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GHOST INTERACTIONS FROM AUXILIARY FIELDS IN $d = 11$ SUPERGRAVITY

A. Van Proeyen ^{*)}
CERN - Geneva

A B S T R A C T

By introducing auxiliary boson fields in 11-dimensional supergravity we can close the commutator linearly on the gravitino. Although the algebra is not closed on the boson fields, the auxiliary fields replace the four-ghost interactions.

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^{*)} Aangesteld navorsers, N.F.W.O., Belgium.

1. - INTRODUCTION

Since the work of Cremmer, Julia and Scherk ¹⁾ it has been known that $N = 1$ supergravity in 11 dimensions provides an elegant way of studying $N = 8$ supergravity in four dimensions. In 11 dimensions the physical fields are the elfbein $e_{a\mu}$, the gravitino ψ_μ^α and an antisymmetric three-index gauge field $A_{\mu\nu\rho}$. The algebra of the supersymmetry transformations does not close on ψ_μ^α . Such a non-closure can present problems for the quantization of the theory. For $N = 1$ ²⁾ and $N = 2$ ³⁾ in four dimensions this has been circumvented by introducing auxiliary fields which close the algebra. However, it is not yet clear whether this is possible for higher N and for the $N = 1, d = 11$ theory. Although some theorems indicate that one needs at least non-trivial central charges ⁴⁾, recent results ^{5),6)} make it very likely that a formulation with auxiliary fields is possible.

In Section 2 of this paper we will show how the introduction of two antisymmetric three-index, and two antisymmetric four-index boson fields in the transformation law of ψ_μ can close the $\{Q, Q\}$ anticommutator linear on ψ_μ^α . One free parameter allows an extra scalar field in the transformation of ψ_μ , but this field is not necessary. After the introduction of these fields the commutator does not close on the gauge field $A_{\mu\nu\rho}$ and on the newly introduced auxiliaries.

In Section 3, we show that terms quadratic in the boson fields can be introduced in the linearized Lagrangian to re-establish its invariance. This is sufficient to prevent four-ghost terms. We will demonstrate the consistency by showing that elimination of the auxiliary fields produces some four-ghost terms such as those obtained from the non-closure function.

2. - CLOSURE OF THE ALGEBRA ON THE ELFBEIN AND ON THE GRAVITINO

The on-shell transformations of 11-dimensional supergravity are, in our notations,

$$\begin{aligned}\delta e_\mu^m &= \frac{\kappa}{2} \bar{\epsilon} \gamma^m \psi_\mu \\ \delta \psi_\mu &= \kappa^{-1} D_\mu(\hat{\omega}) \epsilon + \frac{\sqrt{2}}{288} (\Gamma^{\alpha\beta\gamma\delta}{}_\mu - 8 \delta_\mu^\alpha \Gamma^{\beta\gamma\delta}) \epsilon \hat{F}_{\alpha\beta\gamma\delta} \\ \delta A_{\mu\nu\rho} &= -\frac{\sqrt{2}}{8} \bar{\epsilon} \Gamma_{[\mu\nu} \psi_{\rho]}\end{aligned}\tag{2.1}$$

where $[ ]$ indicates antisymmetrization so that the final weight is 1 ($[\mu\nu] = \frac{1}{2}\mu\nu - \frac{1}{2}\nu\mu$). In the same way, the Γ matrices are the antisymmetrized products with weight 1 of the γ matrices ($\Gamma^{\mu\nu} = \frac{1}{2}(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$), $\hat{\omega}$ is the same expression as in four dimensions and $\hat{F}$ has weight 24 ($\hat{F}_{\alpha\beta\gamma\delta} = 24 \partial[\alpha A_{\beta\gamma\delta}] + \text{covariantization}$). The commutator of two supersymmetry transformations then gives

$$\begin{aligned} [\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] = & \delta_G\left(\frac{1}{2}\bar{\epsilon}_1\gamma^\nu\epsilon_2\right) + \delta_Z\left(-\frac{\sqrt{2}}{24}\bar{\epsilon}_1\Gamma_{\mu\nu}\epsilon_2\right) \\ & + \delta_L\left(\frac{\sqrt{2}}{288}\bar{\epsilon}_1\left(\Gamma^{\mu\nu\alpha\beta\gamma\delta} + 24e^{\mu\alpha}e^{\nu\beta}\Gamma^{\gamma\delta}\right)\epsilon_2\hat{F}_{\alpha\beta\gamma\delta}\right) \end{aligned} \quad (2.2)$$

