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AN EXPLICIT EXAMPLE OF A TUMBLING GAUGE THEORY

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ABSTRACT

We present a class of two-dimensional models with a non-Abelian gauge symmetry. In one model the gauge symmetry is dynamically broken by a fermion bilinear condensate in a $1/n$ expansion. We discuss the differences between a dynamical and elementary Higgs mechanism due to the presence of a non-Abelian axial anomaly.

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I - INTRODUCTION

One of the most interesting features of particle physics today is the idea that the interactions between the elementary particles are described by a unique gauge theory. The observed interactions are obtained by successive breaking of the original grand unified theory¹⁾. This is achieved by the introduction of a set of scalar fields, the Higgs fields, whose potential is constructed such that the original symmetry is broken in successive stages. However, a theory with such fundamental scalar fields is unappealing from an aesthetic point of view, and also unnatural²⁾. For instance, in grand unified theories this leads to the gauge hierarchy problem³⁾.

A possible way out of this problem is to have gauge theories coupled only to fundamental fermions and no elementary scalar fields⁴⁾. It is then assumed that a Lorentz scalar field, bilinear in the fermion fields, would acquire a non-vanishing vacuum expectation value. Hence, the theory would undergo dynamical symmetry breaking. In particular, if the fermions are in a non-real representation of the original gauge group a bilinear fermion condensate, which is not a scalar of the gauge group, will break the original gauge symmetry^{5a),5b)}. It is possible that the theory could break in several successive steps, giving rise to several energy scales. This phenomenon is known as tumbling^{5b)}.

Whilst tumbling is an appealing, intuitive idea, no concrete example of tumbling is known. Hence, it is important to construct explicit models which undergo this phenomenon.

On the other hand, the study of the two-dimensional CP^{n-1} models via the $1/n$ expansion⁶⁾ has proved to be very useful. For instance, it has been used to gain insight into the effect of the $U(1)$ axial anomaly on the dynamics of the pseudoscalar mesons in large n QCD^{7),8)}. Encouraged by the success of the CP^{n-1} model in explicitly revealing mechanisms which are also at work in QCD we construct a class of two-dimensional models. These are a non-Abelian generalization of the CP^{n-1} model with fermions. They have a $U(n)$ global symmetry and a $U(1)$ local symmetry; they are described in Section II. As in the case of the CP^{n-1} model, the quantum properties of these models can be studied via the $1/n$ expansion.

The properties of these models are of intrinsic interest. However, they also provide a laboratory for the explicit study of the properties of dynamical symmetry breaking, produced by a dynamical Higgs mechanism. Recently, the tumbling scenario has been studied in two-dimensions⁹⁾ in fermionic gauge theories. In that theory the tumbling picture is never realized, for two-dimensional reasons. We differ from Ref. 9) because our model has a quartic interaction.

Since the gauge and Higgs bosons are composite particles of the fundamental fermions, the models can also be used to study the properties of composite models for quarks and leptons^{*)}. Of course, the problem of composite fermions in the $1/n$ expansion is subtle¹¹⁾.

In Section III we study the properties of a model where both the left and right fermions transform according to the fundamental representation of a gauge group, $U(\ell)$. We call this model I. Although we have only a vector gauge field, λ_μ , the fermionic part of the Lagrangian is invariant under a chiral $U(\ell)_L \otimes U(\ell)_R$ symmetry. The $U(\ell) \times U(\ell)$ axial symmetry is spontaneously broken to $U(\ell)_{\text{vector}}$ because a fermion bilinear acquires a non-zero vacuum expectation value. It is also explicitly broken to $U(\ell)_{\text{vector}}$ by an axial non-Abelian anomaly, and also by the contribution of the scalar, z , fields. Within the $1/n$ expansion we explicitly write the low energy (energy less than the breaking scale, M) effective Lagrangian for the composite Higgs. This Lagrangian differs from a standard Higgs Lagrangian by an additional term. This term couples the gauge bosons to the Goldstone bosons. It is required by the non-Abelian anomaly. The effect of this term is to give the Goldstone bosons a large mass (of order of the breaking scale, M). It is a non-Abelian generalization of the mechanism in QCD where the singlet, η , gets a mass from the $U(1)$ axial anomaly. Thus, model I shows us an important difference between theories with composite and elementary Higgs, related to the existence of the non-Abelian axial anomaly in composite Higgs theories.

In Section IV we study the properties of a model where the right (left) handed fermions transform as the $\ell(\bar{\ell})$ representation of the $U(\ell)$ gauge group. This is model II. Via the $1/n$ expansion we show that a fermion bilinear, which is not a singlet of the $U(\ell)$ gauge group, acquires a non-zero vacuum expectation value. This dynamically breaks the original gauge symmetry. No light particle is present in the low energy spectrum. The Higgs field acquire a mass either from the dynamical Higgs mechanism or the axial anomaly; whilst the gauge fields get a mass through the dynamical breaking of the gauge symmetry. Since our theory has massless fermions the confining Coulomb force is screened, as in the massless Schwinger model. This results in the gauge bosons of the unbroken subgroup acquiring a mass. However, the leading order in n is unable to give the direction of symmetry breaking. To this order there is a degeneracy of vacuum states, related by a unitary transformation. To obtain the alignment of the vacuum higher order in $1/n$ must be included. This work is in progress¹²⁾.

*) Two-dimensional models have recently been used to study the 't Hooft anomaly conditions²⁾ in this context¹⁰⁾.

In Section II we set up our two models. Sections III and IV are devoted to a study of the properties of the models in a $1/n$ expansion. The results and implications are summarized in Section V. Finally, details of the $1/n$ expansion are given in the Appendix. Also given in the Appendix are the two-point functions for both models.

II - DESCRIPTION OF THE MODELS

In this section we construct explicit two-dimensional models with a non-Abelian gauge symmetry. These are a generalization of the CP^{n-1} model with fermions. In the CP^{n-1} model¹³⁾ there are n complex scalar fields, $z_a(x)$, $a = 1, \dots, n$ which lie on S^{2n-1} . Thus, they are constrained such that

$$\bar{z}^a z_a = \frac{n}{2f} \quad (2.1)$$

where $n/2f$ is the radius of the $2n - 1$ sphere. The Lagrangian has a global $SU(n)$ and local $U(1)$ symmetry. It is

$$\mathcal{L} = \overline{D_\mu z^a} D_\mu z_a \quad (2.2)$$

where

$$D_\mu = \partial_\mu + \frac{i}{\sqrt{n}} \lambda_\mu \quad (2.3)$$

subject to (2.1). The gauge field is an auxiliary field and can be eliminated by its equations of motion. Thus,

$$\lambda_\mu = \frac{if}{\sqrt{n}} \bar{z} \overleftrightarrow{\partial}_\mu z \quad (2.4)$$

using (2.1).

