THE COLEMAN-WEINBERG MECHANISM IN EARLY COSMOLOGY

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A B S T R A C T

The $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ phase transition is examined in the case of zero Higgs bare mass. We find that the Universe supercools by a factor 10^{-15} before the transition to the broken phase is complete. We also study the possibility of an intermediate $SU(4) \times U(1)$ phase.

1. INTRODUCTION

Grand unified theories and their cosmological implications in the framework of standard hot big bang cosmology have recently drawn a lot of attention. Among the many problems not well understood is the dynamics of the symmetry breaking phase transition $GUT \rightarrow SU(3) \times SU(2) \times U(1)$. Even in the case of the simplest GUT, the many existing free parameters allow for different scenarios with different possible consequences on the various relevant questions (monopoles, entropy generation, etc.).

It has been suggested that when the symmetry breaking is driven by radiative corrections, i.e., the elementary Higgs scalars have zero bare mass, large gauge hierarchies seem most natural. Although no theory of this type which gives a large hierarchy without some unnatural adjustment in the Higgs sector is known, it is certainly interesting enough to investigate the consequences of the Coleman-Weinberg type of potential¹⁾. It is the purpose of this paper to elucidate the problem of the symmetry breaking phase transition in the case of the Coleman-Weinberg potential. We restrict ourselves to the simplest GUT $SU(5)$ ^{2),3)} but believe that more complicated models would show similar qualitative behaviour.

As it is known from the study of the Coleman-Weinberg mechanism in the Weinberg-Salam phase transition^{4),5),6)}, the scale invariance of the classical potential forces all these transitions to be strongly first order and consequently very slow. In the case of the electroweak phase transition the Universe would supercool to absurdly low temperatures if it were not forced to the broken phase by the dynamical breakdown of chiral symmetry⁴⁾. Transitions of this kind proceed with the formation of bubbles of true vacuum inside an expanding Universe. The standard procedure is to calculate the bubble formation rate carrying over to finite temperature the work of C. Callan and S. Coleman⁷⁾. For a strictly Coleman-Weinberg potential the progress of the phase transition depends entirely on the bubble nucleation rate since the false vacuum never ceases to exist as a local minimum of the potential. This is not the case when the symmetry is broken at the tree level.

In the case of $SU(5)$ we find that the Universe supercools by a factor $\sim 10^{-15}$ before the phase transition to the broken phase $SU(3) \times SU(2) \times U(1)$ is completed. We also examine the possibility that although at zero temperature the parameters are constrained so that $SU(5)$ breaks down to $SU(3) \times SU(2) \times U(1)$, the Universe passes through an intermediate $SU(4) \times U(1)$ phase. This could happen because for a range of the Higgs parameters the supercooled transition to $SU(4) \times U(1)$ is faster than the transition to $SU(3) \times SU(2) \times U(1)$. In this case, extreme supercooling is expected. However, the generic case is directly to $SU(3) \times SU(2) \times U(1)$, contrary to what may happen if the breaking occurs at the tree level^{8),9)}.

As a result of the supercooling, the superheavy magnetic monopoles that come to existence during the development of the phase transition¹⁰⁾ are considerably diluted. Their density is brought well below the limit found by Preskill^{11),12)}.

In Section 2 we discuss the possible symmetry breaking patterns and examine the different phases. In Section 3 we consider the development of the phase transition through the growth of bubbles and compute the transition rates. Finally, in Section 4 we state our conclusions.

2. THE HIGGS POTENTIAL

The one-loop effective potential of the adjoint Higgs Φ is

$$V(\Phi) = \lambda_M \left[\text{Tr}(\Phi^4) - \frac{7}{30} (\text{Tr} \Phi^2)^2 \right] + \lambda_c \left[\frac{13}{20} (\text{Tr} \Phi^2)^2 - \text{Tr}(\Phi^4) \right] + \frac{3}{64\pi^2} \sum_i M_i^4 \ln \left[\frac{M_i^2}{\mu^2} \right] \quad (1)$$

