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SUPERGRAVITY, R INVARIANCE AND SPONTANEOUS
SUPERSYMMETRY BREAKING

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ABSTRACT

We show that in order for a $U(1)$ gauge theory with a Fayet-Iliopoulos term to be consistently coupled to supergravity, preserving gauge invariance, the superpotential must be R invariant. A supersymmetric cosmological term and therefore an explicit mass-like term for the gravitino is forbidden by gauge invariance. This result severely constrains the possible models for non-gravitational interactions. We comment on possible mass terms for the gauginos induced by gravitational effects.

Supersymmetric theories of electroweak and strong interactions offer an appealing possibility of constraining possible grand unified theory models and even the standard model of non-unified electroweak and strong interactions based on the gauge group $SU(3) \times SU(2) \times U(1)$. In addition, the existence of non-renormalization theorems could solve the so-called gauge hierarchy problem.

However, it has been observed that in order to have realistic super-symmetric models of electroweak and strong interactions at the tree approximation, the gauge group must be larger than $SU(3) \times SU(2) \times U(1)$ ¹⁾.

The simplest possibility, and the only one explored to date, is based on the group $SU(3) \times SU(2) \times U(1) \times \tilde{U}(1)$ where the group $\tilde{U}(1)$ acts chirally on each generation of quarks and leptons. The main reason for having this extra $\tilde{U}(1)$ gauge invariance is to avoid unwanted mass relations²⁾ among quarks, leptons and their supersymmetric scalar partners which must be heavier than their fermionic counterparts in order to have a realistic model.

In this paper we show that the possibility of a consistent coupling to supergravity gives a selection rule in the presence of a Fayet-Iliopoulos D term³⁾ for this extra gauge group. Consistency requires that the superpotential of the globally supersymmetry Lagrangian be R invariant⁴⁾.

Let us consider the general form of a globally supersymmetric gauge theory with group $G' = G \times \tilde{U}(1)$, where G refers to gauge interactions without the Fayet-Iliopoulos term.

In superspace language one has the supersymmetric field strength of the gauge fields⁴⁾

$$W_\alpha = \frac{1}{g} \bar{D} \bar{D} (e^{-gV} D_\alpha e^{gV}) \quad (1)$$

and a set of chiral multiplets S^a transforming under G' .

The over-all action is

$$\int d^4x \int d^4\theta \bar{S} e^{gV} S + \text{Re} \int d^4x d^2\theta [\text{Tr} W^\alpha W_\alpha + f(S^a)] + \int d^4x \int d^4\theta V \quad (2)$$

where $f(S^a)$ is a G' invariant superpotential

$$\frac{\partial f}{\partial S^a} (R^A)_a{}^b S_b = 0 \quad (3)$$

$f(S^a)$ must be a polynomial of at most degree 3 in order to preserve renormalizability.

It is well known that the over-all potential is given by the following general expression

$$\frac{1}{2} D^A{}^2 + \frac{1}{2} |f_{,a}|^2 \quad (4)$$

where $f_{,a} = \partial f / \partial z^a$ and

$$D^A = g_A \bar{Z}_b R^A{}_a{}^b Z^a \quad A \in G, \quad D_{\tilde{Y}} = g \bar{Z} \tilde{Y} Z + \xi \quad (5)$$

ξ is the coefficient of the Fayet-Iliopoulos term, given by the last term in Eq. (2), and $\tilde{Y}$ is the generator for the $\tilde{U}(1)$ gauge group

$$\xi D_{\tilde{Y}}(x) = \xi \int d^4\theta V(x, \theta) \quad (6)$$

In the minimal auxiliary field formulation of supergravity, the above expression can only be extended to local supersymmetry in the form⁵⁾

$$\int d^4x d^4\theta \frac{1}{k^2} e^{\xi k^2 V} E \quad (7)$$

where E is the superspace determinant and $k^2 = 1/M_{Pl}^2$ is the gravitational constant. The local $\tilde{U}(1)$ gauge transformation in superspace which leaves Eq. (7) invariant is a combination of the flat space gauge transformation

$$V \rightarrow V + \frac{i}{g} (\Lambda - \bar{\Lambda}) \quad \bar{D}_\alpha \Lambda = 0 \quad (8)$$

