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e^+e^- PHYSICS BEYOND PETRA ENERGIES

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1. Introduction

Experiments at LEP will allow us to study weak interactions and see what damps the rising weak cross-section (vector bosons? gauge vector bosons? ...)

- to study the spectrum of quarks and leptons
- to study strong interactions
- to see if quarks can be liberated
- ?

Will the rates at LEP be sufficient to carry out these studies? The answer is yes. Fig. 1 shows the ratio of $\sigma(e^+e^+ \rightarrow \text{hadrons})$ to $\sigma_{\text{point}} = \frac{4\pi a^2}{3E^2_{\text{c.m.}}}$ according to the standard model and the expected rates obtained using the LEP luminosities given in the accompanying table of facts. It should be emphasized that the standard model represents a conservative possibility; in most other scenarios something even more spectacular occurs (note that even if there is no $Z$ pole and $\sigma$ keeps rising like $E^2$ at LEP energies, $\sigma_{\text{weak}}(\bar{e}e \rightarrow \bar{e}e + \mu\mu + \tau\tau + \text{known quarks})$ will be of order $4\sigma_{\text{point}}$ by $E_{\text{c.m.}} = 100$ GeV, $20\sigma_{\text{point}}$ at $E_{\text{c.m.}} = 150$ and $70\sigma_{\text{point}}$ at $E_{\text{c.m.}} = 200$).

Next we must ask whether LEP will be needed after PETRA, PEP, $p\bar{p}$ colliding rings at CERN and Isabelle. Again the answer is yes. Whatever discoveries are made with these facilities, we can already identify fundamental questions which will remain and can only be investigated at LEP.

To be sure of these answers we must first examine what we know and do not know today and what we will learn from PETRA, PEP, $p\bar{p}$ and Isabelle. Next we discuss why, when and how $\sigma_{\text{weak}}$ must change with increasing energy and show that fundamental changes will occur in the LEP energy range. We can then consider current expectations for $e^+e^-$ physics at LEP energies. It is obviously extremely dangerous to discuss physics beyond PETRA energies. Forthcoming experiments at PETRA and PEP may produce major surprises and doubtless they will modify our expectations in detail. Nevertheless, I believe that the extremely convincing case for LEP which we can make now will remain valid in outline.

2. What We Know

In the last decade we have learned that

- Hadrons are composed of quarks and gluons (the latter carrying ~ 50% of the momentum)
- the quarks have (effectively?) non-integral charges and behave like (almost) free point-like particles at short distances
- there are at least five flavours of quarks (presumably there are at least six belonging to three weak doublets)
Figure 1: \( \frac{\sigma(\bar{e}e \rightarrow \text{hadrons})}{\sigma\text{ point-like}} \) as a function of the centre of mass energy \( \sqrt{s} \) in the standard model and the corresponding rates with the expected LEP luminosities.
- quarks have three colours
- there is some evidence that the gluons are those expected according to QCD (in particular EEBC data for the moments of the neutrino structure function $F_3$ agree with quantitative predictions of the theory\(^3\)).

**Leptons**
- are point-like
- come in at least 3 families forming 3 weak doublets (in close and suggestive analogy to the quarks).

Weak interactions - are described by the extremely simple and elegant operator

$$L_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left( J^+_{\lambda} J^-_{\lambda} + J^0_{\lambda} J^0_{\lambda} \right)$$

where $J^\pm_{\lambda}$ are the isospin raising lowering components of the "weak isospin" current

$$\tilde{J}_\lambda = \gamma^\mu_1 \gamma^\lambda (1 - \gamma_5) \tilde{\psi}_\lambda$$

where $\psi_\lambda$ represents the weak isospin doublets

$$\psi_\lambda = (\nu_e), (\nu_\mu), (\nu_\tau), (v_e), (u_d), (c_s), (b_l)$$

(actually this structure is only fully established for the $(\nu_e), (\nu_\mu)$ and $(u_d)$ doublets\(^n\)) and

$$J^0_{\lambda} = J^3_{\lambda} - \sin^2\theta_W J^{e.m.}_\lambda$$

where $J^3_{\lambda}$ is the third component of $\tilde{J}_\lambda$ (actually there is as yet no evidence for the $J^{e.m.}_\lambda$ term in $L_{\text{eff}}$ and the coefficient in front of the $J^0_{\lambda} J^0_{\lambda}$ term in $L_{\text{eff}}$ is now known to be one to within 5\%\(^\lambda\)).

One further thing which we know is that the predictions of $L_{\text{eff}}$ are certainly wrong at high energy.

3. **What We Do Not Know**

Although the predictions obtained by using $L_{\text{eff}}$ to generate matrix elements must fail at large $E$ (since they lead to violations of unitarity), we do not know how the predictions are modified (although most theorists are rather confident that they can predict what will happen in outline). Further, we have no theory of flavour (the quark mass spectrum, mixing angles, CP violating phases, etc.). We do not know whether quarks are confined (nor do we know how, if they are confined). Nor do we know ....

4. **What Questions will Remain Unanswered when LEP is Available**

It seems clear that experiments at PETRA (and PEP) will teach us a great deal more about the neutral current. However, although upgraded versions of PETRA and PEP should give hints about the mass scale on which the predictions of $L_{\text{eff}}$ are modified, they will **not** answer the most essential questions about weak interactions at high energy. Experiments at these machines may also discover new fla-
vours of quarks and leptons (there certainly seems to be a good chance that PETRA will discover the top quark — but I know of no convincing theoretical predictions of its mass and the discovery may have to wait for LEP) and they can make many tests of QCD.