We have neglected the scalar loops, anticipating that the Higgs self-couplings are of order g^4 ³⁾. For $\text{Tr}(\Phi^2)$ fixed, the potential is known to have extrema for the critical orbits

$$\Phi = \phi U^\dagger \text{Diag}(1, 1, 1, -3/2, -3/2) U$$

and

$$\Phi = \hat{\phi} V^\dagger \text{Diag}(1, 1, 1, 1, -4) V$$

where U and V are arbitrary $SU(5)$ transformations and ϕ and $\hat{\phi}$ scalars. The first choice breaks $SU(5)$ down to $SU(3) \times SU(2) \times U(1)$ (3-2-1) and the second down to $SU(4) \times U(1)$ (4-1). The masses of the heavy gauge bosons are correspondingly

$$M_i^2 = \frac{25}{8} g^2 \varphi^2 \quad , i=1 \dots 12$$

and

$$M_i^2 = \frac{25}{2} g^2 \hat{\varphi}^2 \quad , i=1 \dots 8$$

In both cases the potential reads

$$V(\varphi) = \left(\frac{5625}{1024} \right) \frac{C_1 g^4}{\pi^2} \varphi^4 \left(\ln \left(\frac{\varphi^2}{\varphi_0^2} \right) - \frac{1}{2} \right) \quad (2)$$

The constant C_1 is unity in the first case and $32/3$ in the second case. In the second case one has to read $\hat{\phi}(\hat{\phi}_0)$ instead of $\phi(\phi_0)$. $\phi_0(\hat{\phi}_0)$ is the minimum of the $SU(3) \times SU(2) \times U(1)$ [$SU(4) \times U(1)$] potential. The minima are related to the couplings via

$$\varphi_0^2 = \frac{8\mu^2}{25g^2} \exp \left[-\frac{64\pi^2 \lambda_c}{15g^4} + 11/3 \right]$$

$$\hat{\varphi}_0^2 = \frac{2\mu^2}{25g^2} \exp \left[-\frac{128\pi^2 \lambda_M}{45g^4} + 11/3 \right]$$

In order to ensure a deeper minimum in the $SU(3) \times SU(2) \times U(1)$ direction, we must have

$$\sigma \equiv \frac{\hat{\varphi}_0}{\varphi_0} \leq \left(\frac{3}{32} \right)^{1/4} \quad (3)$$

Note that the investigation of the gauge hierarchy problem suggests that³⁾
 $\lambda_c(\mu) \approx 0$ and $\lambda_M(\mu) \approx 0.03$ at $\mu = 2.10^{14}$ GeV. This corresponds to $\sigma \approx 5.10^{-3}$,
a number very close to zero.

At non-zero temperature the potential is modified by the addition of the
free energy of the gauge bosons¹³⁾ (we neglect the contribution of the Higgs
bosons since their masses are smaller). Thus we have

$$V(\varphi, T) = V(\varphi) + \frac{c_2 \cdot 18 T^4}{\pi^2} \int_0^\infty dx x^2 \ln \left\{ \frac{1 - e^{-\sqrt{x^2 + \frac{25g^2 c_3}{8T^2}} \varphi}}{1 - e^{-x}} \right\} \quad (4)$$

The constants appearing in (4) are $c_2 = c_3 = 1$ for $SU(3) \times SU(2) \times U(1)$ and
 $c_2 = 2/3$, $c_3 = 4$ for $SU(4) \times U(1)$. The free energy has been normalized so that
the full potential is zero at the origin. Expressing all the dimensional quan-
tities in units of ϕ_0 , the potential can be written as

$$V(\varphi, T) = \left(\frac{5625}{1024} \right) \left(\frac{g^2}{\pi} \right)^2 \varphi^4 \left(\ln \varphi^2 - \frac{1}{2} \right) + \frac{18 T^4}{\pi^2} \int_0^\infty dx x^2 \ln \left\{ \frac{1 - e^{-\sqrt{x^2 + \frac{25g^2}{8T^2}} \varphi}}{1 - e^{-x}} \right\} \quad (5)$$

in the 3-2-1 direction, and

$$V(\varphi, T) = \left(\frac{1875}{32} \right) \left(\frac{g^4}{\pi} \right) \varphi^4 \left(\ln \left(\frac{\varphi^2}{T^2} \right) - \frac{1}{2} \right) + \frac{12 T^4}{\pi^2} \int_0^\infty dx x^2 \ln \left\{ \frac{1 - e^{-\sqrt{x^2 + \frac{25g^2}{2T^2}} \varphi}}{1 - e^{-x}} \right\} \quad (6)$$

for the 4-1 case.

The minimum at non-zero temperature does not occur at $\phi_0(\hat{\phi}_0)$ anymore.
The local extrema of the potential are plotted in Fig. 1 as a function of the
temperature^{*)}. At very high temperatures there is only one minimum at $\phi = 0$.

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*) All numerical results presented are obtained using $g^2/4\pi = 1/42$ and $M_{x,y} = 6.10^{14}$ GeV.