Fermions can be introduced into the Lagrangian in the following manner⁶⁾:

$$\mathcal{L} = \overline{D_\mu z^a} D_\mu z_a + \bar{\psi}^a \not{\partial} \psi_a - \frac{g}{2n} \left[(\bar{\psi}^a \psi_a)^2 + (\bar{\psi}^a \gamma_5 \psi_a)^2 \right] \quad (2.5)$$

Again (2.5) has a global $SU(n)$ and local $U(1)$ symmetry, but now

$$\lambda_\mu = i \frac{f}{\sqrt{n}} \bar{z} \overleftrightarrow{\partial}_\mu z + \bar{\psi} \gamma_\mu \psi \quad (2.6)$$

If the constraint

$$\bar{z}^a \psi_a = \bar{\psi}^a z_a = 0$$

is imposed and $f = g$ then (2.5) is also invariant under supersymmetry transformations. The model can also be extended to include many flavours.

The generalization of (2.2) to a non-Abelian gauge symmetry has also been considered^{14),15)}. In this case the fields, $z_{a,i}$ take values on a Grassmanian manifold. Here, $i, i = 1, \dots, \ell$ is the index of the non-Abelian $U(\ell)$ symmetry, $\ell < n$. The constraint, (2.1) becomes

$$\bar{z}^{a,i} z_{a,j} = \frac{n}{2\ell} \delta_j^i \quad (2.7)$$

A supersymmetric version of this model has also been considered¹⁶⁾.

Introducing fermions into the model with a non-Abelian gauge symmetry is now straightforward. We obtain,

$$\begin{aligned} \mathcal{L} = & \overline{D_{\mu j}^i z^{a,j}} D_{\mu i}^k z_{a,k} + \bar{\psi}^{a,i} \gamma_{\mu}^i \psi_{a,j} \\ & - \frac{g}{2n} \left[(\bar{\psi}^{a,i} \psi_{a,i})^2 + (\bar{\psi}^{a,i} \gamma_5 \psi_{a,i})^2 \right] \end{aligned} \quad (2.8)$$

where,

$$D_{\mu}^i = \partial_\mu \delta_i^j + \frac{i}{\sqrt{n}} \lambda_{\mu}^i{}^j \quad (2.9)$$

and

$$\lambda_{\mu}^i{}^j = \frac{if}{\sqrt{n}} \bar{z}^{a,i} \overleftrightarrow{\partial}_\mu z_{a,j} + \bar{\psi}^{a,i} \gamma_\mu \psi_{a,j} \quad (2.10)$$

We call this model I. The Lagrangian in (2.8) is invariant under a global $SU(n)$ and local $U(l)$ symmetry. We can write (2.8) in terms of left and right components of the fermion fields. Since a Dirac spinor has only two-components in two dimensions left and right-handed spinors are one component objects. These are best described as left or right movers since they move along the left or right part of the light cone. Thus (2.8) can be written

$$\begin{aligned} \mathcal{L} = & (\overline{D_\mu \psi})^{a,i} (D_\mu \psi)_{a,i} + \overline{\psi}_L^{a,i} D_{-i,j} \psi_{La,j} \\ & + \overline{\psi}_R^{a,i} D_{+i,j} \psi_{Ra,j} - \frac{g}{n} (\overline{\psi}_L^{a,i} \psi_{Ra,j} \overline{\psi}_R^{a,j} \psi_{La,i}) \end{aligned} \quad (2.11)$$

where

$$D_\pm = D_0 \pm i D_1 \quad (2.12)$$

and

$$\begin{aligned} \psi_{(L)} &= \left(\frac{1 \pm i\gamma_5}{2} \right) \psi, \quad \gamma^5 = \gamma^0 \gamma^1 \\ \overline{\psi}_{(L)} &= \overline{\psi} \left(\frac{1 \pm i\gamma_5}{2} \right), \quad (i\gamma^5)^2 = 1 \end{aligned} \quad (2.13)$$

From (2.11) we see that ψ_L only couples to λ_- and ψ_R to λ_+ . As far as the fermionic part of (2.11) is concerned λ_- and λ_+ could be two independent fields. Thus, the fermionic part of (2.11) has a $U(l) \otimes U(l)$, rather than a $U(l)$, symmetry. Of course, this is a feature of two-dimensions. For example, if U and V generate the $U(l) \otimes U(l)$ transformations then

$$\begin{aligned} \psi_{Li} &\rightarrow u_i^j \psi_{Lj}, \quad \overline{\psi}_L^i \rightarrow \overline{\psi}_L^j u_j^{+i} \\ \psi_{Ri} &\rightarrow v_i^j \psi_{Rj}, \quad \overline{\psi}_R^i \rightarrow \overline{\psi}_R^j v_j^{+i} \end{aligned} \quad (2.14)$$

and

$$\begin{aligned} (D_-)_i^k &\rightarrow u_i^j D_{-j}^k u_i^{+k} \\ (D_+)_i^k &\rightarrow v_i^j D_{+j}^k v_i^{+k} \end{aligned} \quad (2.15)$$

This leaves the fermion part of (2.11) invariant. However, this invariance is broken to a $U(1)_{\text{vector}}$ by a non-Abelian axial anomaly. [Of course, the bosonic part of (2.11) is invariant only under $U(1)_{\text{vector}}$]. This anomaly plays an important role in the properties of the model.

