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A MODEL INDEPENDENT DESCRIPTION OF
THE DEUTERON ASYMPTOTIC D STATE

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A B S T R A C T

The asymptotic deuteron D state is shown to result nearly model-independently from iterated OPEP yielding a predicted value $\eta = (0.02633 \pm \pm 0.00035)$. Alternatively the result leads to a determination of the πN coupling constant $[r^2 = (0.0792 \pm 0.0012)]$. Attention is drawn to the implications for the size of quark bags. Analogous considerations of the deuteron quadrupole moment permit the first direct determination of its meson exchange part from experiment $\delta Q_{\text{exch}} = (0.005 \pm 0.004) \text{ fm}^2$.

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The deuteron D state component is due to the NN tensor force. It has long been known that the one-pion exchange (OPE) is an important part of this force, and therefore important in phenomena concerning the deuteron D state. In fact, already in the mid 50's, Iwadare et al.¹⁾ used this observation to deduce a value $f^2 = 0.08 \pm 0.01$ for the πN coupling constant recognizing that the quadrupole moment was strongly governed by OPE via the D state amplitude.

Recently, very accurate values for the asymptotic D/S ratio η have become experimentally available using different techniques. Sub-Coulomb stripping yields a value (0.02649 ± 0.00043) ²⁾, while tensor polarization in elastic $(\vec{p}, D)$ scattering gives the values (0.0259 ± 0.0007) ³⁾ and in ${}^2H(\vec{d}, p){}^3T$ (0.0268 ± 0.0007) ⁴⁾. These values are consistent with their averaged value $\eta = (0.0264 \pm 0.0004)$.

The purpose of the present paper is to demonstrate that η can be described to high accuracy and nearly model-independently as strongly dominated by iterated OPEP. The level of accuracy is such that the experimental uncertainty in the πN coupling constant is the major source of uncertainty and that the differences in masses and coupling constants of charged and neutral pions must be included. Even the magnetic moment tensor coupling is taken into account.

As a byproduct, the quadrupole moment of the deuteron is so well understood that the meson exchange contribution to it can be directly deduced from experiments for the first time.

The deuteron S and D wave functions are given by the coupled equations:

$$u'' = (\alpha^2 + U_{00})u + U_{02}w \quad (1a)$$

$$w'' = \left(\alpha^2 + \frac{6}{r^2} + U_{22}\right)w + U_{20}u \quad (1b)$$

where $\alpha^2 = M\varepsilon$ and $U = MV$. The potentials V_{00} , V_{20} , V_{02} and V_{22} are linear combinations of the central, tensor, spin orbit and quadratic spin orbit potentials V_C , V_T , V_{LS} and V_{LL} :

$$V_{00} = V_C; \quad V_{02} = V_{20} = \sqrt{8} V_T \quad \text{and} \quad V_{22} = (V_C - 2V_T - 3V_{LS} - 3V_{LL})$$

The asymptotic wave functions for large r are:

$$u(z) \rightarrow N \sqrt{2\alpha} e^{-\alpha z} . \quad (2a)$$

$$w(z) \rightarrow \eta N \sqrt{2\alpha} e^{-\alpha z} \left(1 + \frac{3}{\alpha z} + \frac{3}{(\alpha z)^2} \right) . \quad (2b)$$

The asymptotic S wave normalization is given to high accuracy by effective range theory⁵⁾

$$(1 + \eta^2) N^2 = \frac{1}{(1 - \alpha \rho(-\epsilon, -\epsilon))} = (1.696 \pm 0.004) . \quad (3)$$

and

$$\alpha^{-1} = (4.31767 \pm 0.00009) \text{ fm} .$$

We find it convenient to work with the formal solution to the D wave function as expressed by the $L = 2$ Green function $G_2(r, r')$

$$w(z) = \int_0^\infty G_2(z, z') U_{20}(z') u(z') dz' . \quad (4)$$

Using the exact $L = 2$ wave functions $J_2(x)$ and $\mathcal{H}_2^{(1)}(x)$ which correspond to spherical Bessel functions $j_2(x)$ and $h_2^{(1)}(x)$, the value for η is then immediately expressible in terms of the wave function $J_2(x)$ regular at $x = 0$

$$\eta N \sqrt{2\alpha} = \int_0^\infty J_2(i\alpha z) z U_{20}(z) u(z) dz . \quad (5)$$

The decisive features which make it possible to describe η to high precision are the following.