The δ_G indicates a general co-ordinate transformation, but where we include all other terms from a covariant translation ⁷⁾. δ_L indicates Lorentz transformations, and δ_Z are the "central charge" transformations which appear as Maxwell transformations on the "photon" $A_{\mu\nu\rho}$.

$$\delta_Z A_{\mu\nu\rho} = 3 \partial_{[\mu} \Lambda_{\nu\rho]} \quad (2.3)$$

However, (2.2) is not satisfied on ψ_μ . The non-closure is explicitly calculated in Ref. 5). Symbolically we write

$$[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)]\psi_\mu = (\text{as in (2.2)}) + \eta_{\mu\nu}^J R^\nu \bar{\epsilon}_1 O^J \epsilon_2 \quad (2.4)$$

where $\bar{\epsilon}_1 O^J \epsilon_2$ refer to the structures (see Appendix)

$$\bar{\epsilon}_1 \gamma^\alpha \epsilon_2, \quad \bar{\epsilon}_1 \Gamma^{\alpha\beta} \epsilon_2, \quad \bar{\epsilon}_1 \Gamma^{\alpha\beta\gamma\delta} \epsilon_2 \quad (2.5)$$

To remedy this non-closure we will add terms to the gravitino transformation. But only those terms are allowed which do not disturb the closure on the elfbein as we will accept that the elfbein transformation remains unchanged. Most extra terms in δe_μ^m such as those occurring in the Fradkin-Vasiliev formulation of $N=2$ supergravity ³⁾ can be absorbed in a redefinition of ψ_μ . Then the only allowed terms in the ψ_μ transformation law are those which do not contribute to the commutator on the elfbein or which result in a Lorentz transformation. We then get

$$\begin{aligned} \delta\psi_\mu = & D_\mu(\hat{\omega})\epsilon + \frac{\sqrt{2}}{288}(\Gamma^{\alpha\beta\gamma\delta}\epsilon_\mu - 8\delta_\mu^\alpha\Gamma^{\beta\gamma\delta}\epsilon) \hat{F}_{\alpha\beta\gamma\delta} \\ & - \frac{1}{72}(\gamma_\mu \in M^{(0)} + \gamma_\mu \Gamma^{\alpha\beta\gamma} \in M_{\alpha\beta\gamma}^{(3)} + \gamma_\mu \Gamma^{\alpha\beta\gamma\delta} \in M_{\alpha\beta\gamma\delta}^{(4)} \\ & + \Gamma^{\alpha\beta} \in N_{\mu\alpha\beta}^{(3)} + \Gamma^{\alpha\beta\gamma} \in N_{\mu\alpha\beta\gamma}^{(4)}) \end{aligned} \quad (2.6)$$

All tensors M and N are completely antisymmetric. This results in Lorentz transformations which change (2.2) in

$$\begin{aligned}
 [\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] = & \delta_4 \left(\frac{1}{2} \bar{\epsilon}_2 \gamma^\nu \epsilon_1 \right) + \delta_2 \left(-\frac{\sqrt{2}}{24} \bar{\epsilon}_2 \Gamma_{\mu\nu} \epsilon_1 \right) \\
 & + \delta_1 \left[\frac{\sqrt{2}}{288} \bar{\epsilon}_2 (\Gamma^{mn\alpha\beta\gamma\delta} + 24 e^{m\alpha} e^{n\beta} \Gamma^{\gamma\delta}) \epsilon_1 \hat{F}_{\alpha\beta\gamma\delta} \right. \\
 & - \frac{1}{72} (\bar{\epsilon}_2 \Gamma^{mn} \epsilon_1 M^{(0)} + \bar{\epsilon}_2 \Gamma^{mn} \Gamma^{\alpha\beta\gamma} \epsilon_1 M_{\alpha\beta\gamma}^{(3)} \\
 & \quad + \bar{\epsilon}_2 \Gamma^{mn} \Gamma^{\alpha\beta\gamma\delta} \epsilon_1 M_{\alpha\beta\gamma\delta}^{(4)} \\
 & \quad \left. - 2 \bar{\epsilon}_2 \gamma_\alpha \epsilon_1 N^{(3)mn\alpha} - 3 \bar{\epsilon}_2 \Gamma_{\alpha\beta} \epsilon_1 N^{(4)mn\alpha\beta} \right]
 \end{aligned} \tag{2.7}$$

