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THE SUPERCURRENT IN $N = 1$ SUPERSYMMETRICAL YANG-MILLS THEORIES

I. The Classical Case

O. Piguet^{*)}

Département de Physique Théorique
Université de Genève

and

K. Sibold^{**)}

CERN - Genève

ABSTRACT

We construct, in the tree approximation, the complete off-shell supercurrent, including Faddeev-Popov fields and gauge contributions for a general supersymmetrical Yang-Mills theory maintaining parity. Due to the conformally non-covariant character of the Faddeev-Popov procedure this supercurrent cannot be improved. The other currents of the super-conformal group can therefore no longer be obtained by simply forming moments.

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^{**) Heisenberg Fellow}

INTRODUCTION

Supersymmetrical theories enjoy a rich variety of structures. Lying "half-way" between internal and geometrical symmetries, supersymmetry induces, via its Ward Identity (WI), not only relations amongst Green's functions - the multiplet structure - but also influences their space-time dependence. Correspondingly, there exist superfields containing the currents of space-time symmetries, together with internal symmetries mediated by supersymmetry transformations: the supercurrents¹⁾. Once a particular one - the supercurrent - is constructed in a model, all other supercurrents can be found as moments and derivatives of it²⁾. Thus, given the supercurrent, the entire symmetry content of the model is known and dealt with in the most concise and elegant fashion one can expect. On the quantized level one has, for example, at one's disposal all anomalies of the respective symmetries and the possible relations amongst them which exist in the model^{2),3)}. This is the key motivation for hunting supercurrents.

In the present (and a subsequent⁴⁾) publication we construct the supercurrent for an arbitrary $N = 1$ supersymmetrical Yang-Mills theory (SYM) with compact, semi-simple gauge group maintaining parity^{1),5)}. The matter fields may lie in any unitary representation of the gauge group. For the quantization we add Faddeev-Popov ($\phi\pi$) fields⁶⁾ and demand BRS-invariance of the supercurrent. We work exclusively with superfields, i.e., supersymmetry is manifest at every stage of the calculation and we stay completely off-shell, i.e., control all off-shell contributions to the respective conservation equations. This is the main novel feature of our approach as opposed to others^{1),7)} and it allows us, for example, to keep track of all anomalies arising with the use of quantized equations of motion.

It turns out that the supercurrent, which is obtained by demanding conservation and BRS-invariance, can never be improved off-shell. Although one can still extract all information on the symmetries of the model from it, this cannot be done in the direct fashion of taking moments.

This paper is specifically devoted to the study of the tree approximation of the theory. In Section 1 the model is defined and placed into the recently found, very general setting of $N = 1$ SYM theories⁸⁾. In Section 2 we derive the contact terms which are the main input for the use of the action principle. In Section 3 we apply the action principle, i.e., we use the equations of motion and decompose the basic insertion into supercurrent and breaking. In Section 4 we discuss this disturbing breaking term and show how the naive dilatational WI can be constructed from it. Computational details of Section 3 are collected in an Appendix.

1. THE ACTION AND THE WARD IDENTITIES DEFINING IT

The most general power counting renormalizable, symmetric, parity-preserving action for $N = 1$ SYM theories with a compact, semi-simple gauge group G and two sets of matter fields is given by^{5),6),8)} (mass terms omitted)

$$\begin{aligned} \Gamma_{\text{YM}} + \Gamma_{\text{matter}} = & \frac{1}{4^2} \text{Tr} \left(\int dS F^\alpha F_\alpha + \text{c.c.} \right) \\ & + \int dV \left(z_1 (\bar{A}_+ e^{\tilde{\phi}} A_+ + A_- e^{-\tilde{\phi}} \bar{A}_-) + z_2 \text{Tr} e^{-\phi} \bar{A} e^{\phi} A \right) \\ & + \text{Tr} \left(\int dS A^3 + \text{c.c.} \right) \end{aligned} \quad (1.1)$$

Here: $F_\alpha \equiv \overline{D\bar{D}}(e^{-\phi} D_\alpha e^{\phi})$

$\phi \equiv \phi^i \tau^i$, τ^i generators in the fundamental representation of G

$\tilde{\phi} \equiv \tilde{\phi}^i T^i$, T^i generators in an arbitrary unitary representation of G

ϕ^i pseudoscalar, vector superfield of dimension zero in the adjoint representation of G

$A_\pm \equiv (A_{\pm a})$, $A_{\pm a}$, A^i chiral superfields of dimension one

$A \equiv A^i \tau^i$

$\bar{A}$ denotes the Hermitean conjugate of A .

This choice of matter field representation covers all cases of interest. Note that on the matter fields, indices referring to an additional group commuting with G and supersymmetry are suppressed. A cubic term of the type $A_+ A_+ A_+$ has also been suppressed by invoking a discrete symmetry, $A_+ \rightarrow -A_+$.

For quantization one has to add gauge fixing and $\phi\pi$ terms^{6),8)}:

$$\Gamma_{\phi\pi} = \frac{2}{\alpha} \text{Tr} \int dV \left(\partial\partial\phi \bar{\partial}\bar{\phi} - \partial\partial c_- Q - \bar{\partial}\bar{\partial} \bar{c}_- \bar{Q} \right) \quad (1.2)$$

where

$$c_\pm \equiv c_\pm^i \tau^i$$

are the $\phi\pi$ (chiral super-) fields

$$Q \equiv R\{c_+\} - \bar{R}\{\bar{c}_+\} \quad (1.3)$$

$$\begin{aligned}
\bar{R}'\{\Lambda\} &\equiv \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} [\phi, [\dots, [\phi, \Lambda] \dots]] \\
\bar{R}'\{\bar{\Lambda}\} &\equiv \sum_{n=0}^{\infty} \frac{1}{(2n+1)!} [\phi, [\dots, [\phi, \bar{\Lambda}] \dots]] \\
R\{\Lambda\} &\equiv \sum_{n=0}^{\infty} a_n [\phi, [\dots, [\phi, \Lambda] \dots]] \\
\bar{R}\{\bar{\Lambda}\} &\equiv \sum_{n=0}^{\infty} (-1)^n a_n [\phi, [\dots, [\phi, \bar{\Lambda}] \dots]]
\end{aligned}
\tag{1.3} \text{ cont.}$$

and the a_n are determined by

$$R\{\bar{R}'\{\Lambda\}\} = \Lambda$$

(In fact, $a_n = 0$ for $n = 3, 5, \dots$). We have chosen a linear gauge function⁸⁾ and thus naive invariance of $\Gamma_{YM} + \Gamma_{\text{matter}} + \Gamma_{\phi\pi}$ under the following BRS-transformation:

$$\begin{aligned}
\Delta \phi &= Q & \Delta A_+ &= -\tilde{c}_+ A_+ \\
\Delta c_- &= \overline{\mathcal{D}} \phi & \Delta A_- &= A_- \tilde{c}_+ \\
\Delta c_+ &= -c_+ c_+ & \Delta A &= [A, c_+] \\
\tilde{c}_+ &\equiv c_+^i T^i
\end{aligned}
\tag{1.4}$$

