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IS SUPERSYMMETRY AFRAID OF CONDENSATES ?

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ABSTRACT

Condensates might appear abundantly in strongly interacting supersymmetric Yang-Mills theories. We argue that they do not break supersymmetry dynamically. A toy model is used to illustrate the mechanism.

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Fermion-antifermion condensates are likely to break global chiral symmetries dynamically. They were also assumed to be a source of supersymmetry breakdown¹⁾⁻³⁾. In this note we consider strongly interacting supersymmetric⁴⁾ gauge theories and assume confinement at a scale Λ . We then investigate these theories at scales small compared to Λ , using 't Hooft's anomaly conditions⁵⁾ in the context of massless composite superfields. We would like to address the following questions in such a scenario: (i) do the scalars screen the fermions⁶⁾ leading to a supersymmetric low energy theory of massless composites and completely unbroken global (chiral) symmetries? (ii) might there be a formation of boson-boson or fermion-antifermion condensates that break chiral symmetries? (iii) do these condensates break supersymmetry? These questions arise in models of supersymmetric technicolour¹⁾⁻³⁾ in connection with an "index theorem" recently formulated by Witten⁷⁾. They might also be relevant in models⁸⁾ where one tries to construct fermionic composites from bosonic and fermionic preons.

We will argue in the course of this paper that the answers to the questions should be: (i) the fundamental scalars do not in general screen all the fermions, i.e., chiral symmetries will at least be partially broken, (ii) this breakdown will be achieved by boson-boson or fermion-antifermion condensation, (iii) supersymmetry will not be broken by this mechanism. Statement (iii) is trivial in the case of scalar-scalar condensates. Our non-trivial result is that whenever a fermion-antifermion condensate is dynamically favoured over the corresponding scalar-scalar condensate, supersymmetry still remains unbroken in the course of the condensation process. A chiral $SU(4) \times SU(8)$ model⁹⁾ with which we have some experience in the non-supersymmetric case is used to illustrate our results.

In models of supersymmetric technicolour¹⁾⁻³⁾ it was assumed that condensation of gauge fermions $\lambda\lambda$ and/or chiral fermions $\psi\bar{\psi}$ break supersymmetry dynamically. In the absence of different supersymmetry breaking mechanisms we would expect the energy of the new "condensed" vacuum to be negative, which might be problematic in models of spontaneously broken supersymmetry¹⁰⁾. Moreover, there is the index theorem⁷⁾ which tells us that in supersymmetric gauge theories with chiral superfields in real (not necessarily irreducible) representations of the gauge group supersymmetry is not broken dynamically. The assumptions in the proof of the index theorem might be subject to criticism^{*)} but this criticism might not apply to the case of gauge theories with semi-simple gauge groups.

*) Consider, e.g., the chiral superfields A , X and Y with superpotential $W = \lambda X(A^2 - M^2) + gYA + (m/2)Y^2 + \text{h.c.}$ In the limit $m = 0$ the index changes by two.

Up to now one has not been able to apply the index theorem in non-left-right symmetric theories, so the question of a dynamical breakdown of supersymmetry is still open.

The model we present is a chiral model with strong gauge group $SU(4)$. It is the supersymmetrized version of the $SU(4) \times SU(8) \times U(1)$ model of Ref. 9). It contains a gauge vector superfield⁴⁾ $V_i^j(x, \theta, \bar{\theta})$ in the adjoint representation of $SU(4)$, which in the Wess-Zumino gauge has components $(A_{\mu j}^i, \lambda_j^i, D_j^i)$ where D_j^i is an auxiliary field. Furthermore there are left-handed chiral superfields $S_{ij}(x, \theta) \equiv (\varphi^{ij}, \chi^{ij}, F^{ij})$ and $T_{i\alpha} = (\eta_{i\alpha}, \psi_{i\alpha}, F_{i\alpha})$ where i, j are $SU(4)$ indices and $\alpha = 1, \dots, 8$. All fermion fields are left-handed Weyl spinors. The symmetry of the model is $SU(4) \times SU(8) \times U(1)_X \times U(1)_Y$ with the following assignments:

$$\begin{aligned} A_{\mu j}^i &= (15, 1, 0, 0) \\ \lambda_j^i &= (15, 1, 0, -1) \\ \chi^{ij} &= (\bar{10}, 1, 8, 4) \\ \varphi^{ij} &= (\bar{10}, 1, 8, 3) \\ \psi_{i\alpha} &= (4, 8, -6, -2) \\ \eta_{i\alpha} &= (4, 8, -6, -3) \end{aligned} \tag{1}$$

An additional $U(1)$ symmetry is broken by the $SU(4)$ instanton process which annihilates eight ψ , six χ and eight λ fields.

Eight superfields T_i were chosen to ensure the absence of anomalies in the gauge group $SU(4)$. There might be indications¹¹⁾ that there are no anomalies at all in supersymmetric Yang-Mills theories which, if true, would allow much more flexibility in constructing models.

