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A NEW EFFECT IN PIONIC ATOMS

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ABSTRACT

We show that a likely explanation of the anomalous energy shifts in heavy pionic atoms is the energy dependence of the s-wave πN scattering amplitudes. This leads to an energy dependence of the π nuclear optical potential. As a result, a non-trivial Coulomb term appears which leads to a remarkably better agreement with experiments without additional free parameters. An anomaly persists for the widths.

Strong interaction level shifts and widths have been observed for a large number of nuclei throughout the periodic system. The data have been obtained for atomic states of very different angular momentum and for different isotopes of the same nuclei. The measured quantities vary systematically over many magnitudes. For example, the 3d absorption widths have been studied from $Z = 15$ to 80 and they vary by no less than 6 magnitudes in this interval. It is therefore very remarkable that nearly all of these observations on shifts and widths in 1s, 2p, 3d, and 4f states can be described semiphenomenologically by a velocity-dependent optical potential with coefficients which are nearly independent of the nuclear specifics [1]:

$$2\mu V = \nabla \cdot \alpha(r) \nabla + q(r) \quad (1)$$

The coefficients in the velocity-dependent potential are closely related to the p-wave πN threshold amplitude as well as p-wave pion absorption on the deuteron, while the coefficients of the local part $q(r)$ are similarly related to the s-wave πN amplitude and the s-wave absorption. The numerical values of these coefficients (see later for exact definitions) are closely linked to the s- and p-wave scattering lengths, although minor variations in the effective amplitudes can be introduced by variations in the theoretical approach. These are of no importance for the further discussion.

The entire philosophy behind Eq. (1) is that the effective potential results from the interaction of the pions of nearly zero kinetic energy.

The very success of this description over the whole range of the periodic system and over such large magnitudes of variation implies that any major systematical deviation must be considered very significant, indicating new physical effects.

Recently shifts and widths have been measured for 3d states in nuclei of considerably higher Z than earlier and have been found to be several times smaller than expected [2,3] (Fig. 1). A tendency towards low values of these quantities was already noticed earlier for lower Z [4].

General indications of the possible origin of the effect are given by the following observations:

- i) The anomaly is observed only for the heaviest 3d states, while the 4f states behave normally. This indicates that the origin is in the *local* part of the potential $q(r)$, since the 4f states are dominated by the velocity-dependent term.
- ii) The anomalous nuclei have strong Coulomb binding of the pion. This suggests that electromagnetic binding effects on the strong interactions may be responsible.
- iii) Further, one notes that the local interaction term $q(r)$ to leading order is proportional to the isoscalar πN scattering length $a^{(+)} = \frac{1}{2}(a_{\pi^-n} + a_{\pi^-p})$

$$2\mu V_{local} = -4\pi \left(1 + \frac{\mu}{M}\right) a^{(+)} \rho(r) \quad (2)$$

As compared to the individual values of the π^-n and π^-p scattering length, this term has enormous cancellations and its value is only a few per cent of its components. As is well known, the local interaction is therefore dominated by the double-scattering contributions from the Pauli correlations [5]. Any effect that would prevent the cancellations from being so accurate might hence give large contributions, and must be carefully considered.

We propose here that the energy dependence of the s-wave πN amplitude is mainly responsible for the anomaly since it gives rise to an energy-dependent potential.

Near the threshold the πN isoscalar scattering amplitude has the form

$$\left(\frac{tg \delta_{l=0}}{k}\right)^{(+)} = a^{(+)} + \beta^{(+)} k^2 + \dots \quad (3)$$

where $k^2 = \omega^2 - \mu^2$. The pion-nuclear potential can be derived in the standard fashion, taking the energy dependence into account. There will also be similar energy-dependent contributions from the p-wave amplitudes. However, since the

energy-dependent isoscalar terms have no cancellations, the contributions to the potential from these terms is particularly strong. It is therefore a good first approximation to express the energy dependence of the potential by

$$\frac{dV}{d\omega} = -4\pi \left(1 + \frac{\mu}{M}\right) \beta^{(+)} \varrho(r) \quad (4)$$

Höhler et al. [6] have deduced the value $\beta^{(+)} = (-0.044 \pm 0.007)m_{\pi}^{-3}$ from πN scattering.