and of a compensating super-Weyl transformation⁶⁾

$$E \rightarrow E e^{\Sigma + \bar{\Sigma}} \quad \bar{D}_\alpha \Sigma = 0 \quad (9)$$

where Σ is related to Λ by

$$\Sigma = -\frac{i}{g} \xi k^2 \Lambda \quad (10)$$

The consequence of Eq. (10) is that the gravitino, as well as all the fermionic partners λ of the gauge fields (gauginos), acquire charges proportional to ξk^2 ⁷⁾. Explicitly we have

$$\tilde{q}(\text{gravitino}) = \tilde{q}(\text{gauginos}) = -\frac{3}{2g} \xi k^2$$

We consider now the local expression which corresponds to Eq. (2). It is obvious that for conformal couplings which are separately invariant under $\tilde{U}(1)$ and super-Weyl transformations, no restriction arises from Eq. (10) ⁵⁾. Therefore, any additional gauge interactions and cubic superpotentials are allowed. However, for an arbitrary superpotential the local expression becomes

$$R e \int d^4x d^4\theta \frac{E}{R} f(S^a) \quad (11)$$

Under a combined $\tilde{U}(1)$ and super Weyl transformation, Eq. (11) transforms as

$$\int d^4x d^4\theta \frac{E}{R} e^{3\Sigma} f(S^a e^{-i q_a \Lambda - \Sigma}) \quad (12)$$

and because of Eq. (10) this requires

$$e^{-i\alpha} f(S^a e^{-i\beta_a}) = f(S^a) \quad (13)$$

with

$$\alpha = \frac{3}{g} \xi k^2 \Lambda, \quad \beta_a = (\delta q_a - \frac{\xi k^2}{g}) \Lambda$$

where δq_a are the differences between the charges of the chiral multiplets ⁸⁾ and those flat space values which are determined by Eq. (13). Note that the charges $q_a = q_a^{\text{flat}} + \delta q_a$ depend on ξk^2 if $f(S)$ is not conformally invariant. For instance, if $f(S) = S^n$ then $q_a = (n-3/n)\xi k^2$, so in this case for $k \rightarrow 0$ we have $q_a \rightarrow 0$. In general the q_a 's do not vanish for $k \rightarrow 0$ but reduce to their flat space values. Equation (13) is nothing but the statement that the superpotential is R invariant. We note that the restriction on $f(S^a)$ is obtained only because we require a consistent coupling to the supergravity Lagrangian which preserves the $\tilde{U}(1)$ gauge invariance. The local $\tilde{U}(1)$ invariance is a combination of the flat superspace $\tilde{U}(1)$ and R invariance.

In the coupling of a matter action to supergravity ⁹⁾ we have that the potential derived from the superpotential is modified as follows

$$\frac{1}{2} |f_{,a}|^2 - \frac{3}{4} \kappa^2 \left| f - \frac{1}{3} z^a f_{,a} \right|^2, \quad f = f(z^a) \quad (14)$$

which in turn implies that f can be at most cubic in order to have a polynomial potential bounded from below. The gravitational action

$$-\int d^4x \frac{1}{\kappa^2} \sqrt{-g} R \quad (15)$$

receives a Brans-Dicke-like modification due to the improvement term

$$\frac{1}{12} \sqrt{-g} |z|^2 R \quad (16)$$

which comes from the conformally invariant part of the matter action. In order to have the correct sign for the Einstein term in the classical Lagrangian we get a bound on the v.e.v. in our choice of scalar variables

$$|z|^2 < 6 M_{Pl}^2 \quad (17)$$

Most importantly, Eq. (13) rules out the possibility of an explicit cosmological term.

The $\tilde{U}(1)$ gauge invariant gravitino Yukawa coupling is¹⁰⁾

$$\frac{1}{2} \sqrt{-g} \bar{\Psi}_R^\mu \sigma_\mu \Psi_R^\nu f(z) \quad (18)$$

Therefore the gravitino gets a mass-like term if and only if $\langle f(z) \rangle \neq 0$, i.e., R symmetry is spontaneously broken. From Eq. (18) we see that an explicit mass term for the gravitino is forbidden by gauge invariance since $\psi_{\mu L}$ transforms under $\tilde{U}(1)$.

