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NUMERICAL ESTIMATE OF THE SU(3) GLUEBALL MASS

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A B S T R A C T

We estimate the mass of the SU(3) glueball 0^+ to be $m(0^+) = (350 \pm 50)\Lambda^L$. With a reasonable relation between the string tension and Λ^L this gives $m(0^+) = (920 \pm 310)$ MeV.

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Early Monte Carlo (MC) estimates^{1),2)} of the SU(2) glueball mass from plaquette-plaquette correlations can be improved considerably by projecting out eigenstates and using a variational method³⁾⁻⁵⁾. However, only the results of Ref. 3), and a recent rather different investigation⁶⁾, give self-consistent results, in the sense that the suspected scaling behaviour is really seen. References 4) and 5) also consider excited glueball states. In the present note we apply the MC variational method [details are given in Ref. 3)] to the SU(3) glueball. As quark corrections are expected to be small⁷⁾⁻⁹⁾, the obtained mass should come close to the experimental value.

Let us first fix the notation. We consider SU(3) lattice gauge theory with the Wilson action. At each link b of a hypercubic four-dimensional lattice there is an element $U(b) \in \text{SU}(3)$ and averages are calculated with the partition function

$$Z = \int \prod_b dU(b) \exp \left\{ -\frac{\beta}{3} \sum_p (1 - \text{Re Tr}(U(p))) \right\} \quad (1)$$

For each plaquette p , $U(p)$ is the ordered product of the four link matrices surrounding the plaquette and dU is the SU(3) Haar measure.

With $\beta = 6/g^2$ the standard definition of the lattice mass scale is

$$\Lambda^L = a^{-1} (\beta_0 g^2)^{-\frac{\beta_1}{2\beta_0^2}} \exp \left(-\frac{1}{2\beta_0 g^2} \right), \quad (2)$$

where

$$\beta_0 = \frac{11}{3} \left(\frac{3}{16\pi^2} \right) \quad \text{and} \quad \beta_1 = \frac{34}{3} \left(\frac{3}{16\pi^2} \right)^2$$

The Λ parameters of perturbation theory can be related to Λ^L . For SU(3) $\Lambda^{\text{MOM}} = 83.5 \Lambda^L$ ¹⁰⁾. Previously the SU(3) glueball mass has been investigated within the strong coupling (SC) expansion by Münster¹¹⁾.

The obtained estimate was

$$m(0^+) = (660 \pm 170) \Lambda^L \quad (3)$$

The reliability of the SC method is, however, questionable. For the SU(2) gauge group some aspects of SC series expansions are discussed in Ref. 3).

We now describe our variational calculation. Relying on our experience with SU(2), we consider the three operators of Fig. 1 in the fundamental representation and perform an MC calculation for the six correlation functions between these operators. The MC calculation is carried out on a $4^3 \cdot 8$ lattice with cyclic boundary conditions. 8 is identified with the time direction. Correlations are measured between different time planes at distance $t = 0, 1, 2$. We sum over all space positions to project onto the zero momentum states and sum over all space-like orientations to select out the $J = 0$ state. This means our correlation functions are

$$g_{ij}(t) = \langle \square_i(0) \square_j(t) \rangle$$

with

$$\square_i(t) = \sum_{\vec{x}, \text{orientations}} (\sigma_i(\vec{x}, t) - \langle \sigma_i(\vec{x}, t) \rangle) \quad (4)$$

The normalization of the operators σ_i is taken such that $\rho_{ii}(0) = 1/18$ for $\beta = 0$.

For the upgrading of the lattice we have used the SU(3) heat bath method of Pietarinen¹²⁾. On the calculation we have spent 17 h CDC 7600 computer time, roughly 2/3 for upgrading and 1/3 for measurements. The whole procedure has turned out to be slower by a factor of 30 than in the similar calculation for SU(2)³⁾. We had spent about the same time on the SU(2) calculation, hence the computer time used for the SU(3) calculation is rather small.

