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NUCLEAR EFFECTS IN PROTONIUM

J.M. Richard

Division de Physique Théorique,
Inst. de Physique Nucléaire,
F-91406 Orsay

and

Université P. et M. Curie, Paris

and

M.E. Sainio

CERN -- Geneva

ABSTRACT

The lifetime and energy shift of the atomic protonium are estimated using an optical potential model which reproduces the experimental NN cross-sections at low energy. Good agreement is found with previous calculations made with a boundary condition model. The short-range distortions of the wave function are also examined and some predictions for the decay properties are provided.

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The formation and the study of the atomic protonium is one of the interesting projects planned for the near future in low energy physics. This will be done in experiments at the LEAR facility at CERN¹⁾. Protonium provides a unique tool for measuring the nucleon-antinucleon ($N\bar{N}$) interaction at rest in each individual partial wave. It also acts as a doorway for producing a variety of mesons. In particular, deeply bound baryonia, if they exist, can be obtained via radiative or pionic transitions. Preliminary evidence for such transitions has already been reported²⁾.

There are already some estimates of the protonium hadronic level shifts and widths. The calculations of Kaufmann and Pilkuhn³⁾ (KP) and Kaufmann⁴⁾ (K) incorporate properly isospin mixing and the neutron to proton mass difference using a set of coupled Schrödinger equations. In KP and K the long-range potential is generated by π , ϵ and ω exchanges, the ρ meson being however omitted. Annihilation is accounted for by applying an absorptive boundary condition to the wave function at short distances. Such a treatment of annihilation seems reasonable since it is rather similar to the method used by Dalkarov and Myhrer⁵⁾ to fit the low energy $\bar{p}p$ cross-section. Alternatively, optical models have been proposed⁶⁾. There the real long-range potential is supplemented by an empirical complex core describing annihilation and short-range elastic forces. Recently, Dover and Richard⁷⁾ (DR) reviewed such a method. They applied the G parity transformation to the Paris NN potential (in the static approximation), adopted a square well cut-off at $r_0 = 0.8$ fm and added a phenomenological core

$$V(r) = (V_R + iV_I) / [1 + \exp a(r-R)]. \quad (1)$$

A good fit to the scattering data is obtained with $V_R = -21$ GeV, $V_I = -20$ GeV, $R = 0$ and $a = 5 \text{ fm}^{-1}$ (model DR1). An alternative choice is $V_R = V_I = -0.5$ GeV, $R = 0.8$ fm and $a = 5 \text{ fm}^{-1}$ (model DR2) which gives a comparable fit with a considerably weaker interaction at very short distances. The advantage of a pure potential picture is obvious since it allows one to make predictions for several processes using always the same elementary $N\bar{N}$ interaction. For instance, our models DR1 and DR2 have already been taken as possible choices for predicting Coulomb interferences⁸⁾, $\bar{p}$ -nucleus interaction⁹⁾ and spin-dependent effects in $\bar{p}p$ collisions¹⁰⁾. The aim of the present paper is to calculate the protonium properties using these DR potentials. For the first levels, we compare our results with the KP and K estimates. We also consider higher levels and compute some branching ratios for the hadronic decays.

Using the DR potentials we have solved the radial equations as written in Eq. (2) of KP. In the unnatural parity states ($L = J$) one has two coupled equations, $p\bar{p}$ and $n\bar{n}$. Natural parity states involve four components with orbital angular momentum $L = J - 1$ or $L = J + 1$. As for our computational strategy, we first solved the pure Coulomb problem and checked the textbook energy and wave function, and then we introduced progressively the real and imaginary nuclear potential in the $p\bar{p}$ channel, the $n\bar{n}$ admixture and finally the orbital mixing when necessary. The numerical stability was tested at each step.