Either the pp collider or Isabelle should discover the Z⁰, but apart from measuring its mass (with considerable errors) they will not allow us to investigate its properties in detail (they may also discover the W⁺ but this looks more difficult).

It therefore seems certain that LEP will be needed to study the behaviour of weak interactions at high energy. Presumably LEP will also be needed to extend our knowledge of the spectrum of quarks and leptons.

5. Why, When, How Must the Weak Interactions Change?

The lepton scattering cross-sections predicted by L_eff violate unitarity at centre-of-mass energies of order 650 GeV for charged currents and 1300 GeV for neutral currents (choosing helicities to get the best bounds). These predictions must therefore be modified at substantially lower energies.

Assuming that L_eff above is correct at low energies, Bjorken has shown as8) on rather general grounds that the scale µ_w on which the point-like structure of the charged current must fail is bounded by

\[ µ_w < \frac{37.4 \text{ GeV}}{\text{sin}^2 \theta_w} \approx 163 \text{ GeV}. \]

This is a conservative bound which can only be saturated if charge renormalization is dominated by weak effects. Consequently the closer µ_w approaches the bound, the larger weak effects in σ_{ee} must become. However, it is very hard to say at what energy these effects occur (although the higher the energy the larger the effects).

An explicit example of these general arguments is provided by a model constructed by Sakurai and Hung8). They examined the simplest model with intermediate vector bosons which reproduces L_eff (without necessarily assuming a local gauge invariance). This requires vector bosons W⁺ and W⁰ and B⁰, with B⁰ and W⁰ mixing to make the photon and Z⁰. The model yields the mass relation

\[ M_z^2 = \frac{M_w^2}{1-(M_w/37.4 \text{ GeV})^2 \sin^2 \theta_w} \]

which is plotted in Fig. 2. Note that M_w satisfies Bjorken's bound. As M_w approaches the bound M_z grows but so do the effects in ee annihilation since

\[ jσ_{ee \to z} ds \sim \alpha \frac{1}{M_z^2}. \]
Figure 2: Relationship of $M_z$ and $M_W$ in the model of Hung and Sakurai

$\sin^2 \theta_W = 0.23$
Although $M_z$ cannot be bounded on general grounds, both $M_w$ and $M_z$ must lie in the range predicted by the standard model if the intrinsic strengths of weak and electromagnetic interactions are similar (and who does not hope that this is the case?). Furthermore, models of the Hung-Sakurai type are very sick in general. The amplitude for $e^+e^- \to W^+W^-$ (for which the relevant diagrams are shown in Fig. 3) grows rapidly at high energies and quickly violates unitarity unless a) the $\gamma W^+W^-$ and $Z^0 W^+W^-$ vertices are those of a gauge theory, b) $M_z$ and $M_w$ are related exactly as in the Weinberg-Salam model (given by the unification point in Fig. 2) (there is still a residual sickness which can only be cured by introducing further ingredients, as discussed in section 8).

It might seem that this conclusion could be avoided by introducing many $Z^0$'s. However, it has been shown by Georgi and Weinberg\(^9\) that in a large class of gauge theories with more than one $Z^0$ which reproduce the successful predictions of the standard model at low energies in a natural way, at least one $Z^0$ must be lighter than the $Z^0$ in the standard model. This is illustrated explicitly by the $SU(2)_L \times SU(2)_R \times U(1) \times U(1)$ model recently discussed by Pati and Rajpoot\(^10\); the relationship of the masses between the two $Z^0$'s in this model is shown in Fig. 4.

We conclude that we can be very confident that spectacular things will happen in the LEP energy range. We now explore detailed expectations in the standard model (bearing in mind that it is conservative in the sense that the effects are more spectacular in most other models).

6. The $Z^0$

In the standard model

$$M_z = \frac{37.4 \text{ GeV}}{\sin^2 \theta_w \cos^2 \theta_w} \approx 90 \text{ GeV}.$$  

The relative branching ratios into a given generation of fermions are

$$\Gamma(Z^0 \to \nu\bar{\nu}), \Gamma(Z^0 \to e\bar{e}), \Gamma(Z^0 \to u\bar{u}), \Gamma(Z^0 \to d\bar{d})$$

$$2 : 1 : \frac{10}{3} : \frac{13}{3}$$

for $\sin^2 \theta_w = \frac{1}{4}$\(^11\). For $N$ generations of fermions,

$$\frac{\Gamma(Z^0 \to e\bar{e})}{\Gamma(Z^0 \to \text{all})} \leq \frac{1}{9.4^N}$$

or 3.5% with the present value of $N = 3$. The total width is given by

$$\Gamma(Z^0 \to \text{all}) \leq 0.75 \text{ N GeV}.$$
Figure 3: Diagrams which contribute to $e^+e^- \rightarrow W^+W^-$

Figure 4: Relationship between the masses $M_1$ and $M_2$ of the two $Z^0$'s in the model discussed by Pati and Rajpoot, which depends on an unknown parameter $\xi$ (the model reproduces the results of the standard model exactly at $q^2 = 0$ but the predictions for forward-backward asymmetries in $e^+e^- \rightarrow \mu^+\mu^-$ at PETRA energies differ substantially for finite $\xi$).