- A) The S wave function $u(r)$ in the driving term of Eq. (5) is very well known everywhere apart from the numerically unimportant region $r \lesssim 0.7$ fm. The long range region is highly accurate due to the very precise asymptotic normalization. At shorter distances its behaviour is mainly controlled by the condition that total normalization must not exceed unity as well as the necessity for an approximately correct r.m.s. radius. In practice, any wave function with reasonably realistic forces will give accurately similar results for η provided the asymptotic normalization is correct (not always the case⁶⁾). The uncertainties from this source on the final answer are less than 0.3%.

- B) The centrifugal potential dominates the elastic $L = 2$ strength for $r \geq 0.8$ fm and will by itself strongly de-emphasize contributions from the inner region. In addition the potential is repulsive which de-emphasizes the inner region even further.
- C) More than 2/3 of the contributions to the asymptotic normalization of $w(r)$ come from the region $r > 2$ fm with very small contributions from the inner region. Consequently the OPEP contributions are very dominant.

We will now give a sequence of successively better approximations to the asymptotic amplitude, starting from very rough but pedagogically useful approximations. They serve to illustrate the confidence to be given to the final result.

1. - BORN APPROXIMATION

We temporarily assume that the centrifugal barrier is so dominant that the rescattering effect of the potential can entirely be neglected. The asymptotic normalization is then given by the replacement of $J_2(i\alpha r)$ by the spherical Bessel function $j_2(i\alpha r)$ in Eq. (5)

$$\eta_{\text{BORN}} N \sqrt{2\alpha} = \sqrt{8} M \int_0^\infty j_2(i\alpha z) z V_T(z) u(z) dz. \quad (6)$$

Since in the small r region $j_2(i\alpha r) \propto r^2$, the effective integrand (6) is approximately proportional to $[r^3 V_T(r)] u(r)$. The characteristic divergence r^{-3} of a tensor potential at short distances is therefore regularized.

For orientation, we now evaluate Eq. (6) assuming pure OPEP^{*)}, for which the tensor potential is

$$V_T(z) = -f^2 \left(1 + \frac{3}{\mu z} + \frac{3}{\mu^2 z^2} \right) \frac{e^{-\mu z}}{z}. \quad (7)$$

*) In all the following, the OPEP potential used is the average

$$V_\pi = \left(\frac{2}{3} V_\pi^{\text{charged}} + \frac{1}{3} V_\pi^{\text{neutral}} \right)$$

with appropriate masses and coupling constants. The evaluations have been performed with $f_c^2 = f_0^2$ using $f_c^2 = 0.0790$ 7).

1.1. - OPEP with asymptotic form for $u(r)$

Assume first that the S wave function can be everywhere approximated by its asymptotic form $N\sqrt{2\alpha}e^{-\alpha r}$. This is a very poor approximation since $u(r)$ is incorrectly normalized by 60%. The region $0 < r < 1.5$ fm is heavily over-emphasized. Replacing $j_2(i\alpha r)$ in Eq. (6) by its value for small r the integral is exactly

$$\eta_{\text{BORN}}^{\text{asy}} = \frac{\sqrt{8}}{15} M \alpha^2 f^2 \left[\frac{3}{(M+\alpha)} + \frac{3M}{(M+\alpha)^2} + \frac{2M^2}{(M+\alpha)^3} \right] = 0.052.$$

This extremely crude approximation gives nearly twice the experimental value. A closed solution results also from a Hultén-type wave function $u(r) = N\sqrt{2\alpha}[e^{-\alpha r} - e^{-\beta r}]$ with an asymptotically correct form. With $u(r)$ normalized to unity so as to be minimally realistic, the previous value is reduced to 0.036.