To analyze the model in a $1/n$ expansion we must make the Lagrangian quadratic in the fields. Thus, we introduce auxiliary fields and (2.11) becomes

$$\begin{aligned} \mathcal{L} = & (\overline{D_\mu z})^{a,i} (D_\mu z)_{a,i} + \overline{\psi}_L^{a,i} D_{-i}^j \psi_{La,j} \\ & + \overline{\psi}_R^{a,i} D_{+i}^j \psi_{Ra,j} - \frac{1}{\sqrt{n}} [\overline{\psi}_L^{a,i} \psi_{Ra,j} \phi_i^j + \overline{\psi}_R^{a,i} \psi_{La,j} \bar{\phi}_i^j] \\ & + \frac{1}{g} \bar{\phi}_i^j \phi_i^j \end{aligned} \quad (2.16)$$

where

$$\begin{aligned} \phi_i^j &= \frac{g}{\sqrt{n}} \overline{\psi}_R^{a,i} \psi_{La,j} \\ \bar{\phi}_i^j &= \frac{g}{\sqrt{n}} \overline{\psi}_L^{a,i} \psi_{Ra,j} \end{aligned} \quad (2.17)$$

Model I can now be analyzed via the $1/n$ expansion. This will be done in the next section.

At this stage, though, we note a few properties of the model. If the Lorentz scalar condensates ϕ and $\bar{\phi}$, acquire non-zero vacuum expectation values this will break the chiral symmetry of the fermion part. (As noted earlier, this is also broken by the presence of the anomaly.) Since the condensate ϕ_i^j can be a scalar of the gauge group we do not expect the gauge symmetry to be broken in this model.

To break the gauge symmetry the condensate must not be a scalar of the gauge group. This requires a generalization of Model I so that left and right fermions are in different representations of the gauge group. Since we require the condensate to be a Lorentz scalar we cannot have arbitrary condensates. Lorentz scalars in two-dimensions are:

$$\bar{\Psi}_L \Psi_R, \bar{\Psi}_R \Psi_L, \bar{\Psi}_L \bar{\Psi}_R, \Psi_L \Psi_R \quad (2.18)$$

A Lorentz scalar cannot be made out of left-handed fermions in two-dimensions. At present we consider models with the first two condensates. For instance, consider a model where the right fermion is in the (ℓ) and the left in the $(\bar{\ell})$ of $U(\ell)$. The Lagrangian is now

$$\begin{aligned} \mathcal{L} = & (\overline{D_\mu \bar{\chi}})^{a,i} (D_\mu \chi)_{a,i} + \bar{\Psi}_L^a D_-^\alpha \Psi_L^b{}_\alpha \\ & + \bar{\Psi}_R^{a,\alpha} D_{+\alpha}{}^\beta \Psi_{R\alpha,\beta} - \frac{g}{n} [\bar{\Psi}_L^a \Psi_{R\alpha,\beta} \bar{\Psi}_R^{a,\beta} \Psi_{L\alpha,\alpha}] \end{aligned} \quad (2.19)$$

where $\alpha, \beta = 1, \dots, \ell$. In terms of auxiliary fields (2.19) becomes

$$\begin{aligned} \mathcal{L} = & (\overline{D_\mu \bar{\chi}})^{a,i} (D_\mu \chi)_{a,i} + \bar{\Psi}_L^a D_-^\alpha \Psi_L^b{}_\alpha \\ & + \bar{\Psi}_R^{a,\alpha} D_{+\alpha}{}^\beta \Psi_{R\alpha,\beta} - \frac{1}{\sqrt{n}} [\bar{\Psi}_L^a \Psi_{R\alpha,\beta} \bar{\Phi}^{\alpha\beta} \\ & + \bar{\Psi}_R^{a,\alpha} \Psi_{L\alpha}{}^\beta \Phi_{\alpha\beta}] + \frac{1}{g} \Phi_{\alpha\beta} \bar{\Phi}^{\alpha\beta} \end{aligned} \quad (2.20)$$

This Lagrangian defines Model II and has a global $U(n)$ and local $U(\ell)$ symmetry. The condensates, $\phi_{\alpha\beta}$ and $\bar{\phi}^{\alpha\beta}$, are singlets of $U(n)$, allowing (2.20) to be analyzed in a $1/n$ expansion. However, they do not contain a group scalar. If $\phi_{\alpha\beta}, \bar{\phi}^{\alpha\beta}$ acquire non-zero vacuum expectation values the gauge group will be broken.

Again we note that, as for Model I, the fermion part of Model II has a $U(\ell) \otimes U(\ell)$ symmetry. As with Model I this symmetry of the fermion part is broken by a non-Abelian axial anomaly. This anomaly has important physical consequences, which will be discussed in Section IV.

We could consider other models where the gauge group would be broken by the condensates, ϕ and $\bar{\phi}$. Requirements for constructing such models are that the condensates are Lorentz scalar and the gauge current anomaly free. The anomaly has to cancel between left and right fermions, and the representations chosen accordingly.

Currents which are not gauged will then have an anomaly, which will have physical consequences as with our models. Such models are under investigation and will not be discussed further here.

III - PHYSICAL PROPERTIES OF MODEL I

In this section model I is analyzed in the large n expansion [$a = 1, \dots, n$, is the index of the global $U(n)$ symmetry], keeping ℓ fixed and $n > \ell$. Details of the $1/n$ expansion are given in the Appendix. After the explicit integration over the fundamental quark and z fields one obtains an effective theory in terms of the composite fields. Thus, the partition function can be written as follows:

$$Z = \int \mathcal{D}\phi \mathcal{D}\bar{\phi} \mathcal{D}\lambda_\mu \mathcal{D}\alpha \exp - S_{\text{eff}} \quad (3.1)$$

where

$$S_{\text{eff}} = n \text{Tr} \log \Delta_B - n \text{Tr} \log \Delta_F + \frac{1}{g} \int d^2x \phi_i^j \bar{\phi}_j^i + i \int d^2x \alpha_i^j g_i^j \frac{n}{2f} \quad (3.2)$$

and

$$\Delta_B = - D_\mu i^k D_\mu k^j + i \alpha_i^j \quad (3.3)$$

$$\Delta_F = \not{D}_i^j - \frac{1}{\sqrt{n}} (\phi_i^j \not{x}_+ + \bar{\phi}_i^j \not{x}_-) \quad (3.4)$$

As shown in Section II the fermion part of (3.2) (i.e., the part which comes from the ψ integration) is invariant under a chiral $U(\ell) \otimes U(\ell)$ symmetry. However,

this is broken to $U(1)_{\text{vector}}$ in the quantum theory because a fermion bilinear acquires a non-vanishing vacuum expectation values:

$$\left\langle \frac{g}{\sqrt{n}} \bar{\psi}_R^{a,i} \psi_L^{a,j} \right\rangle \equiv \langle \phi_i^j \rangle = \sqrt{n} M \delta_i^j \quad (3.5)$$