Figure 5: Diagram which gives a correction to the Weinberg-Salam relationship between $M_W$ and $M_Z$. 
On resonance

\[
\sigma(\bar{e}e + Z^0 + X) = \frac{9}{\sigma^\text{point}} \frac{\Gamma(Z^0 \rightarrow \bar{e}e)}{\alpha^2 \Gamma(Z^0 \rightarrow \text{all})} \frac{\Gamma(Z^0 + X)}{\Gamma(Z^0 + \text{all})}
\]

giving the order of ten thousand events per hour with the LEP luminosity

(\sigma^\text{point} = \frac{4\pi\alpha^2}{3s} \approx 10^{-3}\text{ cm}^2 \text{ at } \sqrt{s} = 90 \text{ GeV}, \text{ which would give } 3.6 \text{ events/hour for } L = 10^{32} \text{ cm}^{-2} \text{ sec}^{-1}).\) With these rates we can produce new quarks, leptons, Higgs mesons ..., measure weak couplings, and study of order \(10^7 \) \(q\bar{q}\) events at \(Q^2 \sim 10^4 \text{ GeV}^2\) in a typical experiment. Some possible measurements are worth considering in more detail:

a) There may exist lepton doublets such that \(M_{e0} < 2M_{L^\pm}\) but the neutral lepton satisfies \(M_{L^0} \ll M_{e0}\) (we would certainly expect \(M_{L^0} \ll M_{L^\pm}\) on the basis of the known leptons). If \(M_{e0}\) were stable or quasi-stable, its existence could be inferred by comparing \(\Gamma_{Z^0}\) and \(\Gamma_{Z^0}\) and hence deducing the width into unobservable channels. For example, if \(n\) such doublets existed (and we assume that corresponding new quark doublets are too heavy to be produced in \(Z^0\) decay) \(R\) and \(\Gamma_Z\) would change as follows (in the standard model)

<table>
<thead>
<tr>
<th>(n)</th>
<th>(R^0_{Z^0})</th>
<th>(\Gamma_{Z^0}(\text{GeV}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3795</td>
<td>2.25</td>
</tr>
<tr>
<td>1</td>
<td>3360</td>
<td>2.39</td>
</tr>
<tr>
<td>5</td>
<td>2200</td>
<td>2.95</td>
</tr>
</tbody>
</table>

Clearly the \(Z^0\) can be used as a sensitive probe of the number of light neutral leptons/neutrinos and hence of their heavy charged partners which may be way beyond the reach of LEP. In fact it might be feared that \(n\) may be so large that the effect of the \(Z^0\) will be largely wiped out. Extending the table above to larger \(n:\)

<table>
<thead>
<tr>
<th>(n)</th>
<th>(R^0_{Z^0})</th>
<th>(\Gamma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2200</td>
<td>2.95</td>
</tr>
<tr>
<td>10</td>
<td>1440</td>
<td>3.66</td>
</tr>
<tr>
<td>25</td>
<td>580</td>
<td>5.55</td>
</tr>
<tr>
<td>50</td>
<td>220</td>
<td>9.29</td>
</tr>
<tr>
<td>100</td>
<td>72</td>
<td>16.3</td>
</tr>
<tr>
<td>200</td>
<td>21</td>
<td>30</td>
</tr>
<tr>
<td>300</td>
<td>10</td>
<td>44</td>
</tr>
</tbody>
</table>

At present we cannot absolutely preclude the possibility that there are 300 or more light \(L^0\)s or neutrinos which could remove the \(Z^0\) peak (cosmological arguments give \(N_{\nu} \ll 3\) or 4 for stable "neutrinos" with \(M_{\nu} \ll M_e^{12}\); the absence of the decay \(K \rightarrow \pi \nu \nu\) gives \(N_{\nu} \ll 6000\) for \(M_{\nu} \ll M_K\) in the standard model; the process
ee -> (t$\bar{t}$) + π(t$\bar{t}$) + ν$\bar{ν}$ may be used to count neutrinos if suitable heavy onia exist$^{13,14}$). However, if $N_ν$ is large we would expect some of the corresponding charged leptons and quarks to lie in the LEP energy range - in which case they will be discovered and their production will restore the $Z^0$ peak if they are lighter than $M_Z/2$ (this argument can be made quantitative in the standard model, as discussed below).

b) Precise measurements of $M_{Z^0}$ and $M_w$ can give very interesting information in the framework of models. For example the Weinberg-Salam model relationship

$$\frac{M_w^2}{\cos^2\theta W M_{Z^0}^2} = 1$$

is only true to leading order. If there exist lepton doublets with $M_L << M_{Z^0}$ they can make substantial corrections through the diagram in Fig. 5 which gives$^{15,16}$

$$\frac{M_w^2}{\cos^2\theta W M_{Z^0}^2} = 1 + \frac{G_F M_L^2}{8\sqrt{2}\pi^2}$$

Existing data interpreted in terms of this model give $0.98 \pm 0.025^5$) (this is the accuracy to which the relative strengths of the two terms in $L_{eff}$ are known), which requires

$$\sum (M_{L^±})^2 < (540 \text{ GeV})^2$$

already a non-trivial result which could be made very stringent by measuring $M_{Z^0}$ and $M_w$ precisely. This bound allows at most 58 $L^±$'s just beyond the reach of the first stage of LEP (all 58 having mass 71 GeV!); this provides quantitative support for the feeling that if enough doublets exist for the $L^±L^-$ channels to threaten to wipe out the $Z^0$, then some of the corresponding $L^±$ would be found in the LEP energy range$^{14}$).