The shifts and widths calculated with the DR1 potential are displayed in Table 1 for the first levels. Comments and additional results are in order.

i) As expected from the study of exotic atoms, the naive perturbative estimate

$$\Delta E = \Delta E_R - i \frac{\Gamma}{2} = \langle \psi_C | V_{N\bar{N}} | \psi_C \rangle \quad (2)$$

where ψ_C is the Coulomb wave function, is definitively inadequate for S and P waves. For the P states the Trueman formula, as used by Green and Wycech⁹⁾, gives very good results. For instance, they obtain a shift of the 2^3P_1 level $\Delta E = (0.037 - i 0.012)$ eV to be compared with our exact computation $\Delta E = (0.036 - i 0.010)$ eV. For S waves, as expected, the agreement is slightly worse. For 1^1S_0 we get $\Delta E = (0.54 - i 0.51)$ keV instead of $\Delta E = (0.59 - i 0.66)$ keV in Ref. 9).

ii) We have studied the effect of isospin mixing by comparing the approximation where the wave function is pure $p\bar{p}$ with the full calculation which includes the coupling to the $n\bar{n}$ channel. The complex energy shift is generally well approximated by the one-channel calculation. The most noticeable exception is the 2^3P_0 state whose width is increased from $\Gamma(p\bar{p}) = 56$ meV to $\Gamma(p\bar{p} + n\bar{n}) = 113$ meV by the charge-exchange potential (using DR1). The isospin content of the decay products is also modified by the charge-exchange potential. In the pure $\bar{p}p$ approximation, the DR model gives equal probability for $I = 0$ and $I = 1$ annihilations. In the complete calculation the two isospin components $u_0(r)$ and $u_1(r)$ are different and it is easy to separate the hadronic width into its two isospin contributions $\Gamma_h = \Gamma_0 + \Gamma_1$ by computing

$$\Gamma_I = -2 \int_0^\infty |u_I(r)|^2 \text{Im } V_{N\bar{N}} dr. \quad (3)$$

The isoscalar piece Γ_0 often turns out to be larger, since the $I = 0$ real potential is generally more attractive¹¹⁾ and thus draws the $I = 0$ wave function more efficiently into the annihilation region. An exception is the 2^3P_1 state, where the $I = 0$ potential has a strong repulsive component¹¹⁾. Of course, these results depend on the simplifying assumption made by DR that the annihilation potential itself is channel-independent.

iii) The hyperfine splitting of the ortho to paraprotonium is only 0.2 keV. The choice of a channel-independent absorption in the DR model tends to wash out the differences induced by the long-range forces which are strongly spin-dependent¹¹⁾. Remember that without the annihilation term (1), one could get a strong hyperfine splitting as large as 2 keV^{12),13)}.

iv) As already noted by KP, the 2P states decay much more often by annihilation than by radiative transition to the 1S levels. On the other hand, the splittings between the different P states with $J^{PC} = 1^{+-}, 0^{++}, 1^{++}$ and 2^{++} are smaller than the electromagnetic splittings as calculated by Barmo et al.¹⁴⁾. An interesting property is that the 3P_0 state is much wider than the others. This illustrates the effect of coherent tensor forces, a point which has been underlined several times¹¹⁾. The differences in the widths of the various 2P states has an indirect experimental consequence which has been noted by Bizot¹⁵⁾. For each 2P state the second X-ray in the cascade $nL \rightarrow 2P \rightarrow 1S$ is anisotropically distributed with respect to the first one. An over-all isotropy is obtained if each of the 2P states contributes with its statistical weight $(2J + 1)$. If the branching ratios for the radiative decays are now different, one gets an over-all anisotropy which is predictable. So it should in principle be possible to check roughly the ordering of the 2P widths in Table 1 even if the resolution of the X-ray detectors does not allow one to explicitly separate the different levels.

v) The effect of strong interactions is very small in D waves. Typically $|\Delta E| < 10^{-6}$ eV, to be compared with the radiative width $\Gamma(3D \rightarrow 2P) \approx 0.4 \times 10^{-4}$ eV. An interesting exception is the 3^3D_1 state since it can be mixed with the 3^3S_1 wave by the strong tensor potential. Remember that the electromagnetic tensor force in the Breit-Fermi Hamiltonian is proportional to r^{-3} and has a vanishing matrix element between the 3D and 3S Coulomb wave functions, a well-known property¹⁶⁾. The presence of the strong tensor force gives better overlap of the wave function with the imaginary potential and increases the hadronic width although the integrated S wave probability remains small. We get approximately $\Gamma_h/\Gamma_{rad} \approx 1\%$ for the 3^3D_1 state. In an experiment with

antiprotons stopped in hydrogen, any precocious annihilation (apart from Stark mixing effects) where the last X-ray lies in the M series, is predicted to occur in a vector state with $J^P = 1^-$. As expected, the isospin content of Γ_h is mainly $I = 0$ ($\approx 80\%$) with DR1. The tensor force is indeed larger for $I = 0$ than for $I = 1$.

