_\sigma < m_\omega \quad (81b)$$

$$\delta_1 \approx \delta_2 \approx \frac{3}{4} \frac{m_\sigma}{m_p} \frac{1}{\eta^3} \quad m_\sigma > m_\omega \quad (81c)$$

where

$$\eta = R_0 \frac{m_\sigma c^2}{\hbar c} \quad (82)$$

In the limit $R_0 \rightarrow 0$ we obtain

$$\delta_1 = 0 \quad \delta_2 \approx \begin{cases} m_\sigma / m_p & m_\sigma < m_\omega \\ \frac{m_\omega}{m_p} \left(\frac{m_\omega}{m_\sigma} \right)^2 & m_\sigma > m_\omega \end{cases} \quad (83)$$

Finally, we remark that the quantities m_A and R_0 are related by

$$R_0 = \sqrt{6} \hbar c / m_A c^2 \quad (84)$$

7. DISCUSSION OF OUR RESULTS

Before we present our numerical results, it may be helpful to focus on some features of our calculation. First we must keep in mind that in order to tackle our problem, certain approximations had to be made. For the benefit of the reader we summarize them here:

- i) All external momenta in the various propagators were neglected.
- ii) The form factors were assumed to be given by a dipole shape [see Eq. (29)].
The external momenta in them were also neglected.
- iii) Whenever necessary, closure was invoked in the summations over intermediate states.
- iv) The quarks were assumed to be non-relativistic particles in S-states uniformly distributed inside the bag.
- v) The spin dependence of the transition operator was extracted by assuming that the interacting nucleons were in singlet states. The one-body nuclear density was assumed to be constant.
- vi) Total branching ratios were computed using closure of the final nuclear states. The four-body pieces in the operator $\Omega^\dagger \Omega$ of Eq. (62) were neglected.

With the above approximations the following qualitative features emerge from the calculation:

The dependence of the total (μ^-, e^+) rate in the bulk nuclear parameters A and Z is simple. In fact this rate is proportional to the number of proton pairs found in the initial nucleus and inversely proportional to the nuclear volume, i.e. proportional to $Z(Z-1)/A$. Since the total muon capture rate is also proportional to Z , the branching ratio will be proportional to

$$\frac{Z-1}{A} \approx \frac{1}{2}$$

The branching ratio is a very sensitive function of the mass m_σ of the Majorana neutrino. For $m_\sigma \rightarrow 0$ the branching ratio $R \rightarrow 0$. For sufficiently large values of m_σ , the branching ratio is proportional to $1/m_\sigma^4$. Thus the branching ratio will become negligible for large values of m_σ . For intermediate values of m_σ , the branching ratio will depend on the reaction mechanism and the other parameters of the theory, i.e. the size parameter m_A and the hard-core radius.

Before we discuss the importance of the various reaction mechanisms, some comments should be made. We notice first of all that the basic transition operator is proportional to Δ/m_σ (time component) or $\hat{r}$ (space component). Thus in the evaluation of the elementary amplitude associated with (Δ^{++}, n) and (π^+, π^-) , the leading contribution tends to be of order m_μ/m_σ . Indeed the time component is, as mentioned above, of this order, and in the space component the leading contribution comes from the dipole term of the multipole expansion associated with the positron plane wave [see Eqs. (23) and (26)]. This is not the case for the reaction mechanism involving only nucleons. Owing to the summation over all final states, the dependence on $\hat{r}$ and the positron wave function has dropped at least in the approximation (v) above. It thus appears that the importance of the (Δ^{++}, n) and (π^+, π^-) mechanisms may be smaller than if we were considering transitions to the ground state of the final nucleus.

We must emphasize, however, that the range of the effective operator associated with the (Δ^{++}, n) and (π^+, π^-) reaction mechanisms is set by m_π . On the other hand, the range of the operator associated with the mode which involves only nucleons is set by $m_R = \min(m_A, m_\sigma)$. Since $m_R > m_\pi$, one expects that the damping

effect of the two-nucleon short-range correlations will be stronger in the mode involving only nucleons. Since the above two effects tend in opposite directions, one will have to examine the relevant parameters in more detail to see which mechanism will actually dominate.