c) By measuring $\sigma(e^+e^- + f\bar{f})$ (where $f$ stands for fermion - lepton or quark) through the $Z^0$ region it is possible to measure the vector ($v_f$) and axial vector ($a_f$) couplings to the $Z^0$ $^1$). In particular the energy dependence of $\sigma$ measures $v_f^2 + a_f^2$ and $v_Q v_f$ and the angular distribution (in which there is a pronounced forward backward asymmetry) measures $v_f^2 + a_f^2$, $v_Q v_f$ and $a_Q a_f$. In principle this allows us to measure all $v_f$ and all $a_f$ and their signs relative to $v_e$ and all $a_e$ and their signs relative to $a_e$. The overall relative $v$-$a$ sign can be settled either by using polarized beams or by measuring the muon polarization in $e^+e^- + μ^+μ^-$ or by measuring the $τ$ polarization in $e^+e^- + τ^+τ^-$ by analysing the decay products, which appears to be easy$^{17,18}$).
The measurement of $d\sigma(\bar{e}e \rightarrow f\bar{f})$ is obviously easy for leptons. (Note
that the forward backward asymmetry above the $Z^0$ is rather sensitive to $Z^0$'s of
heavier mass$^{18}$). It should be rather simple for heavy quarks, whose characteris-
tic decays provide signatures which discriminate between quark and anti-quark jets,
and it will also be possible for light quarks, according to studies based on the
Feynman-Field parameterization of the properties of quark jets$^{19}$).

d) Studies of hadronic physics
e) Heavy lepton production
f) Higgs meson production

7. W. Production

With the current value of $\sin^2\theta_W = 0.23$, the standard model gives
$M_W = 78$ GeV. Single W production is possible in principle, the diagram giving the
largest cross-section being shown in Fig. 5. This process is only of interest for
$s < 2 M_W$ (two W production being much more copious once threshold is reached).
In this range $\sigma_{W^+ W^-} < 5.5 \times 10^{-37}$ cm$^2$ $^{20}$ and discovery of the W in this way
seems very difficult if not impossible$^{18}$).

The diagrams which contribute to $e^+e^- \rightarrow W^+W^-$ are shown in Fig. 3. The
$W^+W^-$ and $Z^0W^+W^-$ couplings cannot be fixed on general grounds. However, they are
uniquely determined in gauge theories and the corresponding cross-section in the
standard model is shown in Fig. 7$^{21}$. For $\sin^2\theta < .3$, $\sigma_{e^+e^- \rightarrow W}\gg 5\sigma$ point for
$s > 2M_W + 35$ GeV$^{22}$. This is sufficiently large to make experimentation involving
the W possible. However, it must be stressed that the cross-section is minimal
in gauge theories, in which the coupling constants satisfy the unique relations
necessary to ensure the large cancellation between different contributions which
are required to prevent violation of unitarity. In non-gauge theories the cross-
section is much larger. This is illustrated in Fig. 8 which shows the separate
contributions of various terms and in Fig. 9 where I have used Fig. 8 to construct
the predictions of the standard model with the $Z^0$ contribution removed. The ef-
eflect of removing the $Z^0$ is very striking and suggests that it should be easy to
see whether the couplings differ greatly from what is expected in a gauge theory.
Furthermore, it may be possible to measure the couplings sufficiently accurately
to distinguish between different gauge theories$^{23}$). In order to do this polarized
beams would be very useful since by using right-handed electrons and/or left-
handed positrons we can switch off the "boring" neutrino exchange contribution
(which can be predicted exactly from our knowledge of $\beta$ decay). In gauge theories
the cross-section should drop by roughly an order of magnitude if the "wrong"
polarizations are used$^{24}$ but this is not true in general, so polarization will
provide an important diagnostic if surprises are in store (note also that with the
"right" polarizations the cross-section should be enhanced by roughly a factor of
Figure 6: Diagram for $e^+e^- \rightarrow W^- \nu e^+$

Figure 7: $\sigma(e^+e^- \rightarrow W^+W^-)$ in the standard model for various values of $\sin^2 \theta_W$. 
Figure 8: The contributions of the different diagrams in fig. 3 to $\sigma(e^+e^- \rightarrow W^+W^-)$ for $\sin^2\theta_w = 3/8$
four). The polarization of the W's is also very sensitive to non-gauge theory couplings \(^{24}\) (in particular it is the coupling of longitudinal W's which grows rapidly giving large cross-sections in non-gauge theories); it may be possible to measure the polarization by studying the angular distribution of the decay products.

The width and branching ratios of the W into known fermions can be calculated exactly. For example

\[
\Gamma(W \to e\nu) = 205 \text{ MeV}
\]

(assuming \(M_W = 78 \text{ GeV}\)) and

\[
\frac{\Gamma(W \to \bar{u}d)}{\Gamma(W \to e\nu)} = 3
\]

(a colour factor). With \(N\) generations of fermions

\[
\Gamma(W \to \text{all}) = 820 N \text{ MeV},
\]

giving 2.5 GeV for the present value of \(N = 3\), and

\[
\frac{\Gamma(W \to e\nu)}{\Gamma(W \to \text{all})} = \frac{1}{4N}.
\]

We therefore expect a reasonable signal from leptonic decays. In addition, the hadronic decays (at least into light quarks) should give two clear jets per W with invariant mass equal to \(M_W\), which should make \(W^+W^-\) production easy to separate\(^{18,25}\).