Let us now consider this potential as an external potential describing the π -nuclear scattering. In the presence of an electromagnetic field, the gauge condition ("local smoothness") is then that *everywhere* in the equations momenta p_{μ} should be replaced by $(p_{\mu} - eA_{\mu})$ in terms of the vector potential A_{μ} . In the present case of an external Coulomb potential $V_C(r)$ we should everywhere replace the pion energy ω by $(\omega - V_C(r))$ *including the potential*. This generates an additional electromagnetic potential from the energy dependence of the strong interaction potential

$$\delta V = -V_C(r) \frac{dV}{d\omega} \simeq V_C(r) 4\pi \left(1 + \frac{\mu}{M}\right) \beta^{(+)} \varrho(r) \quad (5)$$

We note that the range of this potential is the nuclear range.

Alternatively, attributing the energy dependence in the s-wave πN amplitude to a non-local interaction, the Coulomb effect can be obtained by the following argument. To leading order and with a non-locality of short range

$$2\mu V = -4\pi \left(1 + \frac{\mu}{M}\right) \left[\alpha^{(+)} \varrho(r) + \frac{1}{2} \beta^{(+)} \{ \nabla^2 \varrho(r) + \varrho(r) \nabla^2 \} \right]$$

When ∇^2 operates on a wave function $\phi(r)$ in the Coulomb field it generates a term proportional to V_C which is identical to the one in Eq. (5).

The additional contribution to the local potential is repulsive. Its contribution is on the per cent level in light elements increasing to about 35% of the normal local potential in heavy elements.

In order to obtain specific numerical results we used the optical potential in the form:

$$\frac{2\mu}{4\pi} V = - \left[P_1 (\bar{b}_0 \varphi + \bar{b}_1 \Delta \varphi) + P_2 \bar{B}_0 \varphi^2 \right] + \nabla \cdot \frac{\alpha_0}{1 + \frac{4\pi}{3} \alpha_0} \nabla$$

with

$$\alpha_0 = P_1^{-1} (c_0 \varphi + c_1 \Delta \varphi) + P_2^{-1} (C_0 \varphi^2 + C_1 \Delta \varphi \varphi)$$

$$P_1 = 1 + \frac{\mu}{M}, \quad P_2 = 1 + \frac{\mu}{2M}$$

$$\varphi = \varphi_m(r) + \varphi_p(r), \quad \Delta \varphi = \varphi_m(r) - \varphi_p(r)$$

$$\text{Re } B_0 = -\text{Im } B_0$$

$$\bar{B}_0 = B_0 + O(1/A)$$

$$\bar{b}_0 = \frac{1}{3} (\bar{a}_1 + 2\bar{a}_3) + \text{Pauli-correlations}$$

$$\bar{b}_1 = \frac{1}{3} (\bar{a}_3 - \bar{a}_1) + \text{Pauli-correlations}$$

$$\bar{a}_1 = a_1 \left(1 - \frac{1}{2} \tau_{01} a_1 k^2 \right)$$

$$\bar{a}_3 = a_3 \left(1 - \frac{1}{2} \tau_{03} a_3 k^2 \right)$$

$$k^2 = (\omega - e\phi)^2 - \mu^2$$

Pauli-correlations and $1/A$ -terms were accounted for as described by Tauscher and Schneider [7].