In order to deal with the non-linearities of the BRS-transformations we introduce external fields coupled to the variations

$$\Gamma_{\text{ext}} = T_\pi \int dV_P Q - \left(\int dS (T_- \tilde{c}_+ c_+ + YP) + \text{c.c.} \right)$$

$$\begin{aligned}
YP &\equiv Y_- \tilde{c}_+ A_+ + A_- \tilde{c}_+ Y_+ && \text{general representation} \\
&+ T_\pi Y[c_+, A] && \text{adjoint representation}
\end{aligned}$$

and obtain the general non-linear Slavnov identity^{6),8)}

$$\begin{aligned} \Lambda(\Gamma) \equiv & \int dV \left(\frac{\delta \Gamma}{\delta \rho} \cdot \frac{\delta \Gamma}{\delta \phi} + \phi \left(\frac{\delta \Gamma}{\delta c_-} + \frac{\delta \Gamma}{\delta \bar{c}_-} \right) \right) \\ & + \left(\int dS \left(\frac{\delta \Gamma}{\delta \gamma} \cdot \frac{\delta \Gamma}{\delta c_+} + \frac{\delta \Gamma}{\delta \gamma} \cdot \frac{\delta \Gamma}{\delta A} \right) + \text{c.c.} \right) = 0 \end{aligned} \quad (1.5)$$

Here

$$\int dS \frac{\delta \Gamma}{\delta \gamma} \cdot \frac{\delta \Gamma}{\delta A}$$

stands somewhat symbolically for:

$$\int dS \left(\frac{\delta \Gamma}{\delta \gamma} \cdot \frac{\delta \Gamma}{\delta A} \right) + \frac{\delta \Gamma}{\delta \gamma_-} \cdot \frac{\delta \Gamma}{\delta A_+} + \frac{\delta \Gamma}{\delta \gamma_+} \cdot \frac{\delta \Gamma}{\delta A_-}$$

Since even at the quantized level the following ghost equations of motion hold⁸⁾,

$$\frac{\delta \Gamma}{\delta c_-} = - \frac{2}{\alpha} \bar{\partial} \partial \partial \frac{\delta \Gamma}{\delta \rho} \quad (1.6)$$

$$\frac{\delta \Gamma}{\delta \bar{c}_-} = - \frac{2}{\alpha} \partial \partial \bar{\partial} \frac{\delta \Gamma}{\delta \rho}$$

Γ will depend on c_- only through the combination $\eta = \rho - \frac{2}{\alpha}(\partial \bar{\partial} c_- + \bar{\partial} \partial \bar{c}_-)$. It will prove to be convenient to have both sets of variables at one's disposal and we shall distinguish the corresponding functionals also by the notation:

$$\hat{\Gamma}(\rho, c_-, \phi, \dots) = \Gamma(\eta(\rho, c_-), \phi, \dots) = \Gamma(\eta, \phi, \dots) \quad (1.7)$$

The transformation from one set of variables to the other is given by:

$$\frac{\delta \hat{\Gamma}}{\delta \rho} = \frac{\delta \Gamma}{\delta \eta} \quad (1.8)$$

$$\begin{aligned}
\frac{\delta \hat{\Gamma}}{\delta c_-} &= \frac{\delta \Gamma}{\delta c_-} - \frac{2}{\alpha} \overline{D} D \overline{D} D \frac{\delta \Gamma}{\delta \eta} \\
\partial_\alpha \hat{\Gamma} &= \partial_\alpha \Gamma + \frac{2}{\alpha^2} (\overline{D} D c_- + \overline{D} D \bar{c}_-) \frac{\delta \Gamma}{\delta \eta}
\end{aligned}
\tag{1.8}$$

cont.

From the Slavnov identity (1.5) the following set of BRS-transformations for insertions can be deduced:

$$\begin{aligned}
b\phi &= \frac{\delta \hat{\Gamma}}{\delta \rho} & b \frac{\delta \hat{\Gamma}}{\delta \rho} &= 0 \\
b c_- &= \overline{D} D \phi & b^2 c_- &= \overline{D} D \frac{\delta \hat{\Gamma}}{\delta \rho} \\
b c_+ &= \frac{\delta \Gamma}{\delta z} & b \frac{\delta \Gamma}{\delta z} &= 0 \\
b \rho &= \frac{\delta \Gamma}{\delta \phi} & b \frac{\delta \Gamma}{\delta \phi} &= 0 \\
b z &= \frac{\delta \Gamma}{\delta c_+} & b \frac{\delta \Gamma}{\delta c_+} &= 0 \\
b \gamma &= \frac{\delta \Gamma}{\delta A} & b \frac{\delta \Gamma}{\delta A} &= 0
\end{aligned}
\tag{1.9}$$

One notes that $b^2 F = 0$ for any functional F independent of c_- . For the corresponding set of transformations involving η one has

$$\begin{aligned}
b\phi &= \frac{\delta \Gamma}{\delta \eta} & b \frac{\delta \Gamma}{\delta \eta} &= 0 \\
b \eta &= \frac{\delta \bar{\Gamma}}{\delta \phi}
\end{aligned}
\tag{1.10}$$

Here $\bar{\Gamma} = \Gamma - \frac{2}{\alpha} \int dV \overline{D} D \phi \overline{D} D \phi$. All other rules remain the same.

Let us remark that the variation Q in Eqs (1.3), (1.4) and consequently the above Q covariant F_α is not the most general solution of the Slavnov identity (1.5). We have shown in Ref. 8) that there exists an infinity of other variations Q (and Q covariants F_α) which also solve (1.5). But fortunately, all of these are seen to be equivalent to the above by a redefinition of the fields and a change of the gauge fixing (cf. Appendix A of Ref. 8) I). So we can stick to the above "standard" formulation.

For the definition of the model we have, so far, used supersymmetry, BRS-invariance and parity

$$\begin{array}{ll}
 \mathcal{P}: & \theta_\alpha \leftrightarrow \bar{\theta}^{\dot{\alpha}} & D_\alpha \leftrightarrow \bar{D}^{\dot{\alpha}} \\
 & \phi \rightarrow -\phi & A_\pm \leftrightarrow \bar{A}_\mp \quad A \leftrightarrow \bar{A} \\
 & c_+ \leftrightarrow \bar{c}_+ & c_- \leftrightarrow -\bar{c}_- \\
 & \rho \rightarrow -\rho & Z \leftrightarrow \bar{Z} \\
 & \gamma_\pm \leftrightarrow \bar{\gamma}_\mp & \gamma \leftrightarrow \bar{\gamma}
 \end{array} \tag{1.11}$$

But the complete action

$$\Gamma \equiv \Gamma_{\text{gh}} + \Gamma_{\text{matter}} + \Gamma_{\text{gh}} + \Gamma_{\text{ext}} \tag{1.12}$$

turns out to be also C invariant:

$$\begin{array}{ll}
 \mathcal{C}: & \phi \rightarrow -\phi^T & A \rightarrow A^T & \bar{A} \rightarrow \bar{A}^T \\
 & \tilde{\phi} \rightarrow -\tilde{S} \tilde{\phi}^T S & A_+ \rightarrow \tilde{S} A_- & \bar{A}_+ \rightarrow \bar{A}_- S \\
 & & A_- \rightarrow A_+ S & \bar{A}_- \rightarrow \tilde{S} \bar{A}_+ \\
 & c_\pm \rightarrow -c_\pm^T & \bar{c}_\pm \rightarrow -\bar{c}_\pm^T & \\
 & \rho \rightarrow -\rho^T & Z \rightarrow -Z^T & \\
 & \gamma \rightarrow \gamma^T & \gamma_+ \rightarrow \tilde{S} \gamma_- & \bar{\gamma}_+ \rightarrow \bar{\gamma}_- S \\
 & & \gamma_- \rightarrow \gamma_+ S & \bar{\gamma}_- \rightarrow \tilde{S} \bar{\gamma}_+
 \end{array} \tag{1.13}$$

(S is a similarity transformation in the representation space of the T^i s) and only softly broken under R transformations and dilatations provided R weights and dimensions are suitably chosen. Our choice is collected in the table. It is such that R transformations and dilatations commute with the BRS-transformations. It is important to note that one cannot assign dimensions and R weights to c_+ and c_- such that the super-conformal relation $d = -3/2n$ holds for both.