It therefore seems that \(e^+e^- \to W^+W^-\) will be easy to measure and will provide fundamental information about the couplings of the W (gauge theory or non-gauge theory).

8. Beyond the W; Higgs\(^7\)

The interactions of vector bosons (W's and Z's) and fermions are well-behaved when the couplings are those of a gauge theory. However, there is still a residual sickness which shows up, for example, in WW scattering for which the lowest order Feynman diagrams are shown in Fig. 10. With arbitrary couplings (with dimensionless coupling constants) the amplitude for longitudinal W's grows like \(E^6_{c.m.}\); with gauge theory couplings this is reduced to \(E^2_{c.m.} \cdot \text{ still two powers faster than allowed by unitarity as } E_{c.m.} \rightarrow \infty \)\(^{26}\). In a well-behaved renormalizable theory which can be treated perturbatively there must be some new ingredient which cancels the \(E^2_{c.m.}\) term. This is provided by the exchange of a spin zero Higgs meson (Fig. 11).

Without Higgs mesons, W-W interactions must become strong as we approach the unitarity limit for the Born term (at energy \(E_u\)) in the sense that higher order terms must become comparable to the Born term (the Feynman rules give non-renormalizable divergences in higher orders). If \(M_H \gg E_u\), the effects of the
Higgs meson will be unimportant until well beyond the unitarity limit and perturbation theory must also break down; in fact unitarity is violated unless
\[ M_H < \frac{8\pi^2}{3C_F} = 1.2 \text{ TeV.} \]

Presumably, therefore, unless \( M_H \) is small compared to 1.2 TeV there will be large corrections to the lowest order diagrams, e.g. there will be big final state interactions in \( e^+e^- \rightarrow W^+W^- \) and the diagrams in Fig. 3 will not give the right answer. Veltman has argued\(^{28}\) that the critical mass is probably of order 300 GeV in which case either \( M_H < 300 \text{ GeV} \) (there is essentially no lower bound on the mass which may turn out to be very small) and we could hope to produce \( H \) at LEP or \( M_H > 300 \text{ GeV} \) (and perhaps there is no \( H \)) in which case substantial "radiative corrections" should show up in \( e^+e^- \rightarrow W^+W^- \) for \( E_{\text{c.m.}} > 300 \text{ GeV.} \)

It is hard to be precise about the critical mass/energy but clearly this is a powerful argument for pushing for the highest possible energy at LEP.

The reason for the intense theoretical interest in the Higgs sector (or whatever piece of theoretical ignorance it parametrizes\(^{29}\)) is that it is intimately connected with the fundamental problem of the mass spectrum of elementary particles, their mixing angles and \( CP \) violating phases. In the unbroken theory, the vector bosons are massless and members of fermion multiplets are degenerate (being also massless in most models). The coupling to Higgs mesons induces symmetry breaking and generates the mass spectrum - if we understood the Higgs sector we would understand the origin of mass!

It follows that the Higgs meson coupling to a particle \( X \) is proportional to \( M_X \) (or more precisely to the difference of \( M_X \) and the mass before symmetry breaking in the case of fermions\(^{30}\)). Therefore, we expect \( H \) to be produced in conjunction with heavy particles and to decay into them\(^{31,32}\). For example, in the conventional model (in which there is a single \( H^0 \)), for \( M_H < M_T \) the dominant decays are \( H \rightarrow c\bar{c} \) and \( H \rightarrow t\bar{t} \) (in the colour factor ratio 3:1), for \( M_H > M_T \) but \( M_H < 2M_t \) the dominant decay will be \( H \rightarrow b\bar{b} \) etc. We now consider three possible \( H \) production mechanisms at LEP in the conventional model before turning to other models.

a) The decay of heavy vector mesons \( V \) ("onia"), \( V \rightarrow H\gamma \) illustrated in Fig. 12 may have a substantial branching ratio\(^{33}\). For example, the branching ratio
\[ \frac{\Gamma(V \rightarrow H\gamma)}{\Gamma(V \rightarrow \mu^+\mu^-)} = \frac{G_F M_V^2}{4\sqrt{2} \pi a} \left[ \frac{M_H^2}{M_V^2} \right] \]
is expected to be about 6.1%, 4.5% and 2.5% for \( M_H = 15, 20 \) and 25 GeV, respectively, and \( M_V = 30 \text{ GeV} \).
Figure 9: $\sigma(e^+e^- \rightarrow W^+W^-)$ in the standard model with and without the contributions due to $Z^0$ exchange for $\sin^2\theta_W = 3/8$

Figure 10: Lowest order diagrams for $WW \rightarrow WW$

Figure 11: Higgs meson contributions to $WW \rightarrow WW$
It therefore seems that this decay would lead to the discovery of \( H \) with \( M_H < 0.9 \, M_V \) at LEP if suitable onia exist (the mononenergetic photon accompanied by the heavy decay products of \( H \) should provide a signature).

b) The decays \( Z \rightarrow H \bar{u} \bar{d} \) and \( Z \rightarrow H ee \) (Fig. 13) provide a way to search for \( H \). With \( M_Z = 90 \) GeV,

\[
\frac{\Gamma(Z \rightarrow H e \bar{e})}{\Gamma(Z \rightarrow e \bar{e})} = 10^{-3} \text{ for } M_H = 10 \text{ GeV} \\
= 10^{-5} \text{ for } M_H = 50 \text{ GeV}
\]