We now look at the sensitivity of our calculations with respect to the choice of the $N\bar{N}$ interaction. In Table 2 we display the results for the 1S and 2P levels using the Paris potential and the two different cores DR1 and DR2. Also shown are the results of KP and K. Clearly, various treatments of a strong and spread annihilation give similar results. Also, those shifts do not help in distinguishing the dispersion theoretic Paris potential from a simpler one-boson-exchange model. On the other hand, if any future experimental result lies outside the range of the predictions in Table 2, it would require drastic changes in the model of the $N\bar{N}$ dynamics and not only refinements in the choice of the parameters.

In particular, the main property of the DR model used here or of the boundary condition model used by KP or K is the lack of narrow baryonia below the threshold¹⁷⁾, the states created by the attractive real potential being washed out by the absorption. Alternatively, some models for the $N\bar{N}$ dynamics have been proposed in an attempt to restore the existence of narrow baryonia in the nuclear approach.

a) In the scheme derived by Green et al.¹⁸⁾ the most peripheral part of the imaginary potential given by Eq. (1) is attributed to the $N\bar{N} \rightarrow B\pi$ reaction where B is a bound nuclear baryonium. This piece being subtracted we are left with a relatively short-range potential for the direct annihilation $N\bar{N} \rightarrow n\pi$. This could explain why the B's themselves are not too broad. However, the full consistency of such a scenario remains to be established by detailed coupled-channel calculations. By way of illustration we repeated the calculations with the reduced potential suggested by Green and Sainio¹⁸⁾. Our results are not significantly different. One would need, in fact, a more sizeable reduction of the range of the imaginary potential.

b) Another type of attempt has been pursued by Côté et al.¹⁹⁾ who claim to fit the scattering data with a short-range imaginary potential that is inspired by diagrams with two mesons exchanged in the s channel. These diagrams are very short-ranged if nucleons are considered as point-like²⁰⁾. Such a potential would perhaps allow narrow baryonia to exist, with important consequences for the protonium. Smaller widths are expected, as well as sometimes much larger shifts

in the partial wave where a nuclear state lies not too far from the atomic region. An extreme point of view is adopted by Kerbikov et al.¹³⁾. They neglect annihilation and analyze the shift of protonium as a function of the binding of the baryonium. For both baryonium and protonium, annihilation is treated as a small perturbation. We note that their model is not shown to be compatible with the experimental $\bar{p}p$ cross-sections. In any case, if some baryonium state really is close to the threshold, the predictions based on an optical potential with strong annihilation would be greatly modified. The key problem in that field is precisely to understand the coexistence of possible narrow baryonia with large inelastic $N\bar{N}$ cross-sections.

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n	State	ΔE_R	$\Gamma_h/2$	$P(n\bar{n})$	Γ_1/Γ_0
1	1S_0	0.54 keV	0.51 keV	4×10^{-6}	0.68
	$^3S_1 + ^3D_1$	0.76 keV	0.45 keV	3×10^{-6}	0.76
2	1S_0	68 eV	66 eV	0.6×10^{-6}	0.68
	$^3S_1 + ^3D_1$	96 eV	59 eV	0.5×10^{-6}	0.76
	1P_1	-26 meV	13 meV	0.1×10^{-9}	0.96
	3P_0	-74 meV	56 meV	2×10^{-9}	0.03
	3P_1	36 meV	10 meV	0.5×10^{-9}	9.5
	$^3P_2 + ^3F_2$	-5 meV	15 meV	0.1×10^{-9}	0.63
3	1S_0	20 eV	20 eV	0.2×10^{-6}	0.68
	$^3S_1 + ^3D_1$	29 eV	18 eV	0.1×10^{-6}	0.76
	1P_1	-9 meV	4.4 meV	0.4×10^{-10}	0.96
	3P_0	-26 meV	20 meV	8×10^{-10}	0.03
	3P_1	13 meV	3.6 meV	2×10^{-10}	9.4
	$^3P_2 + ^3F_2$	-1.2 meV	5.4 meV	0.3×10^{-10}	0.63

