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AUXILIARY FIELDS FOR LINEARIZED $N = 4$ POINCARÉ SUPERGRAVITY

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A B S T R A C T

A set of auxiliary fields for linearized $N = 4$ Poincaré supergravity in five dimensions is presented. The Lagrangian is given.

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The $N = 4$ supercurrent in five dimensions has 128+128 components and couples in a natural way to a corresponding multiplet of supergravity fields which we shall call the minimal supergravity multiplet¹⁾. In Ref. 1), it was suggested that a multiplet suitable for the purpose of building a supergravity Lagrangian could be obtained by combining this minimal multiplet with five Abelian Yang-Mills multiplets, the latter having an off-shell central charge²⁾. In this note we show that this is indeed the case, at least at the linearized level.

The fields of the combined multiplet are as follows (we use four-dimensional dimensions and give the $Sp(4)$ representations in brackets):

$$\dim 1: h_{\mu\nu}, A_\mu, A_{\mu\nu}^{ij} (\underline{5}), A, A_{ij,kl} (\underline{14})$$

$$\dim \frac{3}{2}: \psi_\mu (\underline{4}), \lambda_i (\underline{4}), \varphi_{i,jk} (\underline{16})$$

$$\dim 2: B, B_{ij} (\underline{10}), B_{ij,kl} (\underline{14}), S_{ij} (\underline{10}), \\ V_\mu^{ij} (\underline{10}), N_{\mu\nu}^{ij} (\underline{5}),$$

$$\dim \frac{5}{2}: \chi_{i,jk} (\underline{16})$$

$$\dim 3: C_{ij,kl} (\underline{14}) \quad (1)$$

$h_{\mu\nu}$ is the graviton field and A_μ the central charge field, the $A_{\mu\nu}$'s are gauge antisymmetric tensors (describing spin 1 in five dimensions), and the ψ_μ 's are the gravitino fields. The singlet dimension 1 scalar A and the quartet of spin $\frac{1}{2}$ fields λ_i are also physical fields, while the others are all auxiliary. The N 's are non-gauge antisymmetric tensors and the V 's are non-gauge vectors. The total number of off-shell degrees of freedom is 208+208. The symmetry properties of the various $Sp(4)$ representations are as follows:

$$\underline{5}: A_{\mu\nu}^{ij} = A_{\mu\nu}^{[ij]}, \quad \eta_{ij} A_{\mu\nu}^{ij} = 0$$

$$\underline{10}: B_{ij} = B_{[ij]}$$

$$\underline{14}: A_{ij,kl} = A_{[kl][ij]}, \quad \eta^{ij} A_{ij,kl} = \eta^{kl} A_{ij,kl} = 0$$

$$\underline{16}: \varphi_{i,jk} = \varphi_{i[jk]}, \quad \eta^{jk} \varphi_{i,jk} = \eta^{ik} \varphi_{i,jk} = 0$$

(2)

where $\eta_{ij} = -\eta_{ji}$ is the $Sp(4)$ "metric". All spinors obey the $Sp(4)$ Majorana condition:

$$\bar{\lambda}^i{}^\tau = C \eta^{ij} \lambda_j \quad (3)$$

where C is the charge conjugation matrix.

The Lagrangian is

$$\begin{aligned} L = & \frac{1}{4} (\partial_\mu h_{\nu\rho} \partial^\mu h^{\nu\rho} + 2 \partial^\rho h_{\rho\mu} \partial^\mu h_{\nu}{}^\nu - 2 \partial^\mu h_{\mu\nu} \partial^\rho h_\rho{}^\nu - \partial^\mu h_\nu{}^\nu \partial_\mu h_\rho{}^\rho) \\ & - \frac{1}{16} F_{\mu\nu}^2 - \frac{1}{6} (G_{\mu\nu\rho}^{ij})^2 - \frac{5}{4} (\partial_\mu A)^2 + \frac{1}{4} A (\Box h_\mu{}^\mu - \partial^\mu \partial_\nu h_{\mu\nu}) \\ & - \frac{1}{4} (\partial_\mu A_{ij,kl})^2 - \frac{1}{2} \bar{\psi}_\mu^i \gamma^{\mu\nu\rho} \partial_\nu \psi_{\rho i} - 5 \frac{1}{2} \bar{\lambda}^i \not{\partial} \lambda_i + 2i \bar{\lambda}^i \gamma^{\mu\nu} \partial_\mu \psi_{\nu i} \\ & + \frac{1}{2} \bar{\varphi}^{ijk} \not{\partial} \varphi_{ijk} - \frac{5}{4} B^2 - \frac{1}{2} (B_{ij})^2 - \frac{1}{4} (B_{ij,kl})^2 + \frac{1}{2} (S_{ij})^2 - \frac{1}{2} (V_\mu^i)^2 \\ & - (N_{\mu\nu}^{ij})^2 - 2 N_{\mu\nu}^{ij} \tilde{G}_{ij}^{\mu\nu} + 4i \bar{\varphi}^{ijk} \chi_{ijk} + 2 A_{ijkl} C^{ijkl} \end{aligned} \quad (4)$$

where

$$\begin{aligned} F_{\mu\nu} &= 2 \partial_{[\mu} A_{\nu]} \\ G_{\mu\nu\rho}^{ij} &= 3 \partial_{[\mu} A_{\nu\rho]}^{ij} \\ \tilde{G}_{\mu\nu}^{ij} &= \frac{1}{3!} \epsilon_{\mu\nu\rho\sigma\lambda} G^{\rho\sigma\lambda ij} \end{aligned} \quad (5)$$