(see Ref. 34). Given the huge rate on the \( Z^0 \) pole, this should provide a way to find \( H \) if \( M_H < 45 \text{ GeV} \).

c) At high energies the cross-section \( \bar{e}e \rightarrow ZH \) is substantial. The ratio

\[
\frac{\sigma(\bar{e}e \rightarrow ZH)}{\sigma\text{point}}
\]

is about one at \( \sqrt{s} = 140 \) GeV for \( M_H < 50 \) GeV (dropping very rapidly for \( M_H > 50 \)) and about 0.5 at \( \sqrt{s} = 200 \) GeV for \( M_H < 90 \) GeV (dropping very rapidly for \( M_H > 90^{35} \)).

LEP provides the best way to look for Higgs mesons (\( H \) production may occur at an appreciable rate in the \( p\bar{p} \) collider and Isabelle but there is no clean signature). However, to have a good chance of exploring the Higgs sector requires the highest possible energy.

So far we have only considered the single \( H^0 \) of the minimal standard model, but probably the real world is more complicated\(^{29}\). As examples, we may consider how the Higgs sector might differ, still within \( SU(2)_L \times U(1) \) with the known flavours. In the simplest case there is a single H doublet (of which \( H^0 \) survives as a physical particle) and this leads to the successful relation

\[
\frac{M^2_W}{M^2_Z \cos^2 \theta_W} = 1.
\]

the simplest ways to preserve this relation are

either to add a second doublet - leaving five physical Higgs particles
\( H^+, H^-, H_1^0, H_2^0, H_3^0 \)
or if we want a single Higgs multiplet, to remove the doublet and introduce a multiplet with weak isospin 3 of which the neutral number has \( I_3 = \pm 2 \). In this case there would be physical Higgs mesons with charges ranging from zero to five!

Charged Higgs mesons would obviously be produced in \( e^+e^- \) collisions - the rate being enormous on the \( Z^0 \). Decays like \( H^+ \rightarrow c\bar{s} \) (leading to \( D^*K^0 \) resonances!) should be easy to detect\(^{32,36} \).
9. New Flavours of Quarks and Leptons

It is clearly likely that the sequence

\[
\begin{pmatrix}
\nu_e \\ e \\
\nu_\mu \\ \mu \\
\nu_\tau \\ \tau
\end{pmatrix},
\begin{pmatrix}
u_d \\ d \\
\nu_s \\ s \\
\nu_b \\ b
\end{pmatrix}
\]

continues. This need not necessarily be the case, of course. For example, in the grand unified SU(5) model of Georgi and Glashow\(^{37}\) the successful predictions\(^{38}\) of \(m_\nu, m_\mu\) and \(\sin^2 \theta_W\) are spoiled by the introduction of more flavours (although one more flavour generation can probably be accommodated). Furthermore, in this model the neutrinos are automatically massless, so cosmological constraints allow at most one more generation. In the SU(5) model, therefore, the only tasks below \(E \sim 10^{15}\) GeV (where unification becomes manifest and substantial baryon and lepton number violations occur) are to discover the \(t, Z^0, W^\pm\) and \(H^0\)!

The search for further quarks and leptons is clearly of fundamental importance. If they exist, they can easily be found at LEP. New leptons show up in \(\mu, e\) events etc. and new quarks through jumps in sphericity at threshold, steps in \(R\), the existence of onia, etc.\(^{19,39}\).

10. Strong Interactions

At present QCD is the only sensible candidate for a theory of the strong interactions. Presumably its validity will have been established by the time LEP is built. However many very interesting predictions of QCD require LEP energies\(^{40}\). For example, the wave function of an "onium" vector meson state (V) is supposed to become Coulomb-like as \(M_V \to \infty\); to see Coulomb-like properties probably requires \(M_V > 50\) GeV. Likewise, to see the expected \(V \to 3\) gluon jet decays may need \(M_V\) substantially bigger than \(M_W\). If heavy quarks exist, QCD predicts that weak Q decay will become increasingly like heavy lepton decay when \(M_Q \to \infty\) as \(\alpha_s(M_Q^2) \to 0\) and strong interaction corrections are switched off\(^{41}\). In addition, the fragmentation function for \(Q \to (Q\bar{Q})\) ... can be predicted in QCD for large \(M_Q\).

The QCD predictions of scaling violations in fragmentation functions will probably be tested at PETRA. Furthermore the predicted 3-jet events should be seen at PETRA, but it will need LEP to see the fraction of these events \((1 - \alpha_s(Q^2) / \pi)\) decreasing with energy and to see the expected broadening of the jets.

There are also very interesting QCD predictions for \(\gamma \gamma \to \text{large } p_T\) particles, for which the rates seem sufficiently high to make experimentation possible (the relevant diagrams are shown in Fig. 15). QCD provides a justification\(^{42}\)
for the simple ansatz proposed long ago\textsuperscript{43}) that there is a two-jet contribution
given by the Born term for $\gamma\gamma \rightarrow q\bar{q}$. QCD also leads to parameter-free predictions
of three-jet processes (two large $p_T$ jets and one jet along the beam axis) and
four-jet processes (two at large $p_T$ and two along the beam axis) which depend in
detail on the predicted properties of gluon bremsstrahlung\textsuperscript{42,44}). It should be
to test the two-jet predictions at PETRA but the 3- and 4-jet processes
may require LEP energies (the two-jet process is very sensitive to quark charges
and will provide definitive tests of models with "funny" - e.g. integer! - charge
assignments e.g. in the Han-Nambu model the cross-section is predicted to be 2.65
times bigger than in the standard model even below colour threshold; furthermore,
since the predictions apply for real photons, they can be used to test the
Pati-Salam model in which the colour charges are effectively "switched off" at large
$q^2$\textsuperscript{45}).