Table 1

	θ	$\bar{\theta}$	D	$\bar{D}$	ϕ	A	c_+	c_-	ρ, η	Z	Y
d	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	0	1	0	1	2	3	2
n	-1	+1	+1	-1	0	$-\frac{2}{3}$	0	-2	0	-2	$-\frac{4}{3}$
$Q_{\phi\pi}$	0	0	0	0	0	0	+1	-1	-1	-2	-1

(indices suppressed, $Q_{\phi\pi} \equiv \phi\pi$ charge)

For completeness we write the field variations for supersymmetry, R and D:

$$\begin{aligned}
 \delta_\alpha^\theta \Phi &= \left(\frac{\partial}{\partial \theta^\alpha} + i \sigma_{\alpha\dot{\alpha}}^\mu \bar{\theta}^{\dot{\alpha}} \partial_\mu \right) \Phi \\
 \delta_{\dot{\alpha}}^{\bar{\theta}} \Phi &= \left(-\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} - i \theta^\mu \sigma_{\mu\dot{\alpha}}^\nu \partial_\nu \right) \Phi \\
 \delta^R \Phi &= i(n + \theta \partial_\theta + \bar{\theta} \partial_{\bar{\theta}}) \Phi \\
 \delta^D \Phi &= (d + x \partial_x + \frac{1}{2} \theta \partial_\theta - \frac{1}{2} \bar{\theta} \partial_{\bar{\theta}}) \Phi
 \end{aligned} \tag{1.14}$$

(Φ : general superfield, n: its R weight, d: its dimension). Let

$$W^A \equiv -i \int d^4z \delta^A \Phi \frac{\delta}{\delta \Phi} \quad A = Q, \bar{Q}, R, D$$

represent the WI operator acting on Γ (the generating functional of one-particle irreducible Green's functions, and in the tree approximation, the action), then the WIs in the tree approximation are summarized by

$$W^A \Gamma \simeq 0 \quad A = Q, \bar{Q}, R, D \tag{1.15}$$

The $\simeq$ sign indicates that the right-hand side is soft, i.e., given by the symmetry variation of the mass terms.

2. THE CONTACT TERMS

Let us first outline the general argument for the construction of the supercurrent in any $N = 1$ supersymmetric model^{2),3),9)}. One starts from the x independent superfield

$$\hat{W} \equiv W^R - i \theta^\alpha W_\alpha^Q + i \bar{\theta}_{\dot{\alpha}} W^{\dot{\alpha}\dot{Q}} - 2 \theta \sigma^\mu \bar{\theta} W_\mu^P \tag{2.1}$$

(W_μ^P is the translation WI operator) and the WIs (1.14) comprised into

$$\hat{W} \Gamma \simeq 0 \quad (2.2)$$

If the superfield w denotes the space-time integrand of $\hat{W}$ one clearly has

$$w \Gamma \simeq \partial^\mu V'_\mu \cdot \Gamma \quad (2.3)$$

where V'_μ can be chosen to be an axial vector superfield since $w\Gamma$ is odd under parity. Suppose, as will be the case, that w can be decomposed

$$w = \mathcal{D}^\alpha u_\alpha - \bar{\mathcal{D}}_{\dot{\alpha}} \bar{u}^{\dot{\alpha}} \quad (2.4)$$

then there follows

$$-2 u_\alpha \Gamma \simeq (\bar{\mathcal{D}}^{\dot{\alpha}} V'_{\alpha\dot{\alpha}} - \mathcal{B}_\alpha) \cdot \Gamma \quad (2.5)$$

with

$$\mathcal{D}^\alpha \mathcal{B}_\alpha - \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\mathcal{B}}^{\dot{\alpha}} \simeq 0 \quad (2.6)$$

One notes that (2.5) is the definition of B_α since both $w_\alpha \Gamma$ and $\bar{\mathcal{D}}^{\dot{\alpha}} V'_{\alpha\dot{\alpha}}$ are known. Due to the locality of the insertions, (2.6) can be reduced to

$$\mathcal{B}_\alpha \simeq \bar{\mathcal{D}} \mathcal{D}_\alpha \mathcal{B} - \mathcal{B}'_\alpha \quad (2.7)$$

where $\bar{\mathcal{B}} = \mathcal{B}$ and

$$\mathcal{D}^\alpha \mathcal{B}'_\alpha = \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\mathcal{B}}'^{\dot{\alpha}} \simeq 0 \quad (2.8)$$

V' or

$$V_{\alpha\dot{\alpha}} = V'_{\alpha\dot{\alpha}} - \frac{2}{3} [\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}] \mathcal{B} \quad (2.9)$$

will then be true supercurrents, i.e., contain the R current, the supersymmetry current and the energy-momentum tensor if, and only if, $B'_\alpha = 0$ (cf. Appendix C of Ref. 2)).

For the actual construction of the supercurrent along these lines one starts with the appropriate contact terms w_α , applies w_α to Γ (i.e., uses the field equations)

$$\frac{1}{2} \psi_\alpha \Gamma = \Delta_\alpha \mathcal{P} \quad (2.10)$$

and then tries to decompose Δ_α into $\bar{D}^{\dot{\alpha}} V_{\alpha\dot{\alpha}}$ and B_α .

In the context of a gauge theory observables have to be gauge invariant; that means the supercurrent - containing the energy-momentum tensor - better be gauge invariant. In the non-Abelian case gauge invariance is replaced by BRS-invariance, so we will require BRS-invariance for the supercurrent. This requirement guides the choice of possible contact terms w_α to which we now turn.