Below some critical temperature there is a maximum B and two local minima C away from the origin, and A at the origin. This plot is indicative of a strongly first order phase transition¹⁴⁾⁻¹⁶⁾. The barrier (B) ceases to exist only at zero temperature. What happens as the Universe cools down is illustrated by Fig. 2 in terms of the 3-2-1 potential.

There is perhaps a distinction to be made between a Coleman-Weinberg type of potential and potentials for which the symmetry breaking is driven by a negative squared mass term. In the former case, the metastability survives down to zero temperature and the duration of the transition cannot be read from the order parameter plot. One has to calculate the rate at which the bubbles of the broken phase fill the Universe as a function of temperature. The duration of the transition is determined by the temperature at which the rate becomes significant.

Another problem is to know whether the Universe goes directly into the 3-2-1 phase or if it passes through our intermediate 4-1 phase. Even if the 3-2-1 minimum is deeper at $T = 0$ it need not be so at non-zero temperature. In Fig. 3 we have plotted the value of the potential at the minimum as a function of temperature at different values of σ . One can see that for most values of σ the 3-2-1 minimum is indeed deeper and the $SU(3) \times SU(2) \times U(1)$ critical temperature ($T_C = 0.3014$) is higher than the $SU(4) \times U(1)$ one ($\hat{T}_C = 0.6028 \sigma$). For example, with the choices of Ref. 3), the $SU(4) \times U(1)$ critical temperature comes out $\hat{T}_C \approx 0.0028$. Of course, since we are talking about a strongly first order phase transition, what is important is not the critical temperature, but the temperature at which the Universe goes into the broken phase. But again, as the calculation will show, in the relevant range of parameters the transition proceeds faster in the 3-2-1 direction.

3. BUBBLE DYNAMICS

The rate of bubble nucleation is obtained solving the Euler-Lagrange equations for the Euclidean theory with the boundary condition that the fields approach the false vacuum at infinity. The tunnelling probability per unit four-volume is given by⁷⁾

$$\gamma = A e^{-S_4} \quad (7)$$

where S_4 is the Euclidean action corresponding to the tunnelling solution (the "bounce"). A has the dimensions of a mass to the fourth power. Its determination would require taking into account quantum fluctuations. We will take for A some typical mass of the problem to the fourth power [e.g., T_C^4 , ϕ_0^4 or $|V(\phi_0)|^4$].

Since the extrema of the potential lie in the 3-2-1 and 4-1 directions, we have two possible classes of solutions interpreted as describing tunnelling towards these two broken vacua. The solution with the least action usually has the maximum symmetry. At zero temperature this means a solution with $O(4)$ invariance.

At finite temperature we must consider the temperature-dependent potential and impose $1/T$ periodicity in time. The solution of highest symmetry is $O(3)$ invariant (and periodic in time). Finding it seems a formidable task and therefore we have to make approximations. If the space extension of the bubble is far bigger than the period $1/T$, then the solution will be approximately static in imaginary time. Thus, assuming that only static configurations are important, we demand only⁴⁾ $O(3)$ invariance. Up to a fixed gauge transformation, the solution is diagonal for all space time. So we are led to the one dimensional problem

$$\frac{d^2 \varphi}{d\rho^2} + \frac{2}{\rho} \frac{d\varphi}{d\rho} = \frac{1}{C} \frac{\partial V(\varphi, T)}{\partial \varphi} \quad (8)$$

With boundary conditions

$$\lim_{\rho \rightarrow \infty} \varphi(\rho) = 0, \quad \left. \frac{d\varphi}{d\rho} \right|_{\rho=0} = 0 \quad (9)$$

C is $15/2$ for $SU(3) \times SU(2) \times U(1)$ and 20 for $SU(4) \times U(1)$. The action is

$$S_4 = \frac{S_3}{T} = \frac{4\pi}{T} \int_0^\infty d\rho \rho^2 \left[\frac{C}{2} \left(\frac{d\varphi}{d\rho} \right)^2 + V(\varphi, T) \right] \quad (10)$$