Chiral symmetry is also explicitly broken to $U(1)_{\text{vector}}$ by an anomaly. Indeed, the fermionic part of S_{eff} is not invariant under a chiral transformation. Instead, one obtains,

$$\delta \left[\frac{i}{g} \phi_i^j \bar{\phi}_j^i - n \text{Tr} \log \Delta_F \right] = -i \frac{\sqrt{n}}{4\pi} \text{Tr} \int d^2x \, \omega \epsilon_{\mu\nu} F_{\mu\nu} \quad (3.6)$$

where ω is the generator of the chiral transformation (2.14) and (2.15) and

$$F_{\mu\nu} = \partial_\mu \lambda_\nu - \partial_\nu \lambda_\mu + \frac{i}{\sqrt{n}} [\lambda_\mu, \lambda_\nu] \quad (3.7)$$

(Of course, chiral symmetry is also broken by the bosonic contribution, as previously mentioned.)

To study the spectrum of composite states we take the leading term in the $1/n$ expansion of S_{eff} . From the Appendix this is

$$\begin{aligned} S_{\text{eff}}^{(2)} = & \frac{1}{2} \int d^2x \int d^2y \int \frac{d^2k}{(2\pi)^2} e^{ik(x-y)} \Gamma(k) \\ & \text{Tr} \left\{ \lambda_\mu(y) (\delta_{\mu\nu} k^2 - k_\mu k_\nu) \lambda_\nu(x) + \partial_\mu \varphi_+(x) \partial_\mu \varphi_+(y) \right. \\ & + 4M^2 \varphi_+(x) \varphi_+(y) + \partial_\mu \varphi_-(x) \partial_\mu \varphi_-(y) + 4iM \epsilon_{\mu\nu} \partial_\mu \lambda_\nu^{(x)} \varphi_-(y) \\ & \left. + \alpha(x) \alpha(y) \right\} \end{aligned} \quad (3.8)$$

where, for simplicity, we have assumed that the two mass scales, m and M , obtained through dimensional transmutation are equal. In (3.8) we have

$$\phi_i^j = \varphi_i^j + \sqrt{n} M \delta_i^j \quad (3.9)$$

$$\alpha_i^j = \alpha_i^j + i\sqrt{n} m \delta_i^j \quad (3.10)$$

and

$$\phi_+^j = \frac{1}{2} (\phi_i^j + \bar{\phi}_i^j) \quad (3.11)$$

$$\phi_-^j = \frac{i}{2} (\phi_i^j - \bar{\phi}_i^j)$$

ϕ_-^j is the multiplet of would-be Goldstone bosons. Finally,

$$\Gamma(k) = \int \frac{d^2 q}{(2\pi)^2} \frac{1}{(q^2 - M^2) [(q+k)^2 - M^2]} \quad (3.12)$$

The resulting two-point functions are explicitly exhibited in the Appendix. The properties of the spectrum can be summarized as follows:

(i) Despite there being no dynamical symmetry breaking of the gauge group $U(1)_{\text{vector}}$, the gauge fields acquire a mass. This screens the confining Coulomb potential. This is a consequence of there being massless quarks in the original Lagrangian. The same phenomenon occurs in the massless Schwinger model. Of course, a long range confining potential would appear if we had inserted a small mass term for the "quarks", as in the CP^{n-1} and Schwinger models.

(ii) Whilst chiral symmetry is spontaneously broken no massless particle results. This is because the anomaly breaks the invariance of the fermionic effective Lagrangian - Eq. (3.6). The anomaly manifests itself by producing the mixed term in (3.8), $\epsilon_{\mu\nu} \partial_\mu \lambda_\nu \phi_-$ and all the ϕ_- 's acquire a mass. This is a straightforward generalization of the mechanism in the Abelian case, where the singlet acquires a mass through the $U(1)$ anomaly^{6),8)}. In our case the entire multiplet of fields $(\phi_-)_i^j$ acquire a large mass (M) via the non-Abelian anomaly.

To show the difference between dynamical and elementary Higgs mesons we consider a low energy approximation of (3.2), assuming that the composite fields are slowly varying functions of x . Limiting ourselves only to terms relevant to the two-point functions we obtain the low energy Lagrangian:

$$\begin{aligned} \mathcal{L} = & \text{Tr} \frac{1}{4} e^2 F_{\mu\nu}^2 + \frac{1}{2} \bar{D}_\mu \bar{\phi} D_\mu \phi \\ & + V_1(\phi, \bar{\phi}) + \epsilon_{\mu\nu} F_{\mu\nu} V_2(\phi, \bar{\phi}) \end{aligned} \quad (3.13)$$

where each field has been redefined by a factor $1/2\sqrt{\pi} M$, this factor coming from the expansion of $\Gamma(k)$ [see (A.22)]. The covariant derivative and field strength are given by,

$$\begin{aligned} (D_\mu \phi)_i{}^j &= \partial_\mu \phi_i{}^j + \frac{i}{\sqrt{n}} [\lambda_\mu, \phi]_i{}^j \\ F_{\mu\nu}{}_i{}^j &= \partial_\mu \lambda_\nu{}_i{}^j - \partial_\nu \lambda_\mu{}_i{}^j + \frac{i}{\sqrt{n}} [\lambda_\mu, \lambda_\nu]_i{}^j \end{aligned} \quad (3.14)$$

where ϕ have been redefined by a factor $1/2\sqrt{\pi} M$ and the two potentials are

$$V_1(\phi, \bar{\phi}) = M^2 \phi \bar{\phi} \left[\log \left(2 \frac{\phi \bar{\phi}}{f^2} \right) - 1 \right] \quad (3.15)$$

$$f = \sqrt{\frac{n}{2\pi}}, \quad e = 2\sqrt{\pi} M$$

$$V_2(\phi, \bar{\phi}) = \frac{\sqrt{n}}{8\pi} [\log(\bar{\phi}) - \log(\phi)] \quad (3.16)$$

V_1 implies that ϕ gets a non-vanishing vacuum expectation value and has been computed in Ref. 17). V_2 is a consequence of the axial anomaly and ensures the transformation properties (3.6).