For a chiral field φ with R weight n , the requirement that for the superfield w_α

$$\int d^4x (\mathcal{D}^\alpha \psi_\alpha - \bar{\mathcal{D}}_{\dot{\alpha}} \bar{\psi}^{\dot{\alpha}}) = \hat{W} \quad (2.11)$$

yields because of dimensional reasons uniquely

$$\psi_\alpha = n \mathcal{D}_\alpha \left(\varphi \frac{\delta}{\delta \varphi} \right) + 2 \mathcal{D}_\alpha \varphi \frac{\delta}{\delta \varphi} \quad (2.12)$$

This is most easily checked by looking at the $\theta = \bar{\theta} = 0$ component of $\int d^4x \mathcal{D}^\alpha w_\alpha$ and using the formula

$$\int d^4x \mathcal{D}^\alpha \Psi = \frac{1}{2} \int d^2S \theta^\alpha \Psi + \frac{1}{2} \theta^\alpha \int d^2S \Psi \quad (2.13)$$

Recall

$$W^R = -i \int d^2S i(n + \theta \partial_\theta) \varphi \frac{\delta}{\delta \varphi} - i \int d^2\bar{S} i(-n + \bar{\theta} \partial_{\bar{\theta}}) \bar{\varphi} \frac{\delta}{\delta \bar{\varphi}}$$

So w_α is fixed for all chiral fields²⁾. But now $[W^R, w_\alpha] = -w_\alpha$, i.e., $w_\alpha \Gamma$ has a well-defined R weight, as it has dimension. Then, for a vector field Φ there are only ten candidates contributing to w_α , namely all independent terms made out of

$$\bar{\mathcal{D}}_{\dot{\alpha}} \dots \bar{\mathcal{D}}^{\dot{\alpha}} \dots \mathcal{D}_\alpha \dots \Phi \dots \frac{\delta}{\delta \Phi}$$

e.g.,

$$\bar{\mathcal{D}} \mathcal{D} \mathcal{D}_\alpha \Phi \frac{\delta}{\delta \Phi}, \quad \bar{\mathcal{D}} \mathcal{D} \mathcal{D}_\alpha \frac{\delta}{\delta \Phi} \quad \text{etc.}$$

Imposing (2.11) yields five relations among them. The remaining arbitrariness for the contact terms of ϕ and ρ will be removed by requiring

$$[w_\alpha, \Delta] = 0 \quad (2.14)$$

This represents the above postulate that the supercurrent be BRS-invariant.

In fact, one first checks that all contributions coming from matter fields and the sources of their variations cancel each other in (2.14); the same is true for c_+ , $\bar{c}_+$, Z , $\bar{Z}$. The c_- dependent part of w_α interferes in (2.14) with the terms arising from ϕ and ρ and finally, fixes all allowed contributions from ϕ and ρ . The result reads as follows

$$\begin{aligned} \frac{1}{2} w_\alpha &= \text{Tr} \left[\phi \bar{\mathcal{D}} \mathcal{D} \partial_\alpha \delta \phi - \bar{\mathcal{D}} \phi \mathcal{D} \partial_\alpha \delta \phi - \bar{\mathcal{D}} \mathcal{D} \partial_\alpha \delta \rho + \mathcal{D} \delta \bar{\mathcal{D}} \bar{\rho} \right. \\ &\quad \left. + \mathcal{D}_\alpha c_+ \delta c_+ - Z \mathcal{D}_\alpha \delta Z - c_- \mathcal{D}_\alpha \delta c_- \right] + \frac{1}{2} w_\alpha^{\text{matter}} \\ \frac{1}{2} w_\alpha^{\text{matter}} &= -\frac{1}{3} \mathcal{D}_\alpha (A_+ \delta A_+) + \mathcal{D}_\alpha A_+ \delta A_+ + \frac{1}{3} \mathcal{D}_\alpha (\gamma_+ \delta \gamma_+) - \gamma_+ \mathcal{D}_\alpha \delta \gamma_+ \\ &\quad + (+ \rightarrow -) \\ &\quad + \text{Tr} \left[-\frac{1}{3} \mathcal{D}_\alpha (A \delta A) + \mathcal{D}_\alpha A \delta A + \frac{1}{3} \mathcal{D}_\alpha (\gamma \delta \gamma) - \gamma \mathcal{D}_\alpha \delta \gamma \right] \end{aligned} \quad (2.15)$$

(where $\delta_\phi \equiv \frac{\delta}{\delta \phi}$).

Comparing with Ref. 3) we see that the contact terms of ϕ differ from the ones used there. Since, furthermore, for c_- the canonical relation $d = -3/2n$ coming from the super-conformal group²⁾ is not maintained, one expects trouble when these contact terms are used to express those of the dilatations obtained in the canonical fashion. Let us investigate this point. Supercurrents obtained up to now¹⁾⁻³⁾ were improved currents. Defining the dilatation current as a moment

$$\hat{\mathcal{D}}_\mu = x^\nu \hat{T}_{\mu\nu} \quad (2.16)$$

$$\hat{T}_{\mu\nu} = V_{\mu\nu} + V_{\nu\mu} - 2\eta_{\mu\nu} V_\lambda{}^\lambda \quad (2.17)$$

$$V_{\mu\nu} = [\mathcal{D}^\rho, \bar{\mathcal{D}}^\rho] V_{\alpha\dot{\alpha}} \sigma_{\mu\rho\dot{\rho}} \sigma_\nu{}^{\alpha\dot{\alpha}} \quad (2.18)$$

leads to

$$\partial^\mu \hat{\mathcal{D}}_\mu = \hat{T}_\mu{}^\mu + x^\nu w_\nu^P \quad (2.19)$$

or

$$\hat{W}^D = \int dx w^D \quad (2.20)$$

with $w^D = w_{Trace}^P + x^\nu w_\nu^P$

$$w_{Trace}^P = \frac{3}{2} i (\mathcal{D}^\alpha u_\alpha + \overline{\mathcal{D}}_{\dot{\alpha}} \bar{u}^{\dot{\alpha}})$$

$$\begin{aligned} w_\nu^P = \frac{1}{16} \bigg\{ & - (\mathcal{D}\mathcal{D}\overline{\mathcal{D}}_{\dot{\alpha}} u_\alpha + \overline{\mathcal{D}}\mathcal{D}\mathcal{D}_\alpha \bar{u}_{\dot{\alpha}}) \sigma_\nu^{\alpha\dot{\alpha}} \\ & - \sigma_\nu^{\dot{\alpha}\alpha} [\mathcal{D}_\mu \overline{\mathcal{D}}_{\dot{\mu}}] (\mathcal{D}^\alpha u_\alpha - \overline{\mathcal{D}}_{\dot{\alpha}} \bar{u}^{\dot{\alpha}}) \\ & + 8 i \mathcal{D}_\nu (\mathcal{D}^\alpha u_\alpha + \overline{\mathcal{D}}_{\dot{\alpha}} \bar{u}^{\dot{\alpha}}) \bigg\} \end{aligned}$$

This $\hat{W}^D$ was the WI operator associated with naive dimensions (or at the renormalized level even with anomalous dimensions). In our case this is not true. Explicitly computing (2.20) with (2.15) yields

$$"W^D" = W^D(d(\phi)=2, d(\psi)=0, d(c_-)=3) \quad (2.21)$$

i.e., a dilatation WI operator but not with the dimensions assigned as in the table [$d(c_-)=3$ checks, of course, with $d = -3/2n$, $n = -2!$]. This means that although the contact terms (2.15) result in the correct contact terms for R, supersymmetry and translations, they do not give, via the moment construction, the canonical dilatation contact terms. We will come back to this question in connection with the corresponding consequences for the currents.