Compared to the non-supersymmetric case⁹⁾ this model has the additional global $U(1)_Y$ symmetry because of the appearance of the gauge fermions λ . Thus $Y(\lambda) \neq 0$ and $U(1)_Y$ is an R symmetry⁴⁾. R symmetries do not commute with the supersymmetry generators and there is evidence¹²⁾ that supersymmetry does not allow the construction of an Adler-Bardeen¹³⁾ current for these symmetries^{*)}.

*) I thank S. Ferrara for discussions on anomalies in supersymmetric theories.

We assume that the $SU(4)$ gauge interactions become strong at a scale Λ and we want to investigate the spectrum at energy $E \ll \Lambda$. The chiral symmetries of the model are supposed to play a major rôle for the low energy spectrum. 't Hooft has given this qualitative statement a mathematical formulation by his anomaly consistency conditions⁵⁾. Even in view of the complexity and the uncertainties with respect to anomalies in supersymmetric theories we believe in the validity of these conditions as a tool for studying the low energy spectrum.

The model of Eq. (1) has no solution to the anomaly conditions at the $SU(8) \times U(1)_X \times U(1)_Y$ level. This statement is true for nearly all the models we have investigated. A discussion of these various models as well as the technical proof of our statement will be presented elsewhere.

The non-existence of solutions to the anomaly conditions is mainly due to the appearance of the additional $U(1)_Y$ symmetry. We are thus led to conclude that the fundamental scalars cannot screen the fundamental fermions completely⁶⁾ and that the chiral symmetries are at least partially broken. Since we know already that the model has a solution to the anomaly conditions at the $SU(8) \times U(1)_X$ level⁹⁾, we speculate that $U(1)_Y$ is broken by the formation of condensates. This could be achieved in principle by multiparticle condensates, but the obvious candidate is a $\lambda_i^j \lambda_j^i$ condensate. This condensate does not break supersymmetry. The argument^{3), 14)} that

$$\{Q_\alpha, \lambda^\alpha \sigma_\mu A^\mu\} = \lambda_\alpha \lambda^\alpha + \dots \quad (2)$$

tells us that $\lambda \sigma_\mu A^\mu$ is the Goldstone spinor of broken supersymmetry is misleading. We first observe that $\lambda \sigma_\mu A^\mu$ is gauge dependent even with respect to the restricted gauge transformations in the Wess-Zumino gauge. For supersymmetry to be broken, we have to remember that $\lambda\lambda$ has to be the auxiliary component of a (gauge invariant) composite superfield (the fermionic component of this superfield would then be the Goldstone spinor). Such a superfield does not exist. The only gauge invariant composite superfield (that contains $\lambda\lambda$) we can construct is $W^\alpha W_\alpha$ where $W^\alpha = \bar{D}^2 D^\alpha V(x, \theta, \bar{\theta})$ ⁴⁾. This superfield, however, contains $\lambda\lambda$ as the scalar component. $\lambda\lambda$ condensation thus breaks $U(1)_Y$ but not supersymmetry. The pseudoscalar component of $W^\alpha W_\alpha$ is a Goldstone boson with fermionic partner $\lambda \sigma^{\mu\nu} F_{\mu\nu}$ (massless but not a Goldstone fermion)^{*)}.

*) The F component of $W^\alpha W_\alpha$ contains $F_{\mu\nu} F^{\mu\nu}$.

Solutions to the anomaly conditions at the $SU(8) \times U(1)_X$ level, transforming as a 28-dimensional representation of $SU(8)$ with $X = -4$, are easily found⁹⁾. There are however some ambiguities. There are three fermion composites

$$\xi_{\alpha\beta} = \chi^{ij} \psi_{i\alpha} \psi_{j\beta} - \chi^{ij} \psi_{i\beta} \psi_{j\alpha} \quad (3)$$

and boundstates of one fermion and two scalars, for example

$$\omega_{\alpha\beta} = \varphi^{ij} \psi_{i\alpha} \varphi_{j\beta} - \varphi^{ij} \psi_{i\beta} \varphi_{j\alpha} \quad (4)$$

Supersymmetry is unbroken; thus $\xi_{\alpha\beta}$ and $\omega_{\alpha\beta}$ have to be the fermionic components of composite superfields. This is, however, not possible for the $\omega_{\alpha\beta}$ states. These states can only be the fermionic component of the left-handed chiral superfield

$$S^{ij} T_{i\alpha} T_{j\beta} - S^{ij} T_{i\beta} T_{j\alpha} \quad (5)$$

which vanishes because of Bose statistics for the chiral superfields. We thus remain with the three fermion boundstate $\xi_{\alpha\beta}$, the same as in the non-supersymmetric model. There one was able to define the concept of complementarity between the confining and the broken (Higgs) phase, both phases have the same spectrum of massless particles. In the broken phase $\chi^{ij} \psi_{i\alpha}$ condensed and broke $SU(4)$ as well as $SU(8)$.