This problem can be easily solved numerically. Nevertheless, following E. Witten⁴⁾, one can obtain an explicit solution under the following approximations. First, keep only the first term of the small ϕ expansion of the vector meson free energy, i.e.,

$$(c_2 c_3) \left(\frac{17}{16} \right) \phi^2 T^2$$

Second, replace ϕ in the argument of the logarithm of the potential by some typical value, i.e.,

$$\frac{\phi^2}{\phi_0^2} \sim \frac{T^2}{M_x^2}$$

where M_x is the mass of the superheavy gauge bosons. As the argument of the logarithm is large, we can also neglect the $-1/2$ term. The approximate potential then reads

$$V(\phi, T) \simeq c_1 \left(\frac{5625}{1024} \right) \left(\frac{g^2}{\pi} \right)^2 \phi^4 \ln \left(\frac{8T^2}{25g^2\sigma^2 c_3} \right) + \frac{17}{16} c_2 c_3 g^2 T^2 \phi^2 \quad (11)$$

i.e., a ϕ^4 potential with negative coupling. The action of the three-dimensional bounce associated with this potential is known¹⁷⁾. Substituting the coefficients appearing in Eq. (11) we obtain

$$S_4 = \frac{S_3}{T} = 42.1 \left(\frac{c_2 c_3 c^3}{c_1^2} \right)^{1/2} \left(\frac{\pi^2}{g^3} \right) \left(\ln \left(\frac{25g^2\sigma^2 c_3}{8T^2} \right) \right)^{-1} \quad (12)$$

Our results for the action are plotted in Fig. 4 as a function of the temperature. S_4 goes very slowly to zero as the temperature goes to zero. The action for the tunnelling to 4-1 is a slowly decreasing function of σ . These two facts are related since S_4 is a function of T/σ . One also sees that for most values of σ , S_4 is bigger for the transition to 3-2-1 than for the transition to

4-1, although the minimum in the 3-2-1 direction is deeper. This fact would favour the transition to 4-1. Note finally that Witten's approximation agrees within 10% with the exact results^{*)}. However, this error is exponentiated in the bubble nucleation rate.

The phase transition develops in an expanding Universe. The standard assumption is that the Universe is homogeneous and isotropic and thus described by the Robertson-Walker metric. The expansion is governed by Einstein's equation

$$\left(\frac{\dot{R}}{R}\right)^2 = \left(\frac{8\pi}{3M_P^2}\right)\rho - \frac{K}{R^2} \quad (13)$$

where $K = -1, 0, 1$ corresponds to an open, flat or closed Universe. The energy density is given by

$$\rho = \left(\frac{\pi^2}{30}\right)\eta T^4 + \rho_0 \quad (14)$$

Here η is the effective number of degrees of freedom at $\phi = 0$ [$\eta = 160.75$ in the standard 3 family SU(5) model] and ρ_0 is the vacuum energy, i.e., the minimum value of $|V(3-2-1)|$ at $T = 0$. (Remember that we have normalized the potential to be zero at the false vacuum.)

At very high temperatures, the temperature term in (14) dominates and the Universe is effectively an ideal gas of massless particles. Einstein's equations can be solved for a flat Universe under the assumption of adiabatic expansion and give

$$R(t) \sim t^{1/2} \quad (15)$$

However, when $T \ll T_C$ the vacuum energy density dominates^{**)} and we derive an exponential rate of expansion

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^{*)} The approximation in the temperature dependent part of the potential is responsible for most of the difference from the exact results. The approximation of the logarithm has virtually no effect.

^{**)} The vacuum density is roughly $\rho = 53 (T^4 + 0.06 T_C^4)$.

$$R(t) \sim \exp \left\{ \frac{\sqrt{8\pi\rho_0/3}}{M_P} t \right\} \quad (16)$$

Let us now consider the rate at which bubbles of the true vacuum fill an expanding Universe characterized by Eq. (14). Following Guth and Tye¹²⁾, the probability that a given point will remain in the symmetric phase at temperature T is given by

$$P(T) = \exp \left\{ -d \int_T^{T_c} \frac{dx}{x^4} \frac{\gamma(x)}{g(x)} \left(\int_T^x \frac{dy}{g(y)} \right)^3 \right\} \quad (17)$$

The function g is

$$g(T) = \sqrt{\rho(T)/\rho_0}$$

and the constant

$$d = \frac{3}{16\pi} \left(\frac{M_P^4}{\rho_0^2} \right) \quad (18)$$

We have computed $p(T)$ under the two extreme assumptions that the Universe goes entirely into the 3-2-1 or into the 4-1 phase. Taking for definiteness the factor A in Eq. (7) to be T_c^4 , we have $d(T_c)^4 \simeq e^{39}$ in the first case and $\simeq e^{42+4\ln \sigma}$ in the second case. Concentrating on the first case, the transition will occur roughly at $S_4 \sim \ln(d \cdot T_c^4) \simeq 39$. Hence we anticipate ~ 15 orders of magnitude of supercooling (see Fig. 4).