For point-like hadrons (nucleons, Δ^{++} , pions) and sufficiently high Majorana mass, i.e. $m_R = m_O$, the (Δ^{++}, n) and (π^+, π^-) will undoubtedly dominate, and fairly large branching ratios are expected with the maximum attained at about $m_O = m_W = 80 \text{ GeV}/c^2$. In the more realistic case with $m_A < \infty$ (i.e. the interacting hadrons have size), the branching ratio is proportional to $(m_A/m_O)^4$ and it will thus be suppressed for fairly heavy Majorana neutrinos (even if $m_O < m_W$). Since typically $m_A = 0.85 \text{ GeV}/c^2$, the range of the operator associated with the mode involving only nucleons is only six times shorter than the corresponding one for (Δ^{++}, n) and (π^+, π^-) , and it has a good chance of dominating the latter.

Before we conclude this section, we mention again that one more step is required in order to be able to compute the branching ratio. This concerns the determination of the combination $(\beta\gamma)^2$ of the two mixing angles β and γ . Short of a complete theory which will shed light on the Higgs mechanism, which is believed to be responsible for such mixings and which will unambiguously determine these angles, they should be treated like free parameters which one day (hopefully) will be determined from experiment. Although so far the determination of these parameters is not complete, we will not elaborate further here. The interested reader is referred to Ref. 9 and to references found therein.

Just as an illustration and as a guide to the experimentalists working in this field, we will give numerical results based on the determination of $(\beta\gamma)^2$ cited above, namely

$$(\beta\gamma)^2 = \begin{cases} 10^{-3} & m_O \leq 0.5 \text{ GeV}/c^2 \\ 3 \times 10^{-3} & m_O > 0.5 \text{ GeV}/c^2 \end{cases}$$

8. CONCLUSIONS

Numerical results obtained in a way described in the previous sections are presented in Tables 2, 3, and 4 for various typical values of the parameters appearing in our theory, namely $(\beta\gamma)^2$, m_A (or R_0), r_c , and m_σ .

Recent gauge theories and grand unification schemes²⁵⁾ tend to favour very large masses for the Majorana neutrinos. Cosmological considerations²⁶⁾ also lead to a definite lower bound for such masses, i.e. $m_\sigma > 9 \text{ GeV}/c^2$. Hence in our calculations we included not only small but also large values of m_σ .

For large values of m_σ the branching ratio becomes exceedingly small regardless of the other parameters of the theory (see Tables 2, 3, and 4 for values of m_σ up to 10^5 GeV). For lighter Majorana neutrinos, various situations can be distinguished.

For point-like hadrons, i.e. $R_0 = 0$ or equivalently $m_A = \infty$, the branching ratios can be as large as 10^{-13} . The maximum is attained around $m_\sigma \approx m_W = 80 \text{ GeV}/c^2$, and for $r_c = 0.4 \text{ fm}$ the process is dominated by the (Δ^{++}, n) mode (see Table 3). Both in the quark model and in the E.P., the (Δ^{++}, n) mode is stronger than the (π^+, π^-) mode. [The (π^+, π^-) mode is comparable to that of the (Δ^{++}, n) in the E.P. However, it should be remembered that in this treatment the (Δ^{++}, n) amplitude may be somewhat underestimated because we ignored the N^{*+} in Fig. 2 and the momentum-dependent terms in Eq. (35)]. The suppression of the mode involving only nucleons depends on the value of m_σ . Even for relatively light Majorana neutrinos, e.g. $m_\sigma = 2 \text{ GeV}/c^2$, we get a branching ratio $R \approx 10^{-17}$, i.e. four orders of magnitude smaller than that of the (Δ^{++}, n) mode. We also note that for this value of m_σ the branching ratio falls two orders of magnitude in going from $r_c = 0.0 \text{ fm}$ to $r_c = 0.4 \text{ fm}$. The above effects become even more dramatic for heavier Majorana neutrinos.