The existence of the term V_2 is the distinguishing feature between a dynamical and elementary Higgs model. There is no such term in an elementary Higgs model because there is no anomaly. The absence of V_2 implies that the

field, ϕ_- , would be left massless in an elementary Higgs model. These fields are the Goldstone bosons of chiral symmetry breaking. They are left massless because the gauge symmetry is unbroken.

In conclusion, the existence of the anomaly produces interesting differences between dynamical and elementary Higgs system. Of course, model I does not generate a real Higgs system because the gauge group is unbroken. However, it does demonstrate interesting mechanisms at work in dynamical models. In the next section we study model II. We demonstrate that this is an explicit model of a tumbling gauge theory.

IV - AN EXPLICIT MODEL OF A TUMBLING GAUGE THEORY

In this section we study the properties of model II in the $1/n$ expansion. Performing the integration over the ψ and z fields and proceeding as in model I we obtain the effective action,

$$S_{\text{eff}} = n \text{Tr} \log \Delta_B - n \text{Tr} \log \Delta_F + \frac{1}{g} \int d^2x \phi_{\alpha\beta} \bar{\varphi}^{\beta\alpha} + i \int d^2x \alpha_i^j S_i^j \frac{n}{2f} \quad (4.1)$$

where

$$\Delta_F = \begin{pmatrix} D_+ \alpha^\beta & \frac{1}{\sqrt{n}} \phi_{\alpha\beta} \\ \frac{1}{\sqrt{n}} \bar{\phi}^{\alpha\beta} & D_- \alpha_\beta \end{pmatrix} \quad (4.2)$$

and Δ_B is as in (3.3). Again, the fermionic part of the classical action, in (2.20) is invariant under a $U(\ell) \otimes U(\ell)$ chiral symmetry. This is again broken by the axial anomaly and the contribution of the z fields to $U(\ell)_{\text{vector}}$.

The novel feature of this model with respect to model I is that the $U(1)$ vector gauge symmetry is dynamically broken in the quantum theory. This is because the "quark" bilinear, $\phi_{\alpha\beta}$, which acquires a non-zero vacuum expectation value, is not a singlet under the gauge group

$$\frac{g}{\sqrt{n}} \langle \bar{\psi}_{R\alpha} \psi_{L\beta} \rangle \equiv \langle \phi_{\alpha\beta} \rangle = \sqrt{n} M_{\alpha\beta} \quad (4.3)$$

The analogous saddle point equation to (A.9)

$$\frac{\delta_\alpha^\gamma}{g} = \int \frac{d^2 p}{(2\pi)^2} \frac{1}{(\delta_\alpha^\gamma p^2 + M_{\alpha\beta} M^{\beta\gamma})} \quad (4.4)$$

gives

$$M_{\alpha\beta} M^{\beta\gamma} = M^2 \delta_\alpha^\gamma \quad (4.5)$$

for the matrix M . This breaks the gauge symmetry. The spectrum of composite Higgs and gauge bosons, as well as the direction of symmetry breaking, can be analyzed in the $1/n$ expansion. Following the procedure previously described for model I the leading order term is,

$$\begin{aligned} S_{\text{eff}}^{(2)} = & \int d^2 x \int d^2 y \int \frac{d^2 k}{(2\pi)^2} e^{ik(x-y)} \Gamma(k) \\ & \text{Tr} \left\{ (\delta_{\mu\nu} k^2 - k_\mu k_\nu) \left[A_{\mu\alpha}^{(y)\beta} A_{\nu\beta}^{(x)\alpha} + B_{\mu\alpha}^{(y)\beta} B_{\nu\beta}^{(x)\alpha} \right] \right. \\ & + 4M^2 B_{\mu\alpha}^{(y)\beta} B_{\mu\beta}^{(x)\alpha} + \partial_\mu \varphi_{-\alpha}^{(y)\beta} \partial_\mu \varphi_{-\beta}^{(x)\alpha} + \partial_\mu \varphi_{+\alpha}^{(y)\beta} \partial_\mu \varphi_{+\beta}^{(x)\alpha} \\ & + 4M^2 \varphi_{+\alpha}^{(y)\beta} \varphi_{+\beta}^{(x)\alpha} - 4M \partial_\mu \varphi_{-\alpha}^{(y)\beta} B_{\mu\beta}^{(x)\alpha} \\ & \left. + 4iM \epsilon_{\mu\nu} \partial_\mu \lambda_{\nu\alpha}^{(y)\beta} \varphi_{-\beta}^{(x)\alpha} + \alpha(y)\alpha(x) \right\} \end{aligned} \quad (4.6)$$

where

$$\left. \begin{aligned} B_{\mu\alpha}{}^{\beta} &= \frac{1}{2} \left(\lambda_{\mu\alpha}{}^{\beta} - \frac{M_{\alpha\gamma} \lambda_{\mu}{}^{\gamma}{}_{\beta} M^{\beta\gamma}}{M^2} \right) \\ A_{\mu\alpha}{}^{\beta} &= \frac{1}{2} \left(\lambda_{\mu\alpha}{}^{\beta} + \frac{M_{\alpha\gamma} \lambda_{\mu}{}^{\gamma}{}_{\beta} M^{\beta\gamma}}{M^2} \right) \end{aligned} \right\} \quad (4.7)$$

$$\left. \begin{aligned} \varphi_{-\alpha}{}^{\beta} &= \frac{i}{2M} \left(\varphi_{\alpha\gamma} M^{\gamma\beta} - M_{\alpha\gamma} \bar{\varphi}^{\gamma\beta} \right) \\ \varphi_{+\alpha}{}^{\beta} &= \frac{1}{2M} \left(\varphi_{\alpha\gamma} M^{\gamma\beta} + M_{\alpha\gamma} \bar{\varphi}^{\gamma\beta} \right) \end{aligned} \right\} \quad (4.8)$$

and

$$\phi_{\alpha\beta} = \sqrt{n} M_{\alpha\beta} + \varphi_{\alpha\beta} \quad (4.9)$$

From (4.6) it is easy to see that the gauge group is broken. The gauge bosons, B_{μ} , acquire a non-zero mass, whilst the A_{μ} gauge bosons are left massless. However, because of (4.5) the unitary matrix, $M_{\alpha\beta}$, enters in the definition of the new fields, Eq. (4.7), but does not appear in $S^{(2)}$. Thus, to leading order in $1/n$ there is no information on the direction of symmetry breaking. At this order we have a degeneracy of vacuum states related to each other by a unitary transformation. To determine the vacuum alignment, and thus the unbroken subgroup, we have to go beyond the leading order; This is in progress¹²⁾.