To close now the commutator on ψ_μ we suppose general expressions for the transformations of the auxiliary fields to R_ν . Symbolically we represent the extra terms in (2.6) by

$$\delta \psi_\mu = \beta_\mu^i \epsilon M^i \tag{2.8}$$

where M^i now stands for all auxiliary fields M and N. Now we take

$$\delta M^i = \bar{\epsilon} \alpha_\nu^i R_\nu \tag{2.9}$$

The arbitrary α_ν^i are parametrized by nine variables : two for each auxiliary field except for the scalar which can only transform as

$$\delta M^{(0)} = \alpha \bar{\epsilon} \gamma \cdot R \tag{2.10}$$

This then has to cancel the non-closure function (2.4)

$$\eta_{\mu\nu}^\gamma R^\nu \bar{\epsilon}_1 \gamma^\gamma \epsilon_2 + (\beta_\mu^i \epsilon_2 \bar{\epsilon}_1 \alpha_\nu^i - (1 \leftrightarrow 2)) R^\nu = 0 \tag{2.11}$$

or

$$(\eta_{\mu\nu}^\gamma)_{\alpha\beta} (\gamma^\gamma)_{\gamma\delta} = [(\beta_\mu^i)_{\alpha\delta} (\gamma^\gamma \alpha_\nu^i)_{\gamma\beta} + (\gamma \leftrightarrow \delta)]$$

For the first expression of (2.5), four tensor structures are possible, the second and the third allow five tensor structures. So if we take just two of the three possibilities for O^J , we have nine or ten equations for the nine parameters. It turns out that a one-parameter solution is possible. But then the remaining four or five equations for the last possibility of O^J turn out to be satisfied

automatically for all values of the parameter. In the light of the δ_Z term of (2.7) one could only impose (2.11) for $O^J = \gamma^\alpha$ and $\Gamma^{\alpha\beta\gamma\delta\epsilon}$. The remaining part with $O^J = \Gamma^{\alpha\beta}$ could then eventually be interpreted as a Z transformation of ψ_μ . But as mentioned we then find that (2.11) is automatically satisfied for $O^J = \Gamma^{\alpha\beta}$. In other words, $\delta_Z \psi_\mu = 0$. The obtained solution is :

$$\begin{aligned} \delta M^{(0)} &= \alpha \bar{\epsilon} \gamma \cdot R \\ \delta M_{\alpha\beta\gamma}^{(3)} &= -\frac{5}{8} \bar{\epsilon} \Gamma_{[\alpha\beta} R_{\gamma]} + \left(\frac{1}{18} + \frac{\alpha}{30}\right) \bar{\epsilon} \Gamma_{\alpha\beta\gamma} \gamma \cdot R \\ \delta M_{\alpha\beta\gamma\delta}^{(4)} &= -\frac{7}{24} \bar{\epsilon} \Gamma_{[\alpha\beta\gamma} R_{\delta]} + \left(\frac{1}{36} - \frac{\alpha}{360}\right) \bar{\epsilon} \Gamma_{\alpha\beta\gamma\delta} \gamma \cdot R \\ \delta N_{\alpha\beta\gamma}^{(3)} &= \frac{27}{4} \bar{\epsilon} \Gamma_{[\alpha\beta} R_{\gamma]} - \frac{5}{8} \bar{\epsilon} \Gamma_{\alpha\beta\gamma} \gamma \cdot R \\ \delta N_{\alpha\beta\gamma\delta}^{(4)} &= 3 \bar{\epsilon} \Gamma_{[\alpha\beta\gamma} R_{\delta]} - \frac{7}{24} \bar{\epsilon} \Gamma_{\alpha\beta\gamma\delta} \gamma \cdot R \end{aligned} \quad (2.12)$$