1.2. - OPEP with realistic form for $u(r)$

As previously noted, the result is very insensitive to the detailed choice of $u(r)$ provided the asymptotic normalization is correct. We have used the Paris wave function⁸⁾. The integrand according to Eq. (6) is shown in the Figure. One notes in the Figure that the main contributions to the integrand come from large r with only 20% from $r < 1.5$ fm. The inclusion of the repulsive potential in the D state, in particular the iteration of the OPEP, will suppress the inner region even further. The value obtained for η is

$$\eta_{\text{BORN}}^{\text{realistic}} = 0.033.$$

This simple, still rather crude approximation, is quantitatively accurate to 20%.

2. - REALISTIC DESCRIPTIONS WITH RESCATTERING

The exact formal solution for the D state amplitude is given by Eq. (5). While a numerical solution can easily be obtained, this will not give much insight into the dependence of the result on the rescattering potential $U_{22}(r)$. We note, however, that an approximate and analytic solution can be obtained in the WKB limit. This approximation is expected to be good, since the centrifugal barrier as well as the potential are repulsive. In this spirit the integrand can be written as a modification of the Born term

$$\eta_{N\sqrt{2}\alpha} = \sqrt{8} M \int_0^\infty \frac{\exp\left[-\frac{1}{5} \int_\infty^z U_{22}(t) dt\right]}{\left[1 + \frac{4}{25} z^2 U_{22}(z)\right]^{1/4}} z \int_2^{(1\alpha z)} V_T(z) u(z) dz. \quad (8)$$

As compared to the Born approximation (6), the crucial feature of the integrand (8) is that it depends exponentially on the integrated value of $r U_{22}(r)$ from the point r to ∞ . The integrand is thus modified by the cumulative effect of the outer part of the potential. This effect as well as the quenching of the inner region are naturally understood as a barrier penetration effect of a wave coming from $r = \infty$. Consequently, the OPEP with its long range will be dominant.

The accuracy of the WKB expression (8) has been explicitly checked using the exact solutions. Realistically the approximation was found to be good to better than 0.3% for $r > 0.5$ fm. Although the approximation introduces no substantial error, an exact evaluation has been used in the numerical results throughout.

We now evaluate the integrand both for iterated OPEP with central and tensor parts as well as for more general interactions.

2.1. - Iterated OPEP

The dominant contribution to η comes from OPEP and its iteration. Since the OPEP rescattering potential is repulsive, the contributions from the inner region are less than for the pure OPEP as indicated by the WKB expression (8). The integrand of Eq. (5) is given in the Figure. The integrated value is

$$\eta_\pi = 0.02762 \quad (9)$$

which differs from the experimental value 0.0264 by only 4%. The inclusion of the OPEP rescattering is therefore a very substantial improvement on the Born approximation and it is still model-independent. One notes in particular in the Figure that the suppression by the iterated OPEP is about 15% even at 2 fm. The region inside 1 fm is heavily suppressed. In both cases the suppression is caused by the cumulative effect of the potential in the outer region [see Eq. (8)].

2.2. - Effect of short and intermediate range potentials

The result for the iterated OPEP is only weakly modified by the contributions from other multimeson exchanges. To demonstrate this we have investigated the effects of two rather different potentials as examples. The first one is the Reid soft core potential⁹⁾. This potential is purely phenomenological in this region. It has partly unrealistic features. The second example is the Paris potential¹⁰⁾ as a representative of modern potentials. The contributions from intermediate range exchange are here theoretically justified by dispersion relations (particularly for 2π exchange with Δ excitation and ρ). The short-range contributions, on the other hand, remain phenomenological.

The contributions to the integrand (5) by the Paris potential^{*)} as compared to the iterated OPEP are shown in the Figure. The over-all effect is a net decrease of η by 4%. The contributions to this decrease come predominantly from the region between 1 to 2 fm. The main effect comes therefore from the 2π contributions in a region where these are theoretically well understood. We estimate the theoretical uncertainty in these contributions to be less than 10% for $r > 1$ fm and less than 30% for $r < 1$ fm.