$p(T)$ is plotted as a function of T in Fig. 5 for the 3-2-1 phase transition. The transition takes place between temperatures of $10^{-15} \phi_0$ and $10^{-16} \phi_0$. Witten's approximation in the determination of S_4 gives temperatures lower by three orders of magnitude. Results for the transition temperature T_S , defined as the temperature for which $p(T_S) \approx 10\%$, are plotted for the 3-2-1 and 4-1 cases in Fig. 6. Our main results can be read on this curve.

i) For $0 < \sigma \lesssim 10^{-2}$, the Universe goes directly into the $SU(3) \times SU(2) \times U(1)$ phase. It supercools for about fifteen orders of magnitude before the transition takes place^{*)}. As already mentioned this range of σ includes the case of Ref. 3) where an analysis of the hierarchy problem indicates that σ is indeed very small.

ii) For $10^{-1} \lesssim \sigma < (3/32)^{1/4}$, the Universe goes first into an $SU(4) \times U(1)$ phase. It will undergo at smaller temperatures a second phase transition to $SU(3) \times SU(2) \times U(1)$. Since this range of values for σ is not theoretically preferred³⁾, the tedious study of this second phase transition is not necessary. A supercooled intermediate phase scenario would certainly lead to much lower final transition temperatures.

iii) Finally for $10^{-2} \lesssim \sigma \lesssim 10^{-1}$ there is a competition between the direct transition $5 \rightarrow 3-2-1$ and the two-step one $5 \rightarrow 4-1 \rightarrow 3-2-1$.

4. CONCLUSIONS

The main result of this analysis is that the Universe would have to supercool down to temperatures of a few hundred MeV where the electroweak phase transition is expected to take place. The influence of the electroweak phase transition may even be essential in the case of large σ . Gravitational effects have also been neglected. However, they may be important close to the unification scale¹⁸⁾.

Putting aside these questions, let us examine some consequences of 15 orders of magnitude of supercooling. Superheavy magnetic monopoles are a common feature in GUTs. Different mechanisms^{12),19),20),21)} have been proposed to bring their present density down to the limit derived by Preskill¹¹⁾.

A supercooled phase transition results in monopole dilution. When the transition takes place at the temperature T_C , the latent heat is released and the Universe is reheated to some temperature T_R comparable to T_C . This dramatically increases the entropy by a factor $(T_R/T_S)^3$ bringing the monopole number to entropy ratio to a value within the bound of Preskill²²⁾.

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^{*)} We disagree with the results of Ref. 21). The disagreement stems from the factor C of Eq. (10) which is absent in Ref. 21).

First order phase transitions present a threat to the baryon number to entropy ratio²³⁾, since the entropy generated might bring the ratio to unacceptably low values. However, barring the intermediate phase scenario, the $SU(5)$ GUT is safe²⁴⁾ as long as the $SU(2) \times U(1) \rightarrow U(1)_{em}$ phase transition does not generate too much entropy.

Finally, we would like to remark that the amount of supercooling found is not enough to support the inflationary scenario proposed by A. Guth²²⁾ for the solution of the horizon and flatness problems. More supercooling induced by the $SU(2) \times U(1) \rightarrow U(1)_{em}$ phase transition would jeopardize the baryon asymmetry.

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FIGURE CAPTIONS

- Fig. 1 : Plot of the maximum (C) and minima (A and B) of the potential in the 3-2-1 and the 4-1 cases. In the second case, the curve is shown for $\sigma = (3/32)^{1/4}$.
- Fig. 2 : Plot of the 3-2-1 potential for various typical temperatures.
- Fig. 3 : Value of the potential at minimum.
- Fig. 4 : The action of the $SU(5) \rightarrow SU(3) \otimes SU(2) \otimes U(1)$ and $SU(4) \otimes U(1)$ bounces. Dashed curves are obtained using Witten's approximation.
- Fig. 5 : $p(T)$ for the $SU(5) \rightarrow SU(3) \otimes SU(2) \otimes U(1)$ transition.
- Fig. 6 : Plot of the transition temperature versus σ . Non-dashed curves correspond to $A = (T_C)^4$ in Eq. (7). Dashed curves are obtained using $A = (\phi_0)^4$ [$(\hat{\phi}_0)^4$ in the 4-1 case]. Curves with $A = |V(\phi_0)|$ lie between the plotted ones. Dotted curves refer to Witten's approximation and $A = (T_C)^4$.

fig. 1

fig. 2

fig. 4

fig. 6