It is obvious, however, that it is quite unrealistic to take $R_0 = 0$ or $m_A = \infty$. The introduction of a bag radius in the quark treatment or a form factor of the type given by Eq. (29) has a great effect on the branching ratio²⁷⁾. For

reasons explained in the previous section, the branching ratio gets greatly suppressed. The (Δ^{++}, n) and (π^+, π^-) reaction mechanisms now become less important (cf. Tables 2b and 3). With $m_A = 0.85 \text{ GeV}/c^2$ ($R_0 = 0.6 \text{ fm}$), the largest branching ratio is 10^{-17} and is obtained when $m_\sigma \lesssim 1 \text{ GeV}/c^2$. For $m_\sigma = 9 \text{ GeV}/c^2$ (lower limit of the cosmological arguments mentioned above) and $r_c = 0.4 \text{ fm}$, the branching ratio becomes 10^{-21} . Even in this case the branching ratio drops an order of magnitude in going from $r_c = 0.0 \text{ fm}$ to $r_c = 0.4 \text{ fm}$. We should also mention that the branching ratio depends somewhat on the precise shape of the form factor, as it is shown in Table 4, by considering monopole and dipole shapes (with appropriate mass parameters). It will also be a sensitive function of the "size" parameter m_A especially for large values of the Majorana neutrino mass. We mention that both the shape of the form factor and the parameter m_A are determined from relatively low-energy experiments²¹⁾.

Before we conclude, we mention that a small branching ratio of the order of 10^{-17} has been obtained in a quite different approach which introduces Higgs particles rather than Majorana neutrinos¹¹⁾. Since in this treatment the intermediate neutrinos are of the usual type, i.e. massless, the uncertainties associated with the treatment of nuclear physics are minimized (long-range operators are involved). Unfortunately, in this approach one has to live with the fundamental uncertainties regarding the Higgs sector.

It may appear, therefore, that the experimental observation of the exotic (μ^-, e^+) process may be doomed because the expected branching ratios are small²⁸⁻²⁹⁾. However, this should not discourage the experimental efforts in this direction. It is quite possible that the reaction may proceed via another hitherto unknown mechanism. It may also happen that theorists have been conservative in the values of the theoretical parameters adopted. Finally, we must mention that it is the view of many people that, at least when the next generation of high-energy accelerators will be completed, experiments such as proton decay, neutrino and neutron oscillations, (μ^-, e^-) and (μ^-, e^+) conversion, double β -decay, etc., are going to be in the forefront of physics. We hope that such interesting -- but unfortunately difficult -- experiments will be pursued.

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Table 1

The form factors associated with the Δ^+_{WP} vertex at zero momentum taken from Ref. 21. The parameter ξ_1 is defined in the text. The quantity C_4^V is not given in the table since $C_4^V = -C_3^V m_p/m^*$.

Model	$C_3^A(0)$	$C_4^A(0)$	$C_5^A(0)$	$C_3^V(0)$	$C_5^V(0)$	$C_6^V(0)$	ξ_1
Salin	0.0	2.7	0.0	2.05	0.0	0.0	10.5
Adler	0.0	-0.3	1.2	"	"	"	18.8
Bijtebier	0.0	3.0	1.2	"	"	"	9.7
Zucker	1.8	-1.8	1.9	"	"	"	8.9

Table 2a

The total branching ratio for the anomalous $(\bar{\mu}, e^+)$ conversion in a model mediated by a Majorana neutrino with mass m_σ is presented. The reaction mechanism involves only nucleons. The nucleon form factors were set equal to unity ($m_A = \infty$); r_c is the hard-core radius and β, γ are the mixing angles. The value of $\langle E_e \rangle \approx 85$ MeV was used. The most appropriate branching ratio is associated with $r_c = 0.4$ fm.

$(\beta\gamma)^2$	m_σ (GeV/c ²)	r_c (fm)	0.0	0.1	0.2	0.3	0.4	0.5	0.6
10^{-3}	0.5		4.7×10^{-16}	4.7×10^{-16}	4.3×10^{-16}	3.7×10^{-16}	3.2×10^{-16}	2.6×10^{-16}	2.0×10^{-16}
3×10^{-3}	1.0		2.8×10^{-15}	2.6×10^{-5}	1.9×10^{-15}	1.2×10^{-15}	6.6×10^{-16}	3.5×10^{-16}	1.7×10^{-16}
"	2.0		5.7×10^{-15}	3.8×10^{-15}	1.3×10^{-15}	3.5×10^{-16}	7.8×10^{-17}	1.3×10^{-17}	2.9×10^{-18}
"	5.0		1.4×10^{-14}	1.8×10^{-15}	3.9×10^{-16}	5.5×10^{-19}	6.6×10^{-21}	7.8×10^{-23}	6.6×10^{-25}
"	10		2.8×10^{-14}	7.8×10^{-16}	1.3×10^{-20}	1.3×10^{-24}	1.0×10^{-28}	7.2×10^{-33}	9.0×10^{-37}
"	80		2.3×10^{-13}	1.3×10^{-44}	-	-	-	-	-
"	100		9.0×10^{-14}	-	-	-	-	-	-
"	10^3		9.0×10^{-18}	-	-	-	-	-	-
"	10^4		9.0×10^{-22}	-	-	-	-	-	-
"	10^5		9.0×10^{-26}	-	-	-	-	-	-