If we were to choose $\alpha = 0$ then $M^{(0)}$ would not be necessary and could be omitted from both (2.6) and (2.7). In fact, the freedom of α indicates that one can add a spinor λ (replace a $\gamma \cdot R$ by λ) to the transformation laws of the auxiliaries without disturbing the closure on ψ_μ . Higher rank antisymmetric spinors $\lambda_{\alpha\beta}^{(2)}$ and $\lambda_{\alpha\beta\gamma}^{(3)}$ are also possible. Furthermore, we note that a linear combination of $M_{\alpha\beta\gamma\delta}^{(4)}$ and $N_{\alpha\beta\gamma\delta}^{(4)}$ transforms as $\bar{\epsilon} \Gamma_{\alpha\beta\gamma\delta} \gamma \cdot R$, and can therefore be the tensor $T_{\alpha\beta\gamma\delta}$ predicted in Ref. 8).

In conclusion, we have now, with (2.6) and (2.12), included some auxiliary fields in 11-dimensional supergravity. The algebra closes now on the elfbein and on the gravitino, but not on the photon and on the auxiliaries.

3. - LAGRANGIAN AND QUANTIZATION

The general Lagrangian quadratic in the auxiliaries is symbolically

$$\mathcal{L}^* = M^i x^j M^j \quad (3.1)$$

We require that the variation of this cancels the variation of the Rarita-Schwinger Lagrangian. In the notation of (2.8) and (2.9) this is

$$\delta \mathcal{L} = - \bar{R}^\mu \beta_\mu^i \epsilon M^i + 2 M^i x^{ij} \bar{\epsilon} \alpha_\mu^j R^\mu = 0 \quad (3.2)$$

This gives equations for the x^{ij}

$$\beta_\mu^i = 2 x^{ij} (\mathcal{E}^{-1} \alpha_\mu^j \mathcal{E}) \quad (3.3)$$

The solution is

$$\begin{aligned} \mathcal{L}^* = \frac{1}{144} \left[\frac{1}{a} M^{(0)2} + \frac{1}{\Delta_1} (b N^{(3)2} - 2c N^{(3)} \cdot M^{(3)} + d M^{(3)2}) \right. \\ \left. + \frac{1}{\Delta_2} (e N^{(4)2} - 2f N^{(4)} \cdot M^{(4)} + g M^{(4)2}) \right] \quad (3.4) \end{aligned}$$

where a is the arbitrary parameter from (2.10) and (2.12)

$$\begin{aligned} b &= \frac{1}{18} + \frac{a}{90} & e &= \frac{1}{36} - \frac{a}{360} \\ c &= -5/8 & f &= -7/24 \\ d &= 27/4 & g &= 3 \\ \Delta_1 &= bd - c^2 = -\frac{1}{64} + \frac{3a}{40} & \Delta_2 &= eg - f^2 = -\frac{1}{576} - \frac{a}{120} \end{aligned} \quad (3.5)$$

If $a = 0$ the term in $M^{(0)2}$ is undetermined or may be omitted.

Let us repeat some formulae for the quantization ⁹⁾. All fields are represented by ϕ^i , α is the index of symmetries

$$\begin{aligned} \delta(\xi) \phi^i &= R^i_\alpha \xi^\alpha \\ [\delta(\xi_1), \delta(\xi_2)] \phi^i &= (R^i_\alpha \xi_1^\alpha \xi_2^\beta + I^{\alpha\beta}_{ij} \eta^{ij}_{\alpha\beta}) \xi_1^\alpha \xi_2^\beta \\ I(q\mu) &= I^{\alpha\beta} + \frac{1}{2} F_\alpha \delta^{\alpha\beta} F_\beta + \bar{C}^{\alpha\beta} F_{\alpha,i} R^i_\beta C^\beta \\ &\quad - \frac{1}{4} \bar{C}^{\alpha\beta} F_{\alpha,i} \bar{C}^{\gamma\delta} F_{\beta,j} \eta^{ij}_{\gamma\delta} C^\gamma C^\delta (-)^{i+\gamma} + O(C^{\alpha\beta} C^\gamma) \end{aligned} \quad (3.6)$$

where F_α is the gauge fixing and C and C^* are ghosts and antighosts which have opposite statistics to their corresponding transformations.