The corresponding action is invariant under the following rigid supersymmetry transformations:

$$\begin{aligned} \delta h_{\mu\nu} &= -i \bar{\xi}^i \delta_{(\mu} \psi_{\nu)i} \quad , \quad \delta A_\mu = -i \bar{\xi}^i \psi_{\mu i} \\ \delta A_{\mu\nu}^{ij} &= \frac{1}{2} \bar{\xi}^k \delta_{\mu\nu} \varphi_k^{ij} + i [\bar{\xi}^{[i} \delta_{\mu\nu} \lambda^{j]} - 2 \bar{\xi}^{[i} \delta_{\mu} \psi_{\nu]}^{j]} \\ &\quad - \text{trace}] \\ \delta A &= i \bar{\xi}^i \lambda_i \quad , \quad \delta A_{ijkl} = \frac{1}{2} \bar{\xi}_i \varphi_{jkl} + \text{sym.} \end{aligned}$$

$$\delta \psi_{\mu i} = -\frac{1}{4} \omega_{\mu, \nu \rho} \gamma^{\nu \rho} \xi_i + S_{ij} \gamma_{\mu} \xi^j - \frac{1}{2} F_{\mu \nu} \gamma^{\nu} \xi_i \\ - \frac{1}{8} \tilde{F}_{\mu \nu \rho} \gamma^{\nu \rho} \xi_i + \tilde{N}_{\mu \nu \rho ij} \gamma^{\nu \rho} \xi^j + V_{\mu ij} \xi^j$$

$$\delta \lambda_i = \frac{2}{5} B_{ij} \xi^j + \frac{8}{5} S_{ij} \xi^j - \frac{1}{2} B \xi_i + \frac{2}{5} V_{ij}^{\mu} \gamma_{\mu} \xi^j \\ - \frac{1}{2} \partial_{\mu} A \gamma^{\mu} \xi_i - \frac{1}{4} F_{\mu \nu} \gamma^{\mu \nu} \xi_i + \frac{6}{5} N_{\mu \nu ij} \gamma^{\mu \nu} \xi^j + \frac{1}{5} \tilde{q}_{\mu \nu ij} \gamma^{\mu \nu} \xi^j$$

$$\delta \varphi_{ijk} = B_{\ell ijk} \xi^{\ell} + [B_{i \ell j} \xi_{k \ell} - S_{i \ell j} \xi_{k \ell} - \text{traces}] \\ + \partial_{\mu} A_{\ell ijk} \gamma^{\mu} \xi^{\ell} + [V_{i \ell j}^{\mu} \gamma_{\mu} \xi_{k \ell} - \text{traces}] \\ + \frac{1}{2} [(\tilde{q}_{jk}^{\mu \nu} + N_{jk}^{\mu \nu}) \gamma_{\mu \nu} \xi_i - \text{traces}]$$

$$\delta B = i \bar{\xi}^i \not{\partial} \lambda_i + 4i \bar{\xi}^i \Psi_i$$

$$\delta B_{ij} = 2i \bar{\xi}_{(i} \not{\partial} \lambda_{j)} + 8i \bar{\xi}_{(i} \Psi_{j)} - i \bar{\xi}^k \not{\partial} \varphi_{(ij)k} - 4i \bar{\xi}^k \chi_{(ij)k}$$

$$\delta B_{ijk\ell} = i \bar{\xi}_i (\frac{1}{2} \not{\partial} \varphi_{j k \ell} + 2 \chi_{j k \ell}) + \text{sym.}$$

$$\delta S_{ij} = -2i \bar{\xi}_{(i} \Psi_{j)} - 4i \bar{\xi}^k \chi_{(ij)k}$$

$$\delta V_{\mu ij} = -i \bar{\xi}^k \partial_{\mu} \varphi_{(ij)k} + 2i \bar{\xi}_{(i} \partial_{\mu} \lambda_{j)} + 3i \bar{\xi}_{(i} \Psi_{\mu j)} \\ + 2i \bar{\xi}_{(i} \gamma_{\mu} \Psi_{j)} - 4i \bar{\xi}^k \gamma_{\mu} \chi_{(ij)k}$$

$$\delta N_{ij}^{\mu \nu} = -i (\bar{\xi}_{(i} \Psi_{j)}^{\mu \nu} - \bar{\xi}_{[i} \gamma^{[\mu} \Psi_{j]}^{\nu]}) - \text{trace} \\ + i \bar{\xi}^k \chi_{kij}$$