11. Exotica

a) $\bar{e}e$ is obviously the best source of quarks if they are not confined\textsuperscript{19,46})
(personally I don't find this possibility as exotic as some people seem to).

b) Historical analogy suggests that our present "elementary" particles -
the quarks - may turn out to have structure (which might explain the proliferation
of flavours in terms of a few subquarks - or "preons"). The effects of substructure
would show up in $R = \frac{\sigma(\bar{e}e \rightarrow \text{hadrons})}{\sigma(\text{point})}$ in the way illustrated in Fig. 16. For
energies of the order of the inverse quark radius, scaling would be violated by
the effects of form factors in $\bar{e}e \rightarrow \bar{q}q^*$ etc. Finally, above preon threshold, $R$
would rescale with a value depending on the charges of the preons. For example,
if quarks are bound states of a flavour preon and a colour preon (making the
number of preons equal to the number of flavours plus three, which is less than
the number of quarks, assuming preons do not have some new attribute) we could have

i) flavour preons with $J = 0$, colour preons with $J = \frac{1}{2}$, so that $R \rightarrow R/3$ above
preon threshold due to the loss of the colour factor;

ii) flavour preons with $J = 0$, colour preons with $J = \frac{1}{2}$, $R \rightarrow R/12$;

iii) as in i) but preons having 137 new attributes (the "charges" of the non-
Abelian force which binds preons) and $R \rightarrow 137 R/3$.

c) Perhaps leptons have form factors (due to substructure?). It turns out
that a sensitive way to look for this, or for other unexpected breakdowns of QED
(due to abnormal non-renormalizable $WW$ interactions?), is in $\bar{e}e \rightarrow \gamma\gamma$ since the
corrections to the Born term are less than or of order $10^{-3}$ in the conventional
model\textsuperscript{47}) (some of the relevant diagrams are shown in Fig. 17).

d) Other exotica include monopoles and the new $R$ hadrons suggested by supersymmetric theories, which could be pair-produced in $\bar{e}e$ collisions\textsuperscript{48}).
Figure 12: Diagrams for the decay of a $l^-(\bar{Q}Q)$ state to $H^0 + \gamma$.

Figure 13: Diagram for the decay $Z^0 \rightarrow H^0 e^+e^-$

Figure 14: Diagram for $e^+e^- \rightarrow Z^0 H^0$

Figure 15: Diagram for $e^+e^- \rightarrow e^+e^- + $ hadrons
Figure 16: Possible appearance of $R = \frac{\sigma(ee \rightarrow \text{hadrons})}{\sigma_{\text{point}}}$ in models with quark sub-structure.

Figure 17: Diagrams for $e^+e^- \rightarrow \gamma\gamma$
12. Conclusions

LEP will be a unique tool to study the properties of matter on a scale $1/100 \text{GeV} \sim 10^{-16} \text{cms}$. By the time LEP is built, the $Z^0$ will probably have been discovered. Nevertheless LEP will be needed to investigate its properties in detail. The $Z^0$ peak will provide a factory for producing new quarks, new leptons, Higgs mesons, free quarks ... and supplying tens of millions of $q\bar{q}$ events, allowing detailed studies of hadron dynamics. Going higher in energy, LEP will allow us to study the $W$ boson and learn whether its properties are those expected in gauge theories. Finally, higher energy yet will very likely be needed to investigate the Higgs meson or whatever mechanism breaks the underlying symmetries of nature and gives rise to the mass spectrum of the observed particles.

Forthcoming experiments at PETRA and other new facilities will doubtless change our expectations for LEP in detail and may make some of our present extrapolations seem foolish (clearly they will also raise quite new questions). Nevertheless I believe that the case for LEP will remain absolutely convincing.

Acknowledgments

I am grateful to all the participants in the LEP Summer Study for stimulating discussions. In particular I thank Mary Gaillard and John Ellis whose previous work on LEP and talks at Les Houches provided a basis for much of this talk.
Table of Basic Facts

\[ \sigma_{\text{point}} = \frac{87 \text{nb}}{(E_{\text{c.m.}}(\text{GeV}))^2} = 10^{-35} \text{cm}^2 \text{ at } E_{\text{c.m.}} = 90 \text{ GeV.} \]

\[ \text{QED} \]

\[ e^+ \rightarrow \mu^+ \]

\[ e^- \rightarrow \mu^- \]

LEP Luminosity: 
\[ L = \left( \frac{E_{\text{c.m.}}}{2x70} \right)^2 10^{32} \text{cm}^{-2}\text{sec}^{-1} \]
for \( E_{\text{c.m.}} < 150 \text{ GeV.} \)
\[ \sim E_{\text{c.m.}}^{-3} \]
for \( 150 < E_{\text{c.m.}} < 200 \text{ GeV.} \)
\[ = 0.5 \times 10^{32} \text{cm}^{-2}\text{sec}^{-1} \text{ for } E_{\text{c.m.}} = 200 \text{ GeV.} \]

With \( R_x \equiv \frac{\sigma(ee \rightarrow x)}{\sigma_{\text{point}}} \)

\[ 40 R_x \text{ events/day for } E_{\text{c.m.}} < 140 \text{ GeV.} \]
\[ 10 R_x \text{ events/day for } E_{\text{c.m.}} = 200 \text{ GeV.} \]

Theorists' 24 hour day.
References

1. See CERN Yellow Report 76-18 "Physics with Very High Energy \(e^+e^-\) Colliding Beams" for earlier work on LEP. The article by J. Ellis and M.K. Gaillard has formed the basis for most subsequent theoretical work.