3. THE SUPERCURRENT

For the actual calculation of the supercurrent we need the equations of motion. They are given by⁸⁾:

$$\begin{aligned}
\frac{\delta \Gamma}{\delta \phi} \approx & -\frac{4}{g^2} \bar{R}' \{ D(\Psi F \bar{\Psi}) \} + \frac{2}{\alpha} L \phi + Q' \\
& + z_1 \tau^i (\bar{A}_+ e^{\tilde{\phi}} \bar{R}' \{ T^i \} A_+ + A_- e^{-\tilde{\phi}} \bar{R}' \{ T^i \} \bar{A}_-) \\
& + z_2 \bar{R}' \{ -\bar{A} \Psi A \bar{\Psi} + \Psi A \bar{\Psi} \bar{A} \}
\end{aligned}$$

$$\frac{\delta \Gamma}{\delta \phi} \equiv \frac{\delta \bar{\Gamma}}{\delta \phi} + \frac{2}{\alpha} L \phi$$

$$Q' \equiv \sum_{n=0}^{\infty} a_n \sum_{k=1}^n [(-1)^k \{ \phi^{n-k}(c_+), \phi^{k-1}(z) \} - (-1)^{n-k} \{ \phi^{n-k}(\tilde{c}_+), \phi^{k-1}(z) \}]$$

$$L \equiv \{ \mathcal{D}\mathcal{D}, \overline{\mathcal{D}\mathcal{D}} \}$$

$$\frac{\delta \hat{\Gamma}}{\delta c_-} \approx -\frac{2}{\alpha} \overline{\mathcal{D}\mathcal{D}} \mathcal{D}\mathcal{D} Q \quad (3.1)$$

$$\frac{\delta \Gamma}{\delta c_+} \approx [z_1 c_+] - \overline{\mathcal{D}\mathcal{D}} \bar{R}' \{ z \} + \gamma_- T A_+ - A_- T \gamma_+ - [\gamma, A]$$

$$\frac{\delta \Gamma}{\delta A_+} \approx z_1 \overline{\mathcal{D}\mathcal{D}} (\bar{A}_+ e^{\tilde{\phi}}) - \gamma_- \tilde{c}_+$$

$$\frac{\delta \Gamma}{\delta A_-} \approx z_1 \overline{\mathcal{D}\mathcal{D}} (e^{-\tilde{\phi}} \bar{A}_-) - \tilde{c}_+ \gamma_+$$

$$\frac{\delta \Gamma}{\delta A} \approx z_2 \overline{\mathcal{D}\mathcal{D}} (\bar{\Psi}' \bar{A} \Psi) + 3L (A^2 - \frac{1}{N} T_A A^2) - \{\gamma, c_+\}$$

$$\frac{\delta \hat{\Gamma}}{\delta S} = Q$$

$$\frac{\delta \Gamma}{\delta \bar{c}} = -c_+$$

$$\frac{\delta \Gamma}{\delta \gamma_+} = A_- \tilde{c}_+$$

$$\frac{\delta \Gamma}{\delta \gamma_-} = -\tilde{c}_+ A_+$$

$$\frac{\delta \Gamma}{\delta \gamma} = [A, c_+]$$

where

$$\psi \equiv e^\phi \quad \bar{\psi} \equiv e^{-\phi}$$

$$\gamma_- T A_+ \equiv \tau^i \gamma_- T^i A_+$$

N = number of generators of G

We now compute

$$\frac{1}{2} u_\alpha \Gamma \simeq \Delta_\alpha \equiv \sum_{k=1}^6 \Delta_\alpha^k$$

$$\Delta_\alpha^1 = \phi \bar{\psi} \mathcal{D}_\alpha \frac{\delta \bar{\Gamma}}{\delta \phi} - \bar{\psi} \mathcal{D}_\alpha \gamma Q$$

$$\Delta_\alpha^2 = -\bar{\psi} \phi \mathcal{D}_\alpha \frac{\delta \bar{\Gamma}}{\delta \phi} + \mathcal{D}_\alpha \gamma \bar{\psi} Q \quad (3.2)$$

$$\begin{aligned} \Delta_\alpha^3 = & -\mathcal{D}_\alpha c_+ \bar{\psi} \bar{\tau} \{ \gamma \} - \gamma_- \mathcal{D}_\alpha \tilde{c}_+ A_+ - A_- \mathcal{D}_\alpha \tilde{c}_+ \gamma_+ \\ & - \mathcal{D}_\alpha c_+ [\gamma, A] \end{aligned}$$

$$\Delta_\alpha^4 = \frac{2}{\alpha} (c_- \mathcal{D}_\alpha L Q + \mathcal{D}_\alpha \bar{\psi} \bar{c}_- \bar{\psi} Q - \bar{\psi} \phi \mathcal{D}_\alpha L \phi)$$

$$\Delta_{\alpha}^5 = -\frac{z_1}{3} \partial_{\alpha} \bar{\partial} (\bar{\alpha}_+ A_+ + A_- \bar{\alpha}_-) \\ - \frac{z_2}{3} \partial_{\alpha} \bar{\partial} (A \bar{\alpha})$$

$$\Delta_{\alpha}^6 = z_1 (\bar{\partial} \bar{\alpha}_+ \partial_{\alpha} A_+ + \partial_{\alpha} A_- \bar{\partial} \bar{\alpha}_- + \bar{\partial} \bar{\alpha}_+ \tilde{f}_{\alpha} A_+ \\ - A_- \tilde{f}_{\alpha} \bar{\partial} \bar{\alpha}_-) - b (f_{\alpha} (\gamma_- \tau A_+ - A_- \tau \gamma_+)) \\ + z_2 \partial_{\alpha} A \bar{\partial} \bar{\alpha} + \partial_{\alpha} c_+ [\gamma, A]$$

Abbreviations:

$$\alpha \equiv \psi A \bar{\psi} \quad \alpha_+ \equiv \tilde{\psi} A_+ \quad \alpha_- \equiv A_- \tilde{\psi}^{-1} \\ \bar{\alpha} \equiv \bar{\psi} \bar{A} \psi \quad \bar{\alpha}_+ \equiv \bar{A}_+ \tilde{\psi}^{-1} \quad \bar{\alpha}_- \equiv \tilde{\psi}^{-1} \bar{A}_- \\ \tilde{\psi} \equiv e^{\tilde{\phi}} \quad \psi \equiv e^{\phi} \quad f_{\alpha} \equiv \bar{\psi}^{-1} \partial_{\alpha} \psi$$

Note that the variable η has been used and that Tr symbols are omitted in front of Δ_{α}^k .