To analyze the spectrum of states we take, for simplicity,

$$M_{\alpha\beta} = M \delta_{\alpha\beta} \quad (4.10)$$

The unbroken subgroup is then $O(\ell)$. [Similar considerations can be made for other unbroken subgroups of $SU(\ell)$, for instance, $Sp(\ell)$.] From (4.10) we can write the fields, (4.7) and (4.8) as follows:

$$\left. \begin{aligned} B_{\mu}^{\alpha} &= B_{\mu}^i (t^i)^{\alpha} \\ A_{\mu}^{\alpha} &= A_{\mu}^a (\tau^a)^{\alpha} \end{aligned} \right\} \quad (4.11)$$

$$\varphi_{-}^{\alpha} = A^a (\tau^a)^{\alpha} + B^i (t^i)^{\alpha} \quad (4.12)$$

$$\varphi_{+}^{\alpha} = c^s (T^s)^{\alpha} \quad (4.13)$$

where T^s are a representation of the generators of $SU(\ell)$, normalized such that

$$\text{Tr} (T^r T^s) = \delta^{rs}$$

whilst

$$(\tau^a)^{\alpha} = \frac{1}{2} (T^a{}^{\alpha} - T^{*a}{}^{\alpha}) \quad (4.14)$$

$$(t^i)^{\alpha} = \frac{1}{2} (T^i{}^{\alpha} + T^{*i}{}^{\alpha}) \quad (4.15)$$

These are respectively the $\ell(\ell-1)/2$ imaginary Hermitian matrices, spanning an $O(\ell)$ algebra, and the $\ell(\ell+1)/2$ real symmetric $\ell \times \ell$ matrices. Notice that the index $a[i]$ runs over the unbroken $O(\ell)$ subgroup [broken part of $U(\ell)$].

Inserting (4.11) to (4.13) into (4.6) we obtain

$$S_{eff}^{(2)} = \frac{1}{2} \int d^2 x \int d^2 y \int \frac{d^2 k}{(2\pi)^2} e^{ik(x-y)} \Gamma(k) \times \quad (4.16)$$

$$\begin{aligned}
 & \times \left\{ (\delta_{\mu\nu} k^2 - k_\mu k_\nu) (B_\mu^i(y) B_\nu^i(x) + A_\mu^a(y) A_\nu^a(x)) \right. \\
 & + 4M^2 B_\mu^i(y) B_\mu^i(x) + \partial_\mu A_\nu^a(y) \partial_\mu A_\nu^a(x) + \partial_\mu B_\nu^i(y) \partial_\mu B_\nu^i(x) \\
 & + 4M^2 C_\mu^f(y) C_\mu^f(x) + \partial_\mu C_\nu^f(y) \partial_\mu C_\nu^f(x) - 4M^2 (\partial_\mu B_\nu^i(y)) B_\mu^i(x) \\
 & \left. + 4i A_\nu^a(y) \epsilon_{\mu\nu} \partial_\mu A_\nu^a(x) + \alpha(x) \alpha(y) \right\} \quad (4.16)
 \end{aligned}$$

We can now see explicitly the dynamical Higgs mechanism. Defining

$$W_\mu^i = B_\mu^i - 2M \partial_\mu B^i \quad (4.17)$$

then (4.16) becomes

$$\begin{aligned}
 S_{\text{eff}}^{(2)} &= \frac{1}{2} \int d^2x \int d^2y \int \frac{d^2k}{(2\pi)^2} e^{ik(x-y)} \Gamma(k) \\
 & \left\{ (\delta_{\mu\nu} k^2 - k_\mu k_\nu) (W_\mu^i(y) W_\nu^i(x) + A_\mu^a(y) A_\nu^a(x)) \right. \\
 & + 4M^2 W_\mu^i(y) W_\mu^i(x) + \partial_\mu A_\nu^a(y) \partial_\mu A_\nu^a(x) + \partial_\mu C_\nu^f(y) \partial_\mu C_\nu^f(x) + 4M^2 C_\mu^f(y) C_\mu^f(x) \\
 & \left. + \alpha(x) \alpha(y) + 4i A_\nu^a(y) \epsilon_{\mu\nu} \partial_\mu A_\nu^a(x) \right\} \quad (4.18)
 \end{aligned}$$

The B^i fields have been eaten by the W_μ^i fields, and thus disappear from the spectrum. Apart from the massive field W_μ^i the action (4.18) is of the same form as (3.7) of model I. Thus the same considerations discussed in Section III apply to the unbroken part of model II. In particular, the A^i fields obtain a mass from the axial anomaly. The diagonalized two-point functions for this model are given in the Appendix.

In conclusion, the main features of model II can be summarized as follows:

(i) The $U(1)_{\text{vector}}$ symmetry is dynamically broken by non-singlet fermion condensates. However, the direction of symmetry cannot be established to leading order in $1/n$.

(ii) The gauge bosons in the broken part of the gauge group acquire a mass through a dynamical Higgs mechanism. This is explicitly shown to happen in a $1/n$ expansion.

(iii) The gauge bosons in the unbroken subgroup also acquire a mass, as in the case of model I. This is because our theory contains massless fermions in the classical Lagrangian. At the quantum level M is the dynamically generated fermion mass. No massless fermions remain.

(iv) Finally, the scalar fields, A^i , not participating in the dynamical Higgs mechanism should remain massless due to chiral symmetry breaking. However, they acquire a mass from the coupling with the gauge bosons in the unbroken sector, due to the non-Abelian axial anomaly, as in model I.

V - DISCUSSION AND CONCLUSION

In this paper we have presented a class of two-dimensional models which are a non-Abelian generalization of the CP^{n-1} model with fermions. These models have several interesting features. In both models discussed the fermion part is invariant under a chiral $U(\ell) \otimes U(\ell)$ symmetry. This is broken to $U(\ell)_{\text{vector}}$ by a non-Abelian axial anomaly and the vacuum expectation value acquired by the fermion bilinear (as well, of course, by the bosonic sector). The presence of this anomaly distinguishes between dynamical and elementary Higgs models.