Table 1 : Complex energy shifts of the protonium levels with model DR1. Also shown are the normalization of the $n\bar{n}$ component and the ratio of the hadronic widths in the two isospin channels.

n (units)	State	DR1	DR2	KP	K
1 (keV)	1S_0	0.54-i0.51	0.58-i0.52	—	0.50-i0.68
	$^3S_1 + ^3D_1$	0.76-i0.45	0.80-i0.46	—	0.75-i0.50
2 (eV)	1P_1	-0.026-i0.013	-0.024-i0.014	-i0.015	-0.029-i0.010
	3P_0	-0.074-i0.056	-0.050-i0.059	-i0.097	-0.070-i0.058
	3P_1	0.036-i0.010	0.037-i0.009	i0.016	0.033-i0.008
	$^3P_2 + ^3F_2$	-0.005-i0.015	-0.002-i0.012	-i0.023	-0.007-i0.014

Table 2 : Comparison of the optical model calculation with two different cores, DR1 and DR2, with the boundary condition model of KP [Ref. 3)] and K [Ref. 4)].

REFERENCES

- 1) U. Gastaldi and R. Klapisch, lectures given at the International School of Physics Enrico Fermi, Varenna (1980).
- 2) P. Pavlopoulos et al., Phys. Lett. 72B (1978) 415;
L. Tauscher, proceedings of the 5th European Symposium on Nucleon-Antinucleon Interactions, Bressanone (1980) (CLEUP, Padova, 1980) p. 575.
- 3) W.B. Kaufmann and H. Pilkuhn, Phys. Rev. C17 (1978) 215.
- 4) W.B. Kaufmann, proceedings of the Kaon Factory Workshop, U.B.C., Vancouver (1979), ed. M.K. Craddock, TRIUMF report TRI-79-1, p. 160;
see also W.B. Kaufmann, Phys. Rev. C19 (1979) 440.
- 5) O.D. Dalkarov and F. Myhrer, Nuovo Cimento A40 (1977) 152.
- 6) M. Gourdin, B. Jancovici and L. Verlet, Nuovo Cimento 8 (1958) 485;
R.A. Bryan and R.J.N. Phillips, Nucl. Phys. B5 (1968) 201.
- 7) C.B. Dover and J.M. Richard, Phys. Rev. C21 (1981) 1466.
- 8) P. Kroll, University of Wuppertal preprint WU.B.81.1 (1981).
- 9) A.M. Green and S. Wycech, to appear in Nucl. Phys. A (1982).
- 10) J.M. Richard, proceedings of the International Symposium on High Energy Physics with Polarized Beams and Polarized Targets, Lausanne (1980), eds. C. Joseph and J. Soffer (Birkhäuser, Basel, 1981) p. 535;
C.B. Dover and J.M. Richard, BNL preprint (1981).
- 11) W.W. Buck, C.B. Dover and J.M. Richard, Ann. Phys. (NY) 121 (1979) 47.
- 12) C.B. Dover, J.M. Richard and M.C. Zabek, Ann. Phys. (NY) 130 (1980) 70.
- 13) B.O. Kerbikov, proceedings of the 5th European Symposium on Nucleon-Antinucleon Interactions, Bressanone (1980) (CLEUP, Padova, 1980) p. 423;
I.S. Shapiro, Phys. Rep. C35 (1978) 129.
- 14) S. Barmo, H. Pilkuhn and H.G. Schlaile, Z. Phys. A301 (1981) 283.
- 15) J.C. Bizot, private communication.
- 16) S. Pasternak and R.M. Sternheimer, J. Math. Phys. 3 (1962) 1280;
T.E.O. Ericson and F. Scheck, Nucl. Phys. B19 (1970) 450.
- 17) F. Myhrer and A.W. Thomas, Phys. Lett. 64B (1976) 59;
F. Myhrer and A. Gersten, Nuovo Cimento A37 (1977) 21.
- 18) A.M. Green and M.E. Sainio, J. of Phys. G6 (1980) 1375;
A.M. Green, to appear in J. of Phys. G8 (1982).
- 19) J. Côté et al., contribution to 9th ICOHEPANS, Versailles (1981).
- 20) A. Martin, Phys. Rev. 124 (1961) 614.