The states that condense in the broken phase (here $\chi^{ij} \psi_{i\alpha}$) appear as constituents of the massless composites in the confining phase^{15),9)} (here $\xi_{\alpha\beta}$). We have to stress here that because of the absence of $\omega_{\alpha\beta}$ it is not $\varphi^{ij} \eta_{i\alpha}$ that can play this role of $\chi^{ij} \psi_{i\alpha}$ in the broken phase. Complementarity even suggests that $SU(8)$ is broken by multifermion condensates⁹⁾, e.g.,

$$\Theta_{\alpha\beta\gamma\delta} = \langle 0 | \chi^{ij} \psi_{i\alpha} \dots \chi^{kl} \psi_{l\delta} | 0 \rangle \quad (6)$$

One might now speculate that in the Higgs phase $\chi^{ij} \psi_{j\alpha}$ condenses. Naively we would expect that this condensate of two left-handed fermions breaks supersymmetry since $X\psi$ can be interpreted as the auxiliary component of the left-

handed chiral superfield $S^{ij}T_{j\alpha}$. But here the interpretation of $\chi\psi$ is linked to the interpretation of $\xi = (\chi\psi)\psi$. It turns out that the state $\xi_{\alpha\beta}$ can only be interpreted as the fermionic component of the chiral left-handed superfield obtained by projection⁴⁾ from the composite vector superfield $(D^2ST)T$, i.e.,

$$\frac{\bar{D}^2 D^2}{\square} [(D^2ST)T] \triangleq ((\chi\psi)\eta, (\chi\psi)\psi, (\chi\psi)F) \quad (7)$$

where $D, \bar{D}$ are covariant derivatives, and we have neglected terms which contain fundamental auxiliary fields. This now implies that $\chi\psi$ is the scalar component of the right-handed chiral superfield $D^2(S^{ij}T_{i\alpha})$, and thus $\chi\psi$ condensation is perfectly consistent with unbroken supersymmetry^{*)}.

This model is just a prototype of several classes of models we have considered. All these models have one property in common: whenever we had a reason that a fermion-antifermion condensate was more likely to be formed than the corresponding scalar-scalar condensate, we inevitably had to identify the fermion-antifermion boundstate as the scalar component of a composite superfield^{**)} .

Although not a proof, this might indicate that condensate formation does not break supersymmetry even in non-left-right symmetric theories. It confirms our intuitive picture of the positive energy of the ground state in supersymmetric Yang-Mills theories. The condensed vacuum state would have to have negative energy if there is not a different mechanism that lifts the energy of the symmetric state. This different mechanism would then be responsible for the supersymmetry breakdown. We could then even be in the amusing situation that the formation of condensates restores supersymmetry.

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^{*)} This holds in a general class of models we have considered. We could however not yet rule out the existence of models where the massless fermion is the partner of a spin one boson.

^{**)} Mass mechanisms due to the combined action of the condensates and the instanton process do not break supersymmetry. Example: the condensate $\theta_{\alpha\beta\gamma\delta}$ in (6) breaks $SU(8)$. Some of the states in $\xi_{\alpha\beta}$ are no longer protected and receive a mass by $\theta, \lambda\lambda$ and the instanton⁹⁾. The scalar partners $(\chi\psi)\eta$ of these states receive a mass through this mechanism as well.

REFERENCES

- 1) S. Dimopoulos and S. Raby, Nucl. Phys. B192 (1981) 353.
- 2) E. Witten, Nucl. Phys. B188 (1981) 513.
- 3) M. Dine, W. Fischler and M. Srednicki, Nucl. Phys. B189 (1981) 575.
- 4) For a review see:
P. Fayet and S. Ferrara, Phys. Rep. 32 (1977) 249.
- 5) G. 't Hooft, in Recent Developments in Gauge Theories, ed. G. 't Hooft et al., (Plenum Press, N.Y., 1980).
- 6) H.P. Nilles and S. Raby, SLAC-PUB 2743 (1981), to be published in Nucl. Phys. B.
- 7) E. Witten, Lectures at Trieste Conference (1981).
- 8) For a review see:
M. Peskin, International Symposium on Lepton and Photon Int., Bonn, ed. W. Pfeil (1981), p. 880.
- 9) H.P. Nilles and S. Raby, Nucl. Phys. B189 (1981) 93.
- 10) M.T. Grisaru, Caltech report, CALT-68-883 (1982).
- 11) O. Piguet and K. Sibold, CERN preprints TH.3171 and TH.3177 (1981);
- 12) S. Ferrara and B. Zumino, Nucl. Phys. 87 (1975) 207;
D.R.T. Jones and J.P. Leveille, Univ. of Michigan preprint UM HE 81-74 (1981).
- 13) S.L. Adler and W.A. Bardeen, Phys. Rev. 182 (1969) 1517.
- 14) M.V. Voloshin and V.U. Zakharov, ITEP preprint (1981).
- 15) S. Dimopoulos, S. Raby and L. Susskind, Nucl. Phys. B173 (1980) 208.

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