For example, in model I no particles remain massless. The would-be Goldstone bosons from chiral symmetry breaking acquire a mass through the anomaly. This is a generalization of the Abelian case when the " η " acquires a mass through the $U(1)$ anomaly. Here the entire multiplet ϕ_i^j acquire a mass M . Because our original Lagrangian contain massless fermions, the gauge bosons acquire a mass. Thus, the long range force is screened, as in the massless Schwinger model.

Model II is an explicit example of the dynamical breaking of a gauge symmetry. A fermion condensate, which is not a group singlet, dynamically breaks the $U(\ell)$ symmetry. However, to leading order the direction of symmetry breaking cannot be established. There is a manifold of vacuum states related by a unitary transformation. However, one can easily see that the gauge bosons in the broken sector acquire a mass through a dynamical Higgs mechanism. The gauge bosons in the unbroken sector also acquire a mass, but through the "screening" mechanism, as in model I. Finally, because of the anomaly the scalar fields not participating in the Higgs mechanism (would-be Goldstone bosons from chiral symmetry breaking) acquire a mass.

Whilst we have only considered two specific models here our results can be generalized to other models. For example, we could take the left and right fields in different representations of the gauge groups, e.g., 5 and $\overline{10}$ of $SU(5)$. The only constraint is that the number of representations have to be chosen so that the anomaly in the gauged current cancels between left and right fermions. The ungauged currents will have anomalies and give masses to the scalars in an analogous way to that described here, the masses acquired being of the order of the scale of symmetry breaking.

Our results could have wider implications. Particles acquiring mass through an (ungauged) anomaly in dynamical symmetry breaking schemes should have a mass of order of the scale of symmetry breaking, as shown by our results. Also, our analysis allows us to write down an effective Lagrangian incorporating the effect of a non-Abelian anomaly. This allows an investigation of masses and interactions of particles in dynamical symmetry breaking schemes, taking into account the gauging of flavour symmetries due to the weak interactions¹⁸⁾. These and related investigations are in progress.

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APPENDIX

In this Appendix we give the details of the $1/n$ expansion for model I and the two-point functions.

The partition function for model I can be written as

$$Z = \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}z \mathcal{D}\bar{z} \mathcal{D}\phi \mathcal{D}\bar{\phi} \mathcal{D}\lambda_\mu \mathcal{D}\alpha \exp - \int d^2x \mathcal{L} \quad (\text{A.1})$$

where

$$\begin{aligned} \mathcal{L} = & (\overline{D_\mu z})^i (D_\mu z)_i + i \bar{z}^i \alpha_i^j z_j - \alpha_i^j \frac{n}{2f} \delta_i^j \\ & + \bar{\psi}^i (\not{D}_i^j - \frac{1}{\sqrt{n}} \phi_i^j \gamma_+ - \frac{1}{\sqrt{n}} \bar{\phi}_i^j \gamma_-) \psi_j \\ & + \frac{1}{g} \phi_i^j \bar{\phi}_j^i \end{aligned} \quad (\text{A.2})$$

The Lagrange multiplier, α_i^j , imposed the constraint (2.7) and

$$\gamma_\pm = \left(\frac{1 \pm i\gamma_5}{2} \right) \quad (\text{A.3})$$

We have omitted the global indices on z and ψ . Since the action is quadratic in the z and ψ fields we can explicitly perform the functional integral over these fields. We obtain,

$$Z = \int \mathcal{D}\phi \mathcal{D}\bar{\phi} \mathcal{D}\lambda_\mu \mathcal{D}\alpha \exp - S_{\text{eff}} \quad (\text{A.4})$$

where

$$\begin{aligned} S_{\text{eff}} = & n \text{Tr} \log \Delta_B - n \text{Tr} \log \Delta_F \\ & + \frac{1}{g} \phi_i^j \bar{\phi}_j^i + i \alpha_i^j \frac{n}{2f} \delta_i^j \end{aligned} \quad (\text{A.5})$$

and

$$\Delta_B = -D_\mu i^k D_{\mu k}^j + i \alpha_i^j \quad (A.6)$$

$$\Delta_F = \not{D}_i^j - \frac{1}{\sqrt{n}} (\phi_i^j \gamma_+ + \bar{\phi}_i^j \gamma_-) \quad (A.7)$$

The functional integrals over the remaining fields are performed using a saddle point approximation. The saddle point conditions give non-zero vacuum expectation values to α_i^j , ϕ , $\bar{\phi}$ as follows

$$\langle \alpha_i^j \rangle = i m \delta_i^j \sqrt{n} \quad (A.8)$$

$$\langle \phi_i^j \rangle = \langle \bar{\phi}_i^j \rangle = M \delta_i^j \sqrt{n}$$

where m and M are the spontaneously generated masses for the z and ψ fields respectively.

The renormalization conditions on the two coupling constants are

$$\frac{1}{g} = \int \frac{d^2 p}{(2\pi)^2} \frac{1}{(p^2 + M^2)} \quad (A.9)$$

and

$$\frac{1}{2f} = \int \frac{d^2 p}{(2\pi)^2} \frac{1}{(p^2 + m^2)} \quad (A.10)$$

Due to dimensional transmutation the two coupling constants disappear from the effective action, being replaced by the generated masses, m and M respectively.

We now expand (A.5) around the saddle point in powers of $1/\sqrt{n}$. The kinetic term for the composites is $O(1)$ and is given by:

$$\begin{aligned}
 S_{eff}^{(2)} = & \frac{1}{2} \int d^2x \int d^2y \int \frac{d^2k}{(2\pi)^2} e^{ik(x-y)} \cdot \\
 & \cdot \left\{ \lambda_\mu(y) \Gamma_{\mu\nu}^\lambda \lambda_\nu(x) + \alpha(y)_i^j \Gamma_\alpha(k) \alpha(x)_j^i \right. \\
 & + \varphi(y)_i^j \Gamma_\varphi(k) \varphi(x)_j^i + \bar{\varphi}(y)_i^j \Gamma_{\bar{\varphi}}(k) \bar{\varphi}(x)_j^i \quad (A.11) \\
 & + \varphi(y)_i^j \Gamma_{\varphi\bar{\varphi}}(k) \bar{\varphi}(x)_j^i + \epsilon_{\mu\nu} \lambda_\mu(y) \Gamma_\nu^{\lambda\varphi}(k) \cdot \\
 & \cdot (\bar{\varphi}(x) - \varphi(x)) \left. \right\}
 \end{aligned}$$