The vacuum expectation value of $f(z)$ in Eq. (18) differs from the true gravitino mass for two reasons. Firstly, due to Eq. (16), the Rarita-Schwinger Lagrangian also has an improvement term¹⁰⁾

$$-\frac{1}{2} \left(1 - \frac{1}{6} \kappa^2 z z^* \right) \varepsilon^{\mu\nu\rho\sigma} \bar{\Psi}_\mu \gamma_5 \gamma_\nu D_\rho \Psi_\sigma$$

so if $\langle z \rangle \neq 0$ f must be scaled. Secondly, it is well known that a massless spin 3/2 field requires a mass-like term if the cosmological constant does not vanish¹¹⁾. Nonetheless, it may turn out that supersymmetry is spontaneously broken when $\langle f \rangle \neq 0$.

The over-all potential of the matter system coupled to supergravity is $\Omega^2 V(\Omega^{-1} z)$ where

$$V(z) = \frac{1}{2} D^A{}^2 + \frac{1}{2} |f_{,a}|^2 - \frac{3}{4} k^2 / f - \frac{1}{3} z^a f_{,a} / f^2 \quad (19)$$

The expression involving $\Omega = (1 + (1/6) k^2 z z^*)^{1/2}$ occurs because one must solve self-consistently the coupled gravity-matter field equations^{*)}. The D term related to $\tilde{U}(1)$ contains new terms coming from fields which were neutral in flat space and which acquired charges proportional to ξk^2 because of local $\tilde{U}(1)$ invariance. Note that the correct normalization of the $\tilde{U}(1)$ D term in the potential is

$$\frac{1}{2} \left(\frac{3}{2} \xi - \frac{g}{4} q_a |z_a|^2 \right)^2$$

with q_a as given by Eq. (13). Because of the last term in Eq. (19) the potential has the possibility of having vanishing vacuum energy even if supersymmetry is spontaneously broken (super Higgs effect)¹⁰⁾. However, because of the restricted form of f due to R invariance it is not evident whether such a circumstance can occur. For instance, for $f(z) = az$ one can make the vacuum energy vanish by relating ξ to a , but this occurs just at the point $|z|^2 = 6 M_{Pl}^2$ which saturates the bound (17). Neglecting for a moment the problem of the cosmological constant in the presence of the D term, one clearly realizes that the breaking of supersymmetry is linked to the breaking of R symmetry. In addition, if supersymmetry is spontaneously broken such that the gravitino receives a mass term, then the gauginos can also have radiatively induced masses¹²⁾ because of gravitational couplings in the local gauge-invariant Lagrangian given by¹³⁾

$$\frac{1}{M_{Pl}^2} \bar{\Psi}_\mu \gamma^\rho \gamma^\sigma \gamma^\lambda \gamma^A \bar{\Psi}_\rho \gamma_\sigma \gamma_\lambda \quad (20)$$

which after Fierz re-arrangement provides a four-fermionic interaction of dimension six of the type

$$\frac{1}{M_{Pl}^2} \lambda_L^A \lambda_L^A \bar{\Psi}_{\mu R} \Psi_R^A + \dots \quad (21)$$

*) We thank Mike Duff for a discussion on this point.

The one-loop diagram in the Figure can give a mass to the λ particle of order $m_\lambda^2 \sim m_{3/2}^2 \Lambda^2 / M_{Pl}^2$. If we set the ultra-violet cut-off $\Lambda = M_{Pl}$ then $m_\lambda \approx m_{3/2}$. If the spin 3/2 particle has a mass in the GeV range¹⁴⁾ instead of an almost vanishing mass as predicted in earlier literature¹⁾, there will be no problem in satisfying the experimental constraints on m_λ . The coupling given by Eq. (20) is a suggestive mechanism for gluino mass generation for which there may well be other mechanisms as well.