$$\delta \chi_{ijk} = C_{\ell ijk} \xi^{\ell} + \frac{1}{4} (\partial^{\nu} N_{\mu \nu jk} \gamma^{\mu} \xi_i - \text{traces}) \\ + \frac{1}{4} (\partial_{\mu} S_{i \ell j} \gamma^{\mu} \xi_{k \ell} - \text{traces}) + \frac{1}{8} (\partial^{\rho} \tilde{N}_{\mu \nu jk} \gamma^{\mu \nu} \xi_i - \text{traces}) \\ - \frac{1}{4} (\partial_{\mu} V_{\nu i \ell j} \gamma^{\mu \nu} \xi_{k \ell} - \text{traces})$$

$$\delta C_{ijk\ell} = \frac{1}{2} \bar{\xi}_i \not{\partial} \chi_{j k \ell} + \text{sym.}$$

where

$$\begin{aligned} \omega_{\mu, \nu\rho} &= \partial_\nu h_{\rho\mu} - \partial_\rho h_{\nu\mu} \\ 2 \partial_{[\mu} \psi_{\nu]i} &= \bar{\Psi}_{\mu\nu i} + 2 \delta_{[\mu} \bar{\Psi}_{\nu]i} + \delta_{\mu\nu} \bar{\Psi}_i \end{aligned} \quad (7)$$

the Ψ 's being γ traceless. "+ sym" is an attractive piece of notation which indicates that appropriate symmetrizations of the given term are to be added to ensure that the left-hand sides remain 14's. The action is also invariant under rigid central charge transformations:

$$\begin{aligned} \delta A_{\mu\nu}^{\bar{ij}} &= c \left(\tilde{q}_{\mu\nu}^{\bar{ij}} + 2 N_{\mu\nu}^{\bar{ij}} \right) \\ \delta A &= c B, \quad \delta A_{ijkl} = c B_{ijkl} \\ \delta B &= c \left(\square A + \frac{1}{10} \left(\square h_{\mu}{}^{\mu} - \partial^{\mu} \partial^{\nu} h_{\mu\nu} \right) \right) \\ \delta B_{ij} &= c \partial^{\mu} V_{\mu ij}, \quad \delta B_{ijkl} = c \left(\square A_{ijkl} + 4 C_{ijkl} \right) \\ \delta V_{\mu ij} &= c \partial_{\mu} B_{ij} \\ \delta \lambda_i &= c \left(\not{\partial} \lambda_i + 4 \bar{\Psi}_i \right) \\ \delta \varphi_{ijk} &= c \left(\not{\partial} \varphi_{ijk} + 4 \chi_{ijk} \right) \end{aligned} \quad (8)$$

and Abelian gauge transformations

$$\begin{aligned} \delta h_{\mu\nu} &= \partial_{(\mu} \xi_{\nu)} \\ \delta A_{\mu} &= \partial_{\mu} \Lambda \\ \delta A_{\mu\nu}^{\bar{ij}} &= 2 \partial_{[\mu} \Lambda_{\nu]}^{\bar{ij}} \\ \delta \psi_{\mu i} &= \partial_{\mu} \rho_i \end{aligned}$$

(9)

The transformation laws (7) and (8) were obtained from superspace³⁾ so that the algebra is guaranteed to close. We have used the fields that arise most naturally in superspace, but it is straightforward to diagonalize the Lagrangian (4). If one does this, then the redefined graviton and gravitino fields will no longer be inert under central charge transformations.

It remains to construct the non-linear theory. Some results in this direction have already been obtained. The superspace torsion constraints relevant for the minimal multiplet have been derived, and the transformation rules for most of the fields of this multiplet given^{1),3)}. Since there are 15 dimension 1 scalars in the combined multiplet, it is likely that they will appear as elements of the coset space $GL(5, \mathbb{R})/Sp(4)$ so that the Lagrangian is expected to exist in a locally $Sp(4)$ invariant form. It is not expected to be globally $GL(5, \mathbb{R})$ invariant, however, since the term $A_{ijkl} C^{ijkl}$ does not have this symmetry even at the linearized level. We anticipate instead a global $Sp(4) \times \mathbb{R}$ invariance which the Lagrangian without auxiliary fields possesses⁴⁾. We note that the off-shell central charge is unlikely to cause problems as the gauge antisymmetric tensors are Abelian gauge fields. The details of the non-linear theory are being investigated and will be presented in a future report.

Finally, we remark that, after dimensional reduction to four dimensions, the Lagrangian (4) will describe $N = 4$ supergravity coupled to $N = 4$ (Abelian) Yang-Mills. It is not possible, however, to separate the two systems and simultaneously retain off-shell closure because of the presence of the five gauge antisymmetric tensors. It therefore seems that some sets of auxiliary fields require more than one physical multiplet, although whether this result is of any relevance for the $N = 8$ problem is not clear.

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