Table 2b

For explanations, see the caption of Table 2a.
 $m_A = 0.85$ GeV was considered in this case.

$(\beta\gamma)^2$	r_c (fm) m_σ (GeV/c ²)	0.0	0.1	0.2	0.3	0.4	0.5
10^{-3}	0.50	1.8×10^{-16}	1.4×10^{-16}	1.1×10^{-16}	7.2×10^{-17}	4.2×10^{-17}	3.4×10^{-17}
3×10^{-3}	0.85	2.4×10^{-16}	1.6×10^{-16}	1.2×10^{-16}	7.8×10^{-17}	4.7×10^{-17}	2.9×10^{-17}
"	1.0	1.6×10^{-5}	9.6×10^{-17}	6.6×10^{-17}	4.0×10^{-17}	2.3×10^{-17}	1.2×10^{-17}
"	2.0	1.5×10^{-17}	1.1×10^{-17}	6.6×10^{-18}	3.7×10^{-18}	1.8×10^{-18}	7.8×10^{-19}
"	5.0	5.4×10^{-19}	3.4×10^{-19}	2.1×10^{-19}	1.0×10^{-19}	4.6×10^{-20}	1.9×10^{-20}
"	10.0	3.5×10^{-20}	2.3×10^{-20}	1.3×10^{-20}	6.6×10^{-21}	2.8×10^{-21}	1.1×10^{-21}
"	80	9×10^{-24}	5.7×10^{-24}	3.2×10^{-24}	1.6×10^{-24}	6.6×10^{-24}	2.8×10^{-25}
"	100	3.7×10^{-24}	2.3×10^{-24}	1.3×10^{-24}	6.6×10^{-25}	2.8×10^{-25}	1.1×10^{-25}
"	10^3	3.7×10^{-28}	2.3×10^{-29}	1.3×10^{-28}	6.6×10^{-29}	2.8×10^{-29}	1.1×10^{-29}
"	10^4	3.7×10^{-32}	2.3×10^{-32}	1.3×10^{-32}	6.6×10^{-33}	2.8×10^{-33}	1.1×10^{-33}
"	10^5	3.7×10^{-36}	2.3×10^{-36}	1.3×10^{-36}	6.6×10^{-37}	2.8×10^{-37}	1.1×10^{-37}

Table 3

The branching ratios for the (μ^-, e^+) reaction associated with the graph of Fig. 3a in the quark model description for $r_c = 0.0$ fm, $A = 60$, and $Z = 30$ are presented. The branching ratio associated with Fig. 3c is approximately $1/3$ of the values given in the table if $P_A = 0.04$. For $r_c = 0.4$ fm the branching ratio is reduced by 40%. The branching ratios associated with the graph of Fig. 5 are only 5% of the values given in the table, and they are further reduced by 60% for $r_c = 0.4$ fm. The quantities m_σ and R_0 are described in the text. The most appropriate value for R_0 is 0.6 fm.