It is now a matter of detailed calculations, reported in the Appendix, to find that Δ_{α} in fact can be split as desired (namely $B'_{\alpha} = 0$):

$$\bar{\partial}^{\alpha} V'_{\alpha\dot{\alpha}} \simeq -2\alpha_{\alpha} \Gamma + \bar{\partial} \partial_{\alpha} \mathcal{B} \quad (3.3)$$

with

$$\mathcal{B} = 4 \left(\mathcal{B}^1 + \frac{4}{\alpha} \mathcal{B}^2 \right) \\ \mathcal{B}^1 \equiv b(\gamma\phi) = \frac{\delta \bar{\Gamma}}{\delta \phi} \phi - \gamma Q \\ \mathcal{B}^2 \equiv \partial \phi \bar{\partial} \phi + \bar{\partial} Q \bar{c} + \partial Q c \quad (3.4)$$

$$V' = -\frac{16}{g^2} V^{YM} + 8 V^1 + \frac{16}{\alpha} (V^2 + V^3) \\ + 8 V^4 + \frac{8}{3} V^5 - 8 V^6$$

$$V_{\alpha\dot{\alpha}}^{YM} \equiv F_{\alpha} \bar{\Psi}^{\dot{\alpha}} \bar{F}_{\dot{\alpha}} \Psi$$

$$V_{\alpha\dot{\alpha}}^1 \equiv \mathcal{L} (\mathcal{D}_{\alpha} \eta \bar{\mathcal{D}}_{\dot{\alpha}} \phi - \bar{\mathcal{D}}_{\dot{\alpha}} \eta \mathcal{D}_{\alpha} \phi)$$

$$V_{\alpha\dot{\alpha}}^2 \equiv \mathcal{D}_{\alpha} \bar{\mathcal{D}} \phi \bar{\mathcal{D}}_{\dot{\alpha}} \mathcal{D} \phi + \mathcal{D}_{\alpha} \bar{\mathcal{D}} Q \bar{\mathcal{D}}_{\dot{\alpha}} \bar{C} - \bar{\mathcal{D}}_{\dot{\alpha}} \mathcal{D} \mathcal{D} Q \mathcal{D}_{\alpha} C$$

$$V_{\alpha\dot{\alpha}}^3 \equiv \mathcal{D}_{\alpha} \bar{\mathcal{D}}_{\dot{\alpha}} \mathcal{D} \mathcal{D} \phi \bar{\mathcal{D}} \phi - \bar{\mathcal{D}}_{\dot{\alpha}} \mathcal{D}_{\alpha} \bar{\mathcal{D}} \mathcal{D} \phi \mathcal{D} \mathcal{D} \phi + \mathcal{D}_{\alpha} \bar{\mathcal{D}}_{\dot{\alpha}} \mathcal{D} \mathcal{D} Q C \\ - \bar{\mathcal{D}}_{\dot{\alpha}} \mathcal{D}_{\alpha} \bar{\mathcal{D}} \mathcal{D} Q \bar{C} + \bar{\mathcal{D}} \mathcal{D} Q \mathcal{D}_{\alpha} \bar{\mathcal{D}}_{\dot{\alpha}} \bar{C} - \mathcal{D} \mathcal{D} Q \bar{\mathcal{D}}_{\dot{\alpha}} \mathcal{D}_{\alpha} C$$

$$V_{\alpha\dot{\alpha}}^4 \equiv \mathcal{L} \left(\mathcal{R} \{ \bar{\mathcal{R}}' \{ \bar{\mathcal{D}}_{\dot{\alpha}} \phi, \mathcal{D}_{\alpha} \phi \} \} \right)$$

$$V_{\alpha\dot{\alpha}}^5 \equiv z_1 [\mathcal{D}_{\alpha}, \bar{\mathcal{D}}_{\dot{\alpha}}] (\bar{A}_+ \tilde{\Psi} A_+ + A_- \tilde{\Psi}^{-1} \bar{A}_-) \\ + z_2 [\mathcal{D}_{\alpha}, \bar{\mathcal{D}}_{\dot{\alpha}}] (\bar{\Psi}^{\dot{\alpha}} \bar{A} \Psi A)$$

$$V_{\alpha\dot{\alpha}}^6 \equiv z_1 \left(\mathcal{D}_{\alpha} (A_- \tilde{\Psi}^{-1}) \tilde{\Psi} \bar{\mathcal{D}}_{\dot{\alpha}} (\tilde{\Psi}^{-1} \bar{A}_-) \right. \\ \left. + \bar{\mathcal{D}}_{\dot{\alpha}} (\bar{A}_+ \tilde{\Psi}) \tilde{\Psi}^{-1} \mathcal{D}_{\alpha} (\tilde{\Psi} A_+) \right) \\ + z_2 \bar{\Psi}^{\dot{\alpha}} \mathcal{D}_{\alpha} (\Psi A \bar{\Psi}^{\dot{\alpha}}) \Psi \bar{\mathcal{D}}_{\dot{\alpha}} (\bar{\Psi}^{\dot{\alpha}} \bar{A} \Psi)$$

where

$$\bar{\mathcal{R}}'\{A, B\} \equiv \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \sum_{k=1}^n \phi^{k-1} \{[A, \phi^{n-k}\{B\}]\}_{+}$$

(Again Tr symbols are omitted in front of V's and B's.)

To prepare for the final discussion we perform the transformation (2.9) and obtain

$$\bar{\mathcal{D}}^{\dot{\alpha}} V_{\alpha\dot{\alpha}} \simeq -2\psi_{\alpha}\Gamma - 2\partial_{\alpha}S \quad (3.6)$$

with

$$V_{\alpha\dot{\alpha}} = V'_{\alpha\dot{\alpha}} - \frac{2}{3} [\partial_{\alpha}, \bar{\mathcal{D}}_{\dot{\alpha}}] B$$

$$S = \frac{1}{6} \bar{\mathcal{D}}\bar{\mathcal{D}} B = \frac{2}{3} \bar{\mathcal{D}}\text{Tr} \left(\ell(\gamma\phi) + \frac{4}{\alpha} (\mathcal{D}\phi\bar{\mathcal{D}}\phi + \bar{\mathcal{D}}Q\bar{c} + \mathcal{D}Qc) \right)$$

The reason why we prefer this form of the supercurrent trace equation will become clear in paper II⁽⁴⁾ where we treat the renormalization. In the tree approximation one can carry through the subsequent considerations quite as well with V' and Eq. (3.3).

4. DISCUSSION AND CONCLUSIONS

S being non-zero in the trace equation (3.6) indicates that the energy-momentum tensor contained in the supercurrent V is not improved⁽²⁾:

$$\hat{T}_{\mu}{}^{\mu} = -\frac{3}{2} i (\mathcal{D}S + \bar{\mathcal{D}}\bar{S}) \neq 0 \quad (4.1)$$

i.e., taking moments does not lead to the canonical WIs of the superconformal group.

Let us study in detail the case of the dilatations since the renormalized dilatation WI will eventually yield the Callan-Symanzik equation. We define, as above, (2.16)

$$\hat{\mathcal{D}}_{\mu} = x^{\nu} \hat{T}_{\mu\nu}$$

and compute

$${}^{\sim}\hat{W}^D\Gamma = \int dX \hat{T}_\mu^{\sim} = -\frac{3}{2}i \left(\int dS S + \int d\bar{S} \bar{S} \right) \quad (4.2)$$

To reveal the content of the right-hand side of (4.2) we recall from Sect. 3 of Ref. 8, I) that

$$\text{Tr} \int dV \mathcal{L}(\psi, \phi) = \mathcal{H}_\phi \Gamma = (N_\phi - N_\eta + 2\alpha \partial_\alpha) \Gamma(\eta, \dots) \quad (4.3)$$

$$\int dV \mathcal{B}^2 = -2\alpha \partial_\alpha \Gamma + \frac{4}{\alpha} \text{Tr} \int dV (\bar{\mathcal{D}}\mathcal{D}Q \bar{c}_- + \mathcal{D}\bar{\mathcal{D}}Q c_-) \Gamma \quad (4.4)$$