The additional contribution to η is therefore

$$\delta\eta = -0.00119 \quad (10)$$

with a theoretical uncertainty of less than 0.00020.

The Reid soft core potential gives results between those of the Paris potential and iterated OPEP. This demonstrates the insensitivity to detailed assumptions about the behaviour in short and intermediate range regions.

2.3. - Magnetic moment and velocity-dependent effects

A small additional contribution to η comes from the electromagnetic tensor force due to interaction of the neutron and proton magnetic moments. This tensor potential is proportional to r^{-3} and has long range. It gives a model-independent decrease of η by 0.000076.

*) Since we are interested in the difference of the potentials we use the same πN coupling constant $f^2 = 0.0790$ in both cases and not $f^2 = 0.0780$ as in the Paris potential. With the latter value we reproduce the result of the Paris group $\eta = 0.02608$.

We note in passing that corrections due to the isobar content of the deuteron are small. These corrections enter mainly as a normalization correction of the wave function. Since $u(r)$ is correctly normalized asymptotically this effect is already implicitly included. Velocity dependent terms in the potential give very small contributions of less than 0.2%.

Collecting the results we conclude that η can be theoretically predicted to better than 1%. Using the πN charged coupling constant of Bugg et al.⁷⁾ $f_c^2 = (0.0790 \pm 0.0010)$ both for charged and neutral pions as well as the weighted mean value for the potential, the predicted value is

$$\eta = 0.02633 + 0.34 (f^2 - 0.0790) \quad (11)$$

The predicted value (11) agrees well with the experimental value $\eta_{\text{exp}} = (0.0264 \pm 0.0004)$.

We may also turn the argument around and use the experimental value of η for a determination of the πN coupling constant. Referring once more to the appropriate charge and neutral average, $\eta_{\text{exp}} = (0.0264 \pm 0.0004)$ gives

$$f^2 = (0.0792 \pm 0.0012) \quad (12)$$

with a theoretical uncertainty of less than ± 0.0008 . While this value is very accurate apparently, we believe some caution should presently be exercised, until sources of systematic experimental uncertainties to η have been re-examined.

The theoretical result (11) for the D/S ratio has indirect consequences for the deuteron quadrupole moment which experimentally is $Q_{\text{exp}} = (0.2816 \pm 0.0015) \text{ fm}^2$. There is in fact a very close link between the value of η and the quadrupole moment in potential models. This relation is particularly obvious in a beautiful recent paper by Klarsfeld et al.¹¹⁾ They assume that the quadrupole moment Q and the r.m.s. radius $\langle r^2 \rangle^{\frac{1}{2}}$ are produced by a potential which in its long range region is OPEP. Applying Schwarz inequalities they demonstrate that a positivity condition gives narrow bounds for η . They conclude from their analysis that $\eta = (0.0271 \pm 0.0007)$, which is consistent with our value (11). The limiting factor in their consideration is the non-potential contribution from meson exchange effects, which are not included.

We are in agreement with their statement that η and Q are closely linked. Our Green function technique permits us to calculate the potential quadrupole moment Q_{pot} straightforwardly in a way that makes it quite clear that its contributions have substantially the same integrand as those contributing to η in Eq. (5) (this analysis will be published elsewhere). The main difference is that in the resulting integrals a model-independent long range contribution has been cut down so that the intermediate and short-ranged contributions to η contribute about 50% more to Q_{pot} . It is now possible to obtain Q_{pot} in two ways. One is to focus on the experimental value of η and ascribe the uncertainty to the short-range region. The uncertainty from f^2 is then small yielding $Q_{\text{pot}} = (0.2815 \pm 0.0040) \text{ fm}^2$. A second way is to calculate Q_{pot} from iterated OPEP with additional contributions from short and intermediate range as in the Paris potential. We then obtain the relation $Q_{\text{pot}} = [0.2815 + 2.9(f^2 - 0.0790)] \text{ fm}^2$. These results include the contribution -0.00044 fm^2 from the magnetic moment interaction.