It is evident that our results rely on the fact that we fixed the flat Lagrangian to be given by Eqs. (2) and (7). This was a consequence of the requirement that we demanded polynomiality in the matter fields S^a of the theory. There exists a new formulation of supergravity¹⁵⁾ in which the potential is still polynomial but is strictly positive. If the Lagrangian in our formulation reduces to this alternative formulation after appropriate Weyl rescalings, this would imply that the cancellation of the cosmological term, in the presence of a Fayet-Iliopoulos term, is not possible. The previous action (2) can be generalized by relaxing the requirement that the superspace Lagrangian is polynomial¹⁰⁾. In fact, it is possible to include $\tilde{U}(1)$ gauge invariant non-polynomial interactions as follows

$$\int d^4x d^4\theta E \left[\frac{1}{k^2} e^{\xi k^2 V} \Phi(X) + \text{Re} \left(W^\alpha W_\alpha + f(S^a) \right) \frac{1}{R} \right]$$

where $f(S)$ is an arbitrary R symmetric superpotential while $\Phi(X)$ is an arbitrary function of the $\tilde{U}(1)$ gauge invariant superfield variable $X = S e^{(gq - \xi k^2)V} \bar{S}$. In order to include the kinetic term of S and the Fayet-Iliopoulos term, $\Phi(X)$ must start with $1 + k^2 X + \dots$. Obviously if S transforms under $G \subset G'$ an appropriate exponential of the relative G gauge field is included in the definition of X .

Finally we want to stress that our restrictions on matter interactions came from the requirement of $\tilde{U}(1)$ gauge invariance in local supersymmetry. However, it may be possible to have an explicit but soft breaking of $\tilde{U}(1)$ symmetry which does not ruin the consistency of the theory¹⁶⁾.

REFERENCES

- 1) P. Fayet, in "Unification of the Fundamental Particle Interactions", eds. S. Ferrara, J. Ellis and P. van Nieuwenhuizen (Plenum Press, N.Y., 1980) p. 587.
- 2) S. Ferrara, L. Girardello and F. Palumbo, Phys. Rev. D20 (1979) 403;
P. Fayet, Phys. Lett. 84B (1979) 416;
S. Dimopoulos and H. Georgi, Nucl. Phys. B193 (1981) 150.
- 3) P. Fayet and J. Iliopoulos, Phys. Lett. 51B (1974) 61.
- 4) See, for instance:
P. Fayet and S. Ferrara, Phys. Rep. 32C (1977) No. 5, 249.
- 5) K.S. Stelle and P.C. West, Nucl. Phys. B145 (1978) 175.
- 6) For a review on superspace-supergravity see, for instance:
B. Zumino, in "Recent Developments in Gravitation", eds. M. Levy and S. Deser (Plenum Press, N.Y., 1978) p. 405.
- 7) D.Z. Freedman, Phys. Rev. D15 (1977) 1173;
A. Das, M. Fischler and M. Rocek, Phys. Rev. D16 (1977) 3427.
- 8) In this connection earlier attempts have been made by:
S. Ferrara, L. Girardello and B. Zumino, unpublished (1978);
M. Rocek and U. Lindstrom, Phys. Lett. 83B (1979) 179.
- 9) S. Ferrara and P. van Nieuwenhuizen, Phys. Lett. 76B (1978) 404;
K.S. Stelle and P.C. West, Phys. Lett. B77 (1978) 376.
- 10) E. Cremmer et al., Nucl. Phys. B147 (1979) 105.
- 11) D.Z. Freedman and A. Das, Nucl. Phys. B120 (1977) 221;
P.K. Townsend, Phys. Rev. D15 (1977) 2808.
- 12) R. Barbieri and D.V. Nanopoulos, Phys. Lett. 91B (1980) 369; Phys. Lett. 95B (1980) 43.
- 13) S. Ferrara, J. Scherk and P. van Nieuwenhuizen, Phys. Rev. Lett. 37 (1976) 1669;
S. Ferrara, J. Scherk, P. van Nieuwenhuizen and F. Gliozzi, Nucl. Phys. B117 (1976) 333;
D.Z. Freedman and J. Schwarz, Phys. Rev. D15 (1977) 1007.
- 14) R. Barbieri, S. Ferrara and D.V. Nanopoulos, CERN preprint TH-3226 (1982).
- 15) M.F. Sohnius and P.C. West, Phys. Lett. 105B (1981) 353.
- 16) J. Wess, talk given at the Second Europhysics Study Conference on Unification of the Fundamental Particle Interactions, Erice (1981).

FIGURE CAPTION: Radiatively induced gaugino mass.