Here $N_{\hat{\Phi}}$ are the counting operators of the field $\hat{\Phi}$. In the variables ρ, c_- these insertions can be rewritten (use (1.8)) as

$$\mathcal{H}_\phi \Gamma = (N_\phi - N_\rho + N_- + 2\alpha \partial_\alpha) \hat{\Gamma}(\rho, c_-, \dots) \quad (4.5)$$

$$\int dV \mathcal{B}^2 = -2\alpha \partial_\alpha \hat{\Gamma}(\rho, c_-, \dots)$$

hence

$$-\frac{3}{2}i \left(\int dS S + \int d\bar{S} \bar{S} \right) = -2i (N_\phi - N_\rho + N_-) \hat{\Gamma} \quad (4.6)$$

Looking back at (2.21) we find that, indeed, we have checked the calculation of the contact terms:

$${}^{\sim}\hat{W}^D\Gamma = -2i (N_\phi - N_\rho + N_-) \hat{\Gamma} \quad (4.7)$$

$${}^{\sim}\hat{W}^D\Gamma = W^D(d(\phi)=2, d(\rho)=0, d(c_-)=3) \quad (2.21)$$

Putting the right-hand side of (4.7) on the left and using (2.21) we recover the naive dilatation WI:

$$\hat{W}^D(d(\phi)=0, d(\rho)=2, d(c_-)=1) \hat{\Gamma} = 0 \quad (4.8)$$

This result shows that the supercurrent (3.6), together with the contact terms, still contains all the information on the symmetry content of the theory, but in a far less direct fashion than in the cases of Refs 2) and 3). To be more precise, off-shell there does not exist a supercurrent which is, at the same time:

- conserved;
- BRS-invariant;
- improved.

The clash originates from the non-conformal nature of the gauge fixing procedure mediated by the $\phi\pi$ field c_- . Since the contact terms are unique, upon imposing R weights and commutation with BRS there is no way out; in SYM theories a supercurrent in the above specified sense exists at most on-shell.

Nevertheless, it is clear that only the completely off-shell treatment allows one to control all the possible anomalies arising in higher orders.

APPENDIX*)

The aim is to decompose Δ_α^k $k = 1, \dots, 6$ given in Eq. (3.2) into the form $\bar{D}^\alpha V_{\alpha\dot{\alpha}} = \bar{D} D_\alpha B$ with V and B BRS-invariant and $V_{\alpha\dot{\alpha}}^P = -V_{\alpha\dot{\alpha}}^\cdot$, $\bar{B} = B$.

As a guide for grouping terms it is useful to express Δ_α as BRS variations wherever possible [they are then automatically invariant; see remark after (1.9)].

$$\Delta_\alpha^1 = b (\bar{D} D \partial_\alpha \chi \phi)$$

$$\Delta_\alpha^2 = -b (\partial_\alpha \chi \bar{D} D \phi)$$

$$\Delta_\alpha^3 = b (-f_\alpha \bar{D} D \bar{\chi} \{ \chi \} + f_\alpha (\chi_- T A_+ - A_- T \chi_+) - f_\alpha [\chi, A])$$

$$\Delta_\alpha^4 = \frac{2}{\alpha} (c_- \partial_\alpha L Q + \partial_\alpha \bar{D} D \bar{c}_- \bar{D} D Q - \bar{D} D \phi \partial_\alpha L \phi)$$

$$b \Delta_\alpha^4 = 0$$

$$\Delta_\alpha^5 = -\frac{1}{3} \partial_\alpha b (\chi_- A_+ + A_- \chi_+) - \frac{1}{3} \partial_\alpha b (\chi A) + b \partial_\alpha A^3$$

$$b \Delta_\alpha^5 = 0$$

$$\Delta_\alpha^6 = z_1 (\bar{D} D \bar{\chi}_+ \partial_\alpha A_+ + \partial_\alpha A_- \bar{D} D \bar{\chi}_- + \bar{D} D \bar{\chi}_+ \tilde{f}_\alpha A_+ - A_- \tilde{f}_\alpha \bar{D} D \bar{\chi}_-) - b (f_\alpha (\chi_- T A_+ - A_- T \chi_+)) + b (\chi \partial_\alpha A) - b \partial_\alpha A^3$$

$$b \Delta_\alpha^6 = 0$$

*) Tr symbols are omitted everywhere.

Furthermore, a - possibly only - partial list of basis elements of B and V is required. Leaving aside, for the moment, matter contributions this is given by:

$$B^1 = b(\gamma\phi)$$

$$B^2 = \partial\partial\phi\bar{\partial}\partial\phi + \bar{\partial}\partial Q\bar{c}_- + \partial\partial Qc_-$$

$$b B^2 = 0$$

(B is characterized by having: $\dim = 2$, R weight $= 0$, $Q_{\phi\pi} = 0$, parity $+$)

$$V_{\alpha\dot{\alpha}}^{\gamma\eta} = F_{\alpha} \bar{\psi} \bar{F}_{\dot{\alpha}} \psi$$

$$\begin{aligned} V_{\alpha\dot{\alpha}}^1 &= \partial_{\alpha} Q \bar{\partial}_{\dot{\alpha}} \gamma - \bar{\partial}_{\dot{\alpha}} Q \partial_{\alpha} \gamma + \partial_{\alpha} \phi \bar{\partial}_{\dot{\alpha}} \frac{\delta \bar{F}}{\delta \phi} - \bar{\partial}_{\dot{\alpha}} \phi \partial_{\alpha} \frac{\delta \bar{F}}{\delta \phi} \\ &= b(\partial_{\alpha} \phi \bar{\partial}_{\dot{\alpha}} \gamma - \bar{\partial}_{\dot{\alpha}} \phi \partial_{\alpha} \gamma) \equiv b \hat{V}_{\alpha\dot{\alpha}}^1 \end{aligned}$$

$$V_{\alpha\dot{\alpha}}^2 = \partial_{\alpha} \bar{\partial}\partial\phi \bar{\partial}_{\dot{\alpha}} \partial\partial\phi + \partial_{\alpha} \bar{\partial}\partial Q \bar{\partial}_{\dot{\alpha}} \bar{c}_- - \bar{\partial}_{\dot{\alpha}} \partial\partial Q \partial_{\alpha} c_-$$

$$b V_{\alpha\dot{\alpha}}^2 = 0$$

$$\begin{aligned} V_{\alpha\dot{\alpha}}^3 &= \partial_{\alpha} \bar{\partial}_{\dot{\alpha}} \partial\partial\phi \bar{\partial}\partial\phi - \bar{\partial}_{\dot{\alpha}} \partial_{\alpha} \bar{\partial}\partial\phi \partial\partial\phi + \partial_{\alpha} \bar{\partial}_{\dot{\alpha}} \partial\partial Q c_- \\ &\quad - \bar{\partial}_{\dot{\alpha}} \partial_{\alpha} \bar{\partial}\partial Q \bar{c}_- + \bar{\partial}\partial Q \partial_{\alpha} \bar{\partial}_{\dot{\alpha}} \bar{c}_- - \partial\partial Q \bar{\partial}_{\dot{\alpha}} \partial_{\alpha} c_- \\ b V_{\alpha\dot{\alpha}}^3 &= 0 \end{aligned}$$

(V : $\dim = 3$, R weight $= 0$, $Q_{\phi\pi} = 0$, parity $-$). These terms are obvious generalizations of non-vanishing Abelian contributions to the supercurrent³⁾ so they are certainly expected to occur.