$(\beta\gamma)^2$	m_σ (GeV/c ²)	R_0 (fm)	0.0	0.2	0.4	0.6	0.8
10^{-3}	0.5		1.6×10^{-17}	3.8×10^{-18}	10^{-18}	3.0×10^{-19}	7.8×10^{-20}
3×10^{-3}	1.0		2.0×10^{-16}	1.2×10^{-17}	9.6×10^{-19}	1.3×10^{-19}	3.4×10^{-20}
"	2.0		7.8×10^{-6}	3.8×10^{-18}	1.3×10^{-19}	1.2×10^{-27}	9.6×10^{-22}
"	5.0		4.9×10^{-15}	2.2×10^{-19}	10^{-22}	9×10^{-24}	1.5×10^{-44}
"	10		2.0×10^{-44}	4.1×10^{-22}	6.6×10^{-24}	5.6×10^{-25}	10^{-15}
"	80		1.3×10^{-12}	10^{-25}	1.6×10^{-27}	1.5×10^{-28}	2.5×10^{-29}
"	10^2		5.1×10^{-13}	4.0×10^{-26}	6.6×10^{-28}	5.7×10^{-29}	10^{-29}
"	10^3		5.1×10^{-17}	4.0×10^{-30}	6.6×10^{-32}	5.7×10^{-33}	10^{-33}
"	10^4		5.1×10^{-21}	4.0×10^{-34}	6.6×10^{-35}	5.7×10^{-37}	10^{-37}
"	10^5		5.1×10^{-25}	4.0×10^{-35}	6.6×10^{-40}	5.7×10^{-41}	10^{-41}

Table 4

The branching ratios are presented for the (μ^-, e^+) reaction associated with the graph of Fig. 3a in the elementary particle treatment for $A = 60$, $Z = 30$, and $r_c = 0.0$ fm. The branching ratio associated with Fig. 3c is about 30% of the values quoted here if $P_\Delta \approx 0.04$. For $r_c = 0.4$ the branching ratios are decreased by approximately 40%. The branching ratios associated with the (π^+, π^-) mechanism are approximately 50% of the values listed at the table, and they are further reduced by 60% for $r_c = 0.4$ fm. The parameter m_A was defined in the text. For illustrative purposes a monopole form factor was considered.

$(B\gamma)^2$	m_σ (GeV/c) ²	$m_A = \infty$	Dipole form factor $m_A = 0.85$ GeV/c	Monopole form factor $m_A = 0.60$ GeV/c
10^{-3}	0.5	1.6×10^{-18}	6×10^{-22}	1.1×10^{-21}
3×10^{-3}	1.0	5.3×10^{-17}	2.9×10^{-21}	2.5×10^{-20}
"	2.0	5.3×10^{-16}	2.7×10^{-21}	2×10^{-20}
"	5.0	9×10^{-15}	1.3×10^{-23}	1.9×10^{-21}
"	10.0	6.6×10^{-4}	1.1×10^{-24}	2.9×10^{-23}
"	80	1.3×10^{-12}	2.1×10^{-28}	3.1×10^{-25}
"	10^2	1.2×10^{-12}	8.4×10^{-29}	1.3×10^{-26}
"	10^3	1.8×10^{-14}	8.4×10^{-33}	3.1×10^{-30}
"	10^4	1.14×10^{-17}	8.4×10^{-37}	6×10^{-34}
"	10^5	2.9×10^{-21}	8.4×10^{-41}	9×10^{-36}

Figure captions

- Fig. 1 : The Feynman diagram describing (μ^-, e^+) in second-order perturbation theory in a model which involves only nucleons. At the quark level the reaction is also a two-step process given by a similar diagram with $p \rightarrow u$ and $n \rightarrow d$ (u = up quark, d = down quark).
- Fig. 2 : Diagrammatic representation for the process $\mu^- + \Delta^{++} \rightarrow n + e^+$ mediated by the Majorana neutrino as explained in the text. The intermediate hadron is either the proton p or the singly positively charged isobar N^{*+} .
- Fig. 3 : Diagrammatic representation of (μ^-, e^+) conversion in the presence of a virtual Δ^{++} . The full diagram (a) may be split into two parts. The first one (b) yields the probability of finding the Δ^{++} in the nucleus. The second one (c) is nothing but shorthand notation for the diagram of Fig. 2.
- Fig. 4 : Diagrammatic representation for the elementary process $\pi^+ + \mu^- \rightarrow \pi^- + e^+$ mediated by the Majorana neutrino N_0 as explained in the text.
- Fig. 5 : Diagrammatic representation of the (μ^-, e^+) reaction mediated by a π^+ in flight between two nucleons. The box is just shorthand notation for the diagram of Fig. 4.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5