The ghost Lagrangian is determined by transformations of the gauge fixings. Therefore to obtain the ghost terms, we only have to consider fields which enter in these gauge fixings. For supersymmetry we can take a function of ψ_μ only as

$$F_\alpha = (\gamma \cdot \psi)_\alpha \quad (3.7)$$

Then the transformation of ψ_μ will determine the two-ghost interaction, and the non-closure function of ψ_μ determines the four-ghost interaction. It is known that higher ghost terms do not appear ⁵⁾. But with the auxiliary fields which close the commutator on ψ_μ , we have no non-closure function from fields which enter in gauge fixing terms to field equations from such fields (no non-closure $\psi_\mu \rightarrow R_\mu$). Indeed, the photon enters in the gauge fixing for its Maxwell symmetry, but the non-closure function of the photon involves only auxiliary fields. Therefore

$$F_{\alpha,i} \eta^{\dot{j}} F_{\beta,j} \quad (3.8)$$

is identically zero, which prevents the four-ghost term in (3.6). So with these auxiliary fields we have already only two-ghost terms. The elimination of the auxiliaries reproduces the four-ghost term obtained before. This will now be shown explicitly.

We will rewrite the supersymmetry part of (3.6). We take the F_α of (3.7) and

$$\gamma^{\alpha\beta} = -\frac{1}{2} (\not{C} \not{D})^{\alpha\beta} \quad (3.9)$$

Comparing (2.4) with the second line of (3.6) $(I^{cl})_{(\nu\beta)} = -\bar{R}_{\nu\beta} = -(\bar{R}_\nu^T C)_\beta$

$$\eta^{(\mu\alpha)(\nu\beta)}_{\gamma\delta} = \left(\eta_{\mu\nu}^T \not{C}^{-1} \right)_{\alpha\beta} (\not{C} O^T)_{\delta\gamma} \quad (3.10)$$

Then (3.6) gives

$$\begin{aligned} \mathcal{L}(q\mu) = & \mathcal{L}^d + \frac{1}{4} \bar{\psi} \cdot \gamma \not{C} \gamma \cdot \psi + \bar{C}^* \not{C} C - \frac{\sqrt{2}}{288} \bar{C}^* \Gamma \cdot \hat{F} C + \bar{C}^* \gamma \cdot \beta^i C M^i \\ & - \frac{1}{4} \bar{C}^* \gamma^\mu \eta_{\mu\nu}^T \gamma^\nu C^* \bar{C} O^T C \end{aligned} \quad (3.11)$$

Without the auxiliary fields we have to use the non-closure function of Ref. 5) to obtain the four-ghost term [the term with M^i is then of course absent in (3.11)]

$$\begin{aligned} \mathcal{L}^{(4-gh)} = & -\frac{1}{4608} \left[245 \bar{C}^* \gamma^\alpha C^* \bar{C} \gamma_\alpha C + \frac{43}{2} \bar{C}^* \Gamma^{\alpha\beta} C^* \bar{C} \Gamma_{\alpha\beta} C \right. \\ & \left. + \frac{1}{24} \bar{C}^* \Gamma^{\alpha\beta\gamma\delta\epsilon} C^* \bar{C} \Gamma_{\alpha\beta\gamma\delta\epsilon} C \right] \end{aligned} \quad (3.12)$$

When using the auxiliary fields this four-ghost term is absent, but there is the two-ghost term

$$\begin{aligned} \mathcal{L}^{*(gh)} = & -\frac{1}{72} \left(\bar{C}^* \Gamma^{\alpha\beta\gamma} C N_{\alpha\beta\gamma}^{(3)} + \bar{C}^* \Gamma^{\alpha\beta\gamma\delta} C N_{\alpha\beta\gamma\delta}^{(4)} \right) \\ & - \frac{11}{72} \left(\bar{C}^* C M^{(0)} + \bar{C}^* \Gamma^{\alpha\beta\gamma} C M_{\alpha\beta\gamma}^{(3)} + \bar{C}^* \Gamma^{\alpha\beta\gamma\delta} C M_{\alpha\beta\gamma\delta}^{(4)} \right) \end{aligned} \quad (3.13)$$