The constants $b_1 = (a_3 - a_1)/3$, c_0 , and c_1 were taken from πN phase-shift analyses [6], whereas the parameters b_0 , B_0 , C_0 , and C_1 were adjusted so as to fit the $2p-1s$ and $3d-2p$ transitions of a few selected light pionic atoms (B , ^{12}C , O , ^{32}S , ^{40}Ca , ^{44}Ca) for $k^2 = 0$ (normal potential). The data were taken from Tauscher [8]. The parameters of the potential are listed in Table 1. Shifts and widths obtained

with the normal potential are shown in Table 2 and in Fig. 1. Using the parameters of the normal potential but now with $k^2 \neq 0$ yields shifts which are significantly lower (cf. Table 2 and Fig. 1). The influence of the k^2 terms not being entirely negligible for the light elements, the parameter b_0 was readjusted (cf. Table 1). This lifts the shifts up somewhat as shown in Fig. 1 and Table 2.

From these numerical results we conclude that there is a spectacular overall improvement in the description of energy shifts, not only for the anomalous 3d states of high-Z elements but also throughout the periodic system. In the spirit that overall consistency of the data is very important, we made an overall adjustment of the effective parameters which gave even better agreement, in spite of only minor changes in parameters. One notes in particular that the 3d anomaly seems associated with a gradual replacement of the attraction of the velocity-dependent potential by the repulsion of the local potential, as indicated by the results for $Z \approx 92$. This is analogous to the corresponding effect for 2p states near $Z = 35$.

While the description of energy shifts is much improved, there is no corresponding improvement of the description of the anomalously small 3d widths for high-Z nuclei. These remain a theoretical challenge.

Finally we note that the present effect should give rise to observable effects in the strong interaction quadrupole splitting of heavy nuclei [10].

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Table 1
Optical potential parameters*)

	I Normal	II As I, but with finite range correction	III As II, but local part readjusted
$b_0 \quad [m_\pi^{-1}]$	-0.00532	unchanged	-0.00398
$b_1 \quad [m_\pi^{-1}]$	-0.0873	unchanged	unchanged
$B_0 \quad [m_\pi^{-4}]$	$-0.0489 + 0.0489 i$	unchanged	unchanged
$c_0 \quad [m_\pi^{-3}]$	0.2087	unchanged	unchanged
$c_1 \quad [m_\pi^{-3}]$	0.1779	unchanged	unchanged
$C_0 \quad [m_\pi^{-6}]$	$0.1988 + 0.0879 i$	unchanged	unchanged
$C_1 \quad [m_\pi^{-6}]$	$0.737 + 0.0 i$	unchanged	unchanged
$r_{01} \quad [m_\pi^{-1}]$	0	0.869	0.869
$r_{03} \quad [m_\pi^{-1}]$	0	10.68	10.68
ξ	1	unchanged	unchanged

*) Comments:

- i) Potential includes Pauli correlation corrections and $1/A$ -terms (cf. Ref. 7).
- ii) Potential has isospin term $C_1(\rho_n + \rho_p) \times (\rho_n - \rho_p)$.
- iii) Nucleons are point-like.

Table 2
Comparison of experimental shifts ϵ and widths Γ
with calculations for three different nuclei

	I Experiment	II Normal optical potential	III As II, but with finite range correction	IV As III, but local part readjusted
As ϵ [keV]	4.6 ± 0.9 [9]	8.55	4.96	5.97
As Γ [keV]	14.5 ± 4.0 [9]	23.9	21.1	21.8
Ta ϵ [keV]	18.1 ± 1.2 [3] 13.5 ± 1.0 *) [2]	25.6	18.1	19.4
Ta Γ [keV]	19.0 ± 5.0 [3] 17.7 ± 3.3 [2]	41.2	32.9	34.1
Bi ϵ [keV]	20.6 ± 1.0 *) [2]	38.3	21.1	24.0
Bi Γ [keV]	25.4 ± 5.3 *) [2]	87.3	67.1	69.9

*) Note that shifts are with respect to extended nuclei and not point nuclei.

Figure caption

Fig. 1 : Level shifts ϵ (a) and level widths Γ (b) for the pionic 2p, 3d, and 4f states as a function of atomic number Z . The lines represent the various calculations. They connect calculated points and only serve to guide the eye. Points are measured values, taken from Refs. 2, 3 and 8.

Fig. 1a

Fig. 1b