where

$$\begin{aligned}
 \Gamma_{\mu\nu}^\lambda(k) = & - \int \frac{d^2q}{(2\pi)^2} \text{Tr} \left\{ \frac{(i\not{q} + M) \gamma_\mu (i\not{q} + k + M) \gamma_\nu}{(q^2 + M^2) ((q+k)^2 + M^2)} \right\} \quad (A.12) \\
 & + \int \frac{d^2q}{(2\pi)^2} \frac{-2\delta_{\mu\nu}}{q^2 + m^2} + \frac{(2q+k)_\mu (2q+k)_\nu}{(q^2 + m^2) ((q+k)^2 + m^2)}
 \end{aligned}$$

$$\Gamma^\alpha(k) = \int \frac{d^2q}{(2\pi)^2} \frac{1}{(q^2 + m^2) ((q+k)^2 + m^2)} \equiv \Gamma(k, m) \quad (A.13)$$

$$\Gamma(k)^\varphi = \Gamma(k)^{\bar{\varphi}} \equiv \Gamma(k, M) \cdot M^2 \quad (A.14)$$

$$\Gamma^{\varphi \bar{\varphi}}(k) = (k^2 + 2M^2) \Gamma(k, M) \quad (\text{A.15})$$

and

$$\Gamma_{\nu}^{\lambda \varphi}(k) = -2 k_{\nu} M \Gamma(k, M) \quad (\text{A.16})$$

We have defined

$$\begin{aligned} \alpha_i^j &= \sqrt{n} m \delta_i^j + \varphi_i^j \\ \phi_i^j &= \sqrt{n} M \delta_i^j + \varphi_i^j \end{aligned} \quad (\text{A.17})$$

To obtain the physical spectrum we must diagonalize (A.11). We first define the scalar and pseudoscalar fields from ϕ and $\bar{\phi}$. These are

$$\begin{aligned} \varphi_{+i}^j &= \frac{1}{2} (\varphi_i^j + \bar{\varphi}_i^j) \\ \varphi_{-i}^j &= \frac{i}{2} (\varphi_i^j - \bar{\varphi}_i^j) \end{aligned} \quad (\text{A.18})$$

Then (A.11) becomes

$$\begin{aligned} S_{\text{eff}}^{(2)} &= \frac{1}{2} \int d^2x \int d^2y \int \frac{d^2k}{(2\pi)^2} e^{ik(x-y)} \Gamma(k) \cdot \\ &\cdot \left\{ (k^2 \delta_{\mu\nu} - k_{\mu} k_{\nu}) \lambda_{\mu}(y) \lambda_{\nu}(x) \right. \\ &+ \partial_{\mu} \varphi_{+i}^j(y) \partial_{\mu} \varphi_{+j}^i(x) + 4M^2 \varphi_{+i}^j(y) \varphi_{+j}^i(x) \\ &+ \partial_{\mu} \varphi_{-i}^j(y) \partial_{\mu} \varphi_{-j}^i(x) - 4M i \epsilon_{\mu\nu} \partial_{\mu} \lambda_{\nu}(y) \varphi_{-}(x) \\ &\left. + \mathcal{L}(x) \mathcal{L}(y) \right\} \end{aligned} \quad (\text{A.19})$$

where

$$\Gamma(k) = \int \frac{d^2 q}{(2\pi)^2} \frac{1}{(q^2 + M^2)((k+q)^2 + M^2)} \quad (\text{A.20})$$

$$\xrightarrow{k \rightarrow 0} \frac{1}{4\pi M^2} \quad (\text{A.21})$$

and we have taken $m = M$ in (A.19). This now corresponds to (3.8).

We can now obtain the physical two-point functions for model I. These are:

$$\langle \lambda_{\mu}(x)_i^j \lambda_{\nu}(y)_j^k \rangle = \int \frac{d^2 k}{(2\pi)^2} e^{ik(x-y)} \frac{\delta_i^k \left(\delta_{\mu\nu} - \frac{k_{\mu} k_{\nu}}{k^2} \right)}{\Gamma(k) (k^2 + 4M^2)} \quad (\text{A.22})$$

$$\langle \varphi_+(x)^a \varphi_+(y)^b \rangle = \int \frac{d^2 k}{(2\pi)^2} e^{ik(x-y)} \frac{\delta^{ab}}{\Gamma(k) (k^2 + 4M^2)} \quad (\text{A.23})$$

$$\langle \varphi_-(x)^a \varphi_-(y)^b \rangle = \int \frac{d^2 k}{(2\pi)^2} e^{ik(x-y)} \frac{\delta^{ab}}{\Gamma(k) (k^2 + 4M^2)} \quad (\text{A.24})$$

$$\langle \alpha(x) \alpha(y) \rangle = \int \frac{d^2 k}{(2\pi)^2} e^{ik(x-y)} \frac{1}{\Gamma(k)} \quad (\text{A.25})$$

Following the same procedure for model II we obtain the analogue to (A.19). This is Eq. (4.6). The physical two-point functions are obtained from (4.18). These are

$$\langle W_\mu(x)^i W_\nu(y)^j \rangle = \int \frac{d^2 k}{(2\pi)^2} e^{ik(x-y)} \frac{\delta^{ij} \left(\delta_{\mu\nu} + \frac{k_\mu k_\nu}{4M^2} \right)}{\Gamma(k) (k^2 + 4M^2)} \quad (A.26)$$

$$\langle A_\mu(x)^a A_\nu(y)^b \rangle = \int \frac{d^2 k}{(2\pi)^2} e^{ik(x-y)} \frac{\delta^{ab} \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right)}{\Gamma(k) (k^2 + 4M^2)} \quad (A.27)$$

$$\langle c^r(x) c^s(y) \rangle = \int \frac{d^2 k}{(2\pi)^2} e^{ik(x-y)} \frac{\delta^{rs}}{\Gamma(k) (k^2 + 4M^2)} \quad (A.28)$$

$$\langle A^a(x) A^b(y) \rangle = \int \frac{d^2 k}{(2\pi)^2} e^{ik(x-y)} \frac{\delta^{ab}}{\Gamma(k) (k^2 + 4M^2)} \quad (A.30)$$

As noted in Section IV all particles obtain a mass, M .

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