Since the deuteron quadrupole moment has additional contributions from meson-exchange currents we do not expect a perfect agreement with experiments. We should rather look at the difference between the experimental value and the present model-independent potential value as a direct determination of the meson exchange contribution

$$\Delta Q_{\text{meson}} = (Q_{\text{exp}} - Q_{\text{pot}}) = [0.0045(15) - 2.9(f^2 - 0.0790)] \text{ fm}^2.$$

with the theoretical uncertainty of less than ± 0.0040 . This is the first direct determination of the meson exchange contribution. Its value compares to typical predictions of $+0.0010$ ¹²⁾. The present experimental indications suggest a smaller correction.

Finally, we note that the present precise quantitative description of the D/S ratio in the deuteron raises an important issue for any model attempting a description of the NN force using quark bags. While the contribution to this ratio from the region inside 1 fm is nearly negligible, there is about 10% coming from the region $1 \text{ fm} < r < 1.5 \text{ fm}$ and about 20% from the region $1.5 \text{ fm} < r < 2 \text{ fm}$. It is clear that even minor modifications in the contributions from the region $1 < r < 2 \text{ fm}$ will destroy the present quantitative parameter-free description. It is naively plausible that such modifications could occur from the overlap of the quark bags, particularly in the case in which each of these bags are larger than 1 fm. It is therefore important to examine the consequences of the bag radius for the D/S ratio as well as to establish the limits it imposes on the bag size.

Note

After the completion of this work, Dr. W. Gruebler has informed us that his group has just completed a new determination of η using the $^2\text{H}(\vec{d}, p)^3\text{H}$ reaction with the result $\eta = (0.0275 \pm 0.0004)^{13)}$. In addition they have improved their previous analysis²⁾ and introduced Coulomb effects according to Santos¹⁴⁾ with the new value $\eta = (0.0275 \pm 0.0007)$ instead of 0.0259. These new values for η on the one hand confirm the very quantitative level of the theoretical description as discussed above. On the other hand, the linkage between various experimental quantities and theory is now so strong that there are significant and disturbing inconsistencies on the level of several per cent. We briefly explore the consequences of a high value for η as reported by Dr. Gruebler.

Examining the D/S ratio alone, a high value may either result from a larger value of f^2 or from an anomalous contribution from 2π and other exchanges. If the origin is the coupling constant, the value must be at least $f^2 = 0.081$. This is possible but uncomfortable to reconcile with $f_c^2 = 0.0790$. It is not realistic to blame it on the difference in charged and neutral pion couplings since this implies large breaking of isospin symmetry.

We believe our estimated uncertainty of the intermediate and short-range contributions to be rather conservative, but would, under pressure, be willing to concede up to double the uncertainty. An explanation of the high η values on these lines would, however, require a nearly 100% uncertainty in these terms. This is extremely difficult to reconcile with dispersion relations.

When η is considered simultaneously with the quadrupole moment there are additional difficulties. Since the value of η governs that of the quadrupole moment, an increased η leads to an increased value of the potential quadrupole moment, quite irrespective whether this is due to coupling constant or to the short-range contributions. Characteristic increases are $+0.010 \text{ fm}^2$ due to f^2 or 0.015 fm^2 due to short range. This has then the consequence that the deduced mesonic effects to the quadrupole moment are about -0.005 to -0.010 fm^2 . They have then even the opposite sign to the normally expected value $+0.010 \text{ fm}^2$. The large reported value for η is thus uncomfortable although as such barely consistent with the predicted η . A thorough re-examination of all the relevant aspects of both theoretical and experimental analyses concerning both η and Q is urgently needed.

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FIGURE CAPTION

Plot of the function $\eta(r)$: OPEP contribution without rescattering (dotted line); OPEP with rescattering (dashed-dotted line); modified Paris potential with $f^2 = 0.0790$ (full line).