If one wants to reconstruct the four-ghost interactions from this formulation, one has to solve the field equations from $\mathcal{L}^* + \mathcal{L}^{*(gh)}$ which gives

$$\begin{aligned} N_{\alpha\beta\gamma}^{(3)} &= -\frac{1}{8} \bar{C}^* \Gamma_{\alpha\beta\gamma} C ; \quad N_{\alpha\beta\gamma\delta}^{(4)} = -\frac{5}{24} \bar{C}^* \Gamma_{\alpha\beta\gamma\delta} C ; \\ M^{(0)} &= 11\alpha \bar{C}^* C ; \quad M_{\alpha\beta\gamma}^{(3)} = \left(-\frac{1}{72} + \frac{11\alpha}{90} \right) \bar{C}^* \Gamma_{\alpha\beta\gamma} C ; \quad M_{\alpha\beta\gamma\delta}^{(4)} = \left(\frac{1}{72} - \frac{11\alpha}{360} \right) \bar{C}^* \Gamma_{\alpha\beta\gamma\delta} C \end{aligned} \quad (3.14)$$

If this is used in $\mathcal{L}^* + \mathcal{L}^{*(gh)}$ then $\mathcal{L}^* = -\frac{1}{2} \mathcal{L}^{*(gh)}$ and we obtain

$$\begin{aligned} \mathcal{L}^{*(4-gh)} &= \frac{1}{2592} \left[5 \left(\bar{C}^* \Gamma_{\alpha\beta\gamma} C \right)^2 + \left(\bar{C}^* \Gamma_{\alpha\beta\gamma\delta} C \right)^2 \right] \\ &\quad - \frac{121\alpha}{144} \left[\left(\bar{C}^* C \right)^2 + \frac{1}{90} \left(\bar{C}^* \Gamma^{\alpha\beta\gamma} C \right)^2 - \frac{1}{360} \left(\bar{C}^* \Gamma_{\alpha\beta\gamma\delta} C \right)^2 \right] \end{aligned} \quad (3.15)$$

After Fierzing, the α terms cancel. The four-ghost term is thus just the first line of (3.15). This can be Fierzed to the form (3.12), showing the consistency of the two methods. This consistency can be proved quite general, once (2.11) allows a solution for α and β . This solution has eventually to satisfy restrictions imposed by other commutators (here by the commutator on the elfbein). Moreover one requires that (3.3) allows a solution for the x^{ij} . This determines the Lagrangian (3.1). Then we suppose a gauge fixing function which depends only on ψ_μ . We obtain

$$\mathcal{L}^{*(ga)} = M^i x^{ij} M^j + \bar{C}^{*\alpha} F_{\alpha,(\mu\beta)} \left(\beta_\mu^i \right)_{\beta\gamma} M^i C^\gamma \quad (3.16)$$

The field equation of the M fields

$$2 x^{ij} M^j + \bar{C}^{*\alpha} F_{\alpha,(\mu\beta)} \left(\beta_\mu^i \right)_{\beta\gamma} C^\gamma = 0 \quad (3.17)$$

implies that the first term of (3.16) is $(-\frac{1}{2}) \times$ the second term. In this second term we then use (3.3) for the β_μ before using the field equation (3.17) for eliminating the M fields [Eq. (3.3) implies that the x^{ij} are symmetric.]

$$\begin{aligned} \mathcal{L}^{*(4gh)} &= -\frac{1}{2} \bar{C}^{*\alpha} F_{\alpha,(\mu\beta)} \bar{C}^{*\delta} F_{\delta,(\nu\epsilon)} (\beta_\nu^j)_{\epsilon\delta} C^5 (C^{-1} \alpha_\mu^T C)_{\beta\delta} C^\gamma \\ &= +\frac{1}{2} \bar{C}^{*\alpha} F_{\alpha,(\mu\beta)} \bar{C}^{*\delta} F_{\delta,(\nu\epsilon)} C_{\beta\gamma}^{-1} (\beta_\nu^j)_{\epsilon\delta} (C \alpha_\mu)_{\delta\gamma} C^5 C^\delta \end{aligned} \quad (3.18)$$

The evident symmetry $\zeta \leftrightarrow \gamma$ allows us to use (2.11). With (3.10) we then obtain

$$\mathcal{L}^{*(4gh)} = -\frac{1}{4} \bar{C}^{*\alpha} F_{\alpha,(\mu\beta)} \bar{C}^{*\delta} F_{\delta,(\nu\epsilon)} \eta^{(\nu\epsilon)(\mu\beta)}_{\delta\gamma} C^5 C^\delta \quad (3.19)$$

The symmetry of the upper indices of the non-closure function can be read off from (3.20) or can be proved in general ⁹⁾. So we obtain the four-ghost term of (3.6). This shows that if auxiliary fields exist which close the commutator on ψ_μ and allow an invariant Lagrangian, the four-ghost term obtained by eliminating them is the same as that obtained by (3.6).