In the first step we try to establish a relation amongst the non-variations, i.e., Δ_{α}^4 , B^2 , V^2 , V^3 and find indeed:

$$\frac{\alpha}{4} \Delta_{\alpha}^4 = \bar{\partial}\partial \partial_{\alpha} B^2 - \bar{\partial}^{\dot{\alpha}} (V_{\alpha\dot{\alpha}}^2 + V_{\alpha\dot{\alpha}}^3) \quad (A.2)$$

In the second step we link the terms which are variations, i.e., $\Delta^1, \Delta^2, \Delta^3, B^1$ and V^1 and introduce - again in analogy to the Abelian case -

$$C_\alpha \equiv \sum_{k=1}^3 \Delta_\alpha^k - \overline{\mathcal{D}\mathcal{D}} \mathcal{D}_\alpha B^1 + 2 \overline{\mathcal{D}}^{\dot{\alpha}} V_{\alpha\dot{\alpha}}^1 \quad (\text{A.3})$$

With $C_\alpha = \mathcal{L} \hat{C}_\alpha$ we have on the level before variation

$$\hat{C}_\alpha = -\overline{\mathcal{D}\mathcal{D}} \mathcal{D}_\alpha \eta + \mathcal{D}_\alpha \phi \overline{\mathcal{D}\mathcal{D}} \eta - \overline{\psi}^{\dot{\alpha}} \mathcal{D}_\alpha \psi \overline{\mathcal{D}\mathcal{D}} \overline{\mathcal{R}} \{ \eta \} \quad (\text{A.4})$$

(Abelian limit: $\hat{C}_\alpha \rightarrow -\overline{\mathcal{D}\mathcal{D}} \mathcal{D}_\alpha \phi \eta$, i.e.,

$$\begin{aligned} C_\alpha &\rightarrow -\overline{\mathcal{D}\mathcal{D}} \mathcal{D}_\alpha \phi \frac{\delta \overline{\mathcal{R}}}{\delta \phi} = -\overline{\mathcal{D}\mathcal{D}} \mathcal{D}_\alpha \phi \overline{\mathcal{D}}_{\dot{\alpha}} \mathcal{D}\mathcal{D} \overline{\mathcal{D}}^{\dot{\alpha}} \phi \\ &= -\overline{\mathcal{D}}^{\dot{\alpha}} (F_\alpha \overline{F}_{\dot{\alpha}}) \end{aligned}$$

If we now introduce

$$\begin{aligned} E_\alpha &\equiv \overline{\mathcal{D}}^{\dot{\alpha}} V_{\alpha\dot{\alpha}}^{\gamma\eta} = \mathcal{L} \hat{E}_\alpha \\ \hat{E}_\alpha &\equiv -F_\alpha \overline{\mathcal{R}} \{ \eta \} \end{aligned} \quad (\text{A.5})$$

we can check that

$$\hat{C}_\alpha - \hat{E}_\alpha = -2 \overline{\mathcal{D}}^{\dot{\alpha}} \hat{V}_{\alpha\dot{\alpha}}^4 \quad (\text{A.6})$$

where

$$\hat{V}_{\alpha\dot{\alpha}}^4 \equiv \overline{\mathcal{R}}' \{ \overline{\mathcal{D}}_{\dot{\alpha}} \phi, \mathcal{D}_\alpha \phi \} \overline{\mathcal{R}} \{ \eta \}$$

$V_{\alpha\dot{\alpha}}^{4\bullet} = \mathcal{L} \hat{V}_{\alpha\dot{\alpha}}^{4\bullet}$ is then the last element of the basis of $V_{\alpha\dot{\alpha}}^\bullet$ needed in the tree approximation without matter.

We have thus far obtained the following results:

$$\begin{aligned} \Delta_\alpha &= \frac{4}{3} \overline{\mathcal{D}\mathcal{D}} \mathcal{D}_\alpha B^2 - \frac{4}{\alpha} \overline{\mathcal{D}}^{\dot{\alpha}} (V^2 + V^3)_{\alpha\dot{\alpha}} + \overline{\mathcal{D}\mathcal{D}} \mathcal{D}_\alpha B^1 \\ &\quad - 2 \overline{\mathcal{D}}^{\dot{\alpha}} (V^1 + V^4)_{\alpha\dot{\alpha}} + \overline{\mathcal{D}}^{\dot{\alpha}} V_{\alpha\dot{\alpha}}^{\gamma\eta} \end{aligned} \quad (\text{A.7})$$

Let us now incorporate the matter contributions. To be dealt with are Δ^5 , Δ^6 , and an additional term in Δ^3 . The Abelian limit³⁾, the Wess-Zumino model²⁾, respectively, suggest

$$V_{\alpha\dot{\alpha}}^5 = z_1 [\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}] (A_- \bar{\mathcal{D}}_- + \bar{A}_+ \mathcal{D}_+) \quad (A.8)$$

$$+ z_2 [\mathcal{D}_\alpha, \bar{\mathcal{D}}_{\dot{\alpha}}] (\bar{\Psi}' \bar{A} \Psi A)$$

$$V_{\alpha\dot{\alpha}}^6 = z_1 \mathcal{D}_\alpha (A_- \tilde{\Psi}^{-1}) \tilde{\Psi} \bar{\mathcal{D}}_{\dot{\alpha}} (\tilde{\Psi}^{-1} \bar{A}_-) + z_1 \bar{\mathcal{D}}_{\dot{\alpha}} (\bar{A}_+ \tilde{\Psi}) \tilde{\Psi}^{-1} \mathcal{D}_\alpha (\tilde{\Psi} A_+) \quad (A.9)$$

$$+ z_2 \tilde{\Psi}' \mathcal{D}_\alpha (\Psi A \bar{\Psi}') \Psi \bar{\mathcal{D}}_{\dot{\alpha}} (\bar{\Psi}' \bar{A} \Psi)$$

as further basis elements of the current and actual calculation of $\bar{\mathcal{D}}_{\alpha\dot{\alpha}}^{\dot{\alpha}}$ shows that

$$\frac{2}{3} \bar{\mathcal{D}}^{\dot{\alpha}} V_{\alpha\dot{\alpha}}^5 - 2 \bar{\mathcal{D}}^{\dot{\alpha}} V_{\alpha\dot{\alpha}}^6$$

accounts for the matter contributions Δ^5 , Δ^6 and in Δ^3 . Collecting the results we have

$$\bar{\mathcal{D}}^{\dot{\alpha}} V'_{\alpha\dot{\alpha}} = -2 \kappa_\alpha \Gamma + \mathcal{D} \bar{\mathcal{D}} \mathcal{D}_\alpha \mathcal{B} \quad (A.10)$$

$$\mathcal{B} = 4(\mathcal{B}^1 + \frac{4}{\alpha} \mathcal{B}^2)$$

$$V' = -\frac{16}{g^2} V^{\gamma\eta} + \frac{16}{\alpha} (V^2 + V^3) + 8(V^1 + V^4)$$

$$+ \frac{8}{3} V^5 - 8 V^6$$

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