2. CERN/ISR-LEP/78-17.


4. It is now fairly well established for the \(\tau\) (for a review and references see G. Flügge, DESY preprint DESY 78/42). The spectrum in \(D \rightarrow e\nu\ldots\) provides evidence that the current in charmed particle decay is largely left-handed; furthermore charmed particles are known to decay mostly to strange particles.

5. L.M. Sehgal. Invited talk at the "Neutrino 78" conference at Purdue Univ. (III Physikalisches Institut Technische Hochschule Aachen preprint PITHA-102 (1978)). For other thorough reviews of the interpretation of neutral current data see J.J. Sakurai UCLA preprints UCLA/78/TEP/9 (paper presented at the Conference in honour of P.A.M. Dirac) and UCLA/78/TEP/19 (paper presented at the Oxford Neutrino Conference).


7. Unless otherwise stated, I use \(\sin^2 \theta_w = 0.23\) in this paper.


11. Detailed formulae as a function of \(\sin^2 \theta_w\) may be found in Ref. 1.


14. For a discussion and further references on neutrino counting and the effect of many neutrinos on \(e^+e^-\) annihilation see K.J.F. Gaemers, R. Gastmans and F.M. Renard, ECFA/LEP 45.


18. M. Davier, these proceedings. LEP Summer Study/1-3.

19. K. Winter, these proceedings. LEP Summer Study/1-4.

20. This result is taken from Ref. 1 where the cross-section as a function of energy calculated by J. Prentki and G. Preparata is quoted. See W.D. Schlatter ECFA/LEP 10, 1978 for a detailed discussion of backgrounds, etc.

22. The heavier the W the larger the cross-section, relative to $\sigma_{\text{point}}$, since $g_w^2 = \frac{G_{\text{F}} M_w^2}{\sqrt{2}}$.


29. In the standard model with N generations of quarks there are $2N + (N-1)^2$ arbitrary parameters which characterize the Higgs-quark couplings which must be adjusted to reproduce the $2N + (N-1)^2$ parameters which characterizes the masses, mixing angles and relative phases of the quarks. This disgusting fact raises grave doubts about the Higgs sector (at least in its simplest form).

30. Thus although the H-electron coupling $-e M_e / M_H$ in most models, it may be much larger e.g. in the Georgi-Glashow model (which is now ruled out by experiment) the coupling $-e M_L / M_W$ where $M_L$ is a heavy lepton mass, and $e e H^0$ might have been observable (J.R. Primack and H.R. Quinn. Phys. Rev. D6, 3171, 1972 Section V).


32. For an update of some of the considerations in Ref. 31 and an introduction to the Higgs meson see M.K. Gaillard CERN TH 2461, 1978 (to be published in Comments in Nuclear and Particle Physics). See also J. Ellis SLAC-PUB-2177 1978 (these lecture notes contain an extensive review of many recent developments associated with gauge theories).


39. For a discussion of changes of sphericity at thresholds and references to the original literature see the article by J. Ellis in these proceedings LEP Summer Study/1-14.

40. For a thorough discussion of tests of QCD in $e^+e^-$ annihilation experiments see the article by J. Ellis in these proceedings LEP Summer Study/1-14.

41. The first order strong corrections have been given by M. Suzuki. LBL preprint LBL 7948, 1978.


44. The phenomenology of the two–jet process has been discussed in detail by K. Kajantie, ECFA/LEP 5, 1978. The phenomenology of all three processes has been discussed by S.J. Brodsky, T.A. de Grand, J.F. Gunion and J.H. Weis Phys. Rev. Lett. 41, 672, 1978 and SLAC-PUB in preparation. However their model for the three and four–jet processes does not take into account the gluon corrections (discussed in Ref. 42) with the result that the predicted cross-sections are somewhat too large at large $p_T$ (see C.H. Llewellyn Smith, T. Robinson and T. Weiler – in preparation).

45. Defining $R^1 = \frac{d\sigma(\gamma_1\gamma_2 \to \text{large } p_T \text{ jets})}{d\sigma(\gamma_1\gamma_2 \to \mu^+\mu^-)}$ we have $R^1 = 17/27$ per flavour doublet in the standard model and $5/3$ per doublet in Han-Nambu below colour threshold. In the Pati Salam model $R^1 = 3$ per doublet for $q_1^2 = q_2^2 = 0$ (3 is the Han Nambu value above colour threshold) but $R^1$ drops to $\frac{17}{27}$ as $q_1^2$ and $q_2^2 \to \infty$.

46. See J.C. Pati and A. Salam. ICTP Trieste preprint IC/78/54 and references therein for a discussion of the signatures of integrally charged quarks.