4. - CONCLUSION

Even without a full closure of the gauge algebra, the quantization of 11-dimensional supergravity can be facilitated by introducing some auxiliary fields. We only have to introduce bosonic fields which close the commutator on the gravitino. These extra bosonic fields are included in the action, which makes the four-ghost terms unnecessary. Indeed, the proof of BRST invariance needs only closure on fields which enter in gauge fixing conditions. However, by introducing these bosonic terms, the commutator does not close on the photon $A_{\mu\nu\rho}$ and on the auxiliary fields. Therefore the BRST transformation of the action contains a term $\sim C_2^* M C_Q C_Q$. This can be canceled by BRST transformations of the auxiliaries involving terms with a Z antighost and two supersymmetry ghosts. If these auxiliaries do not enter in gauge fixing conditions then the BRST invariance of the action is already assured.

We found that we need two antisymmetric three-tensors and two antisymmetric four-tensors to obtain such a closure (a scalar can be added). This means that we have already 1165 or 1166 boson degrees of freedom, which is much more than the 768 which enter in Taylor's proposal ⁶⁾ for auxiliary fields of $N = 8$ supergravity. Such a small number is impossible in 11 dimensions also on the fermionic side. Indeed, a pair of vector-spinors, which seems unavoidable, brings the fermionic degrees of freedom already much higher.

APPENDIX - SPINORS IN 11 DIMENSIONS

In 11 dimensions one can introduce a charge conjugation matrix C and Majorana spinors ψ which satisfy

$$\begin{aligned} C \gamma_\mu C^{-1} &= -\gamma_\mu^T & C^T &= -C \\ \bar{\psi} &= \psi^T C \end{aligned} \quad (A.1)$$

The matrices 1 , γ_μ , $\Gamma_{\mu\nu}$, $\Gamma_{\mu\nu\rho}$ and $\Gamma_{\mu\nu\rho\sigma}$ form an independent set.

$$\begin{aligned} \Gamma_{\alpha_1 \dots \alpha_n} &= \frac{i}{(11-n)!} \epsilon_{\alpha_1 \dots \alpha_{11}} \Gamma^{\alpha_{(n+1)} \dots \alpha_{11}} & \text{for } n=0, 1 \bmod 4 \\ \Gamma_{\alpha_1 \dots \alpha_n} &= \frac{-i}{(11-n)!} \epsilon_{\alpha_1 \dots \alpha_{11}} \Gamma^{\alpha_{(n+1)} \dots \alpha_{11}} & \text{for } n=2, 3 \bmod 4 \end{aligned} \quad (A.2)$$

For anticommuting spinors the Fierzing rule reads

$$\bar{\lambda} M \chi \bar{\psi} N \varphi = -\frac{1}{32} \sum_{m=0}^5 \frac{1}{m!} \bar{\psi} \Gamma^{(m)} \chi \bar{\lambda} M \Gamma^{(m)} N \varphi \quad (-)^{m=2,3} \quad (A.3)$$

From (A.1) we find that

$$C \gamma_\mu, C \Gamma_{\mu\nu}, \text{ and } C \Gamma_{\mu\nu\rho\sigma}$$

are symmetric, while

$$C, C \Gamma_{\mu\nu\rho}, \text{ and } C \Gamma_{\mu\nu\rho\sigma}$$

are antisymmetric. Therefore for anticommuting spinors only

$$\bar{\lambda}_1 \gamma_\mu \lambda_2, \bar{\lambda}_1 \Gamma_{\mu\nu} \lambda_2, \text{ and } \bar{\lambda}_1 \Gamma_{\mu\nu\rho\sigma} \lambda_2$$

are antisymmetric in $(1 \leftrightarrow 2)$.

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