Our main results are given in Figs. 2 and 3 and in the tables. They are more encouraging than even an optimist might have hoped. Figure 2 represents MC data for the glueball mass from plaquette-plaquette correlations at a distance $t = 1$, and Fig. 3 gives the glueball mass after minimization over contributions from all the six considered correlations (again at a distance $t = 1$) has been carried out. The three solid lines are parallel to the weak coupling renormalization group (RG) prediction (2), and represent the glueball estimates from Figs. 2 and 3.

In Fig. 2 we have also included the spin wave result (tree approximation for $\beta \rightarrow \infty$ on the finite lattice), and (as in Fig. 3) the first two orders of the SC expansion ($\beta \rightarrow 0$). See the broken lines. From the tables the interested reader may read off the correlations of the operators and the number of sweeps used for measurements. We have first used 600 sweeps to bring one lattice "adiabatically" into the considered scaling region, and then spent at each value of β an additional number of 200 sweeps for equilibrium. For plaquette-plaquette correlations at distance $t = 0$ we have checked our MC data in the SC limit, and for the high value $\beta = 10$ we have checked our MC results on the spin wave result of Fig. 2.

In Figs. 2 and 3 and in the tables the mean values are from all data and the error bars are obtained by comparison relative to 10 samples of consecutive events. To prevent statistical fluctuations of the error bars, the number of subsamples has to be reasonably large. On the other hand, as explained in Ref. 3), subsamples have to be rather big to give a good wave function and to minimize the bias brought about by the minimization procedure. Our way of calculating the error bar in some sense also accounts for the bias. The bias of mean values from averages over rather large samples of size 400 lowers for $5.2 \leq \beta \leq 5.6$ the result by 5% in the worst case ($\beta = 5.2, 5.6$). We have also obtained results at distance $t = 2$. They are still afflicted by rather large error bars and do not yet allow a reliable minimization. For the plaquette-plaquette correlations alone the mean values for the glueball mass from distance $t = 2$ are (as expected) systematically below the mean values from distance $t = 1$, and the result is well compatible with Eq. (5) below.

The results of Fig. 3, as well as of Fig. 2, fit very well the weak coupling RG prediction and allow the rather accurate relation

$$m(0^+) = (350 \pm 50) \Lambda^L \quad (5)$$

The error bar in Eq. (5) comes from a trajectory of points which follow the weak coupling RG behaviour, and tries to estimate (in a somewhat subjective manner) the order of magnitude of possible corrections in the approach to the continuum limit. This kind of error bar should not be confused with the statistical error bar coming from a single point, as used in the final results of Refs. 4)-8).

There are several notable points. First of all, contrary to our SU(2) results, the glueball mass from correlations of 0^+ plaquette eigenstates at a distance $t = 1$, already exhibits scaling in a certain region of β . An estimate

of the glueball mass from these correlations alone (Fig. 2) would give a value $m(0^+) = (380 \pm 50)\Lambda^L$, which is already close to the value (5). That the one-plaquette operator gives a reasonable result in a certain range of the scaling region comes presumably from the higher complexity of the SU(3) gauge group. Before starting this investigation we had conjectured this, but we did not expect the effect to be so drastic. After minimization the glueball wave functions turn out to be rather flat in the involved operators. Our SU(3) results are self-consistent in the sense that the data follow the predicted scaling behaviour. With respect to this they are even better than our SU(2) results³⁾.

Our glueball mass data undershoot (in the region where we observe scaling) the strong coupling series of Ref. 11) (except the PADE [2,5]). Therefore the value (5) is smaller than the SC extrapolation (3). There is, however, an overlap within two error bars, and it is notable that the lowest value within Münster's error bars gives a tangent to the highest order of the SC expansion.

Our value (5) is in disagreement (i.e., no overlap within two error bars) with a value $M = (720 \pm 100)\Lambda^L$ quoted in Ref. 7). No details are given there. We would like to remark that reliable results cannot be obtained from finite size scaling effects without having a detailed understanding of the degeneracy and the excited levels.

A subtle question is to extract a GeV value for the glueball mass from our result (5). If SU(3) hadron spectroscopy^{7),9)} continues to make rapid progress, the best attitude is certainly to make a comparison with experimentally well-known quark bound states. We here rely on the MC estimates of the SU(3) string tension. The values are

$$\Lambda^L = (5 \pm 1.5) \cdot 10^{-3} \sqrt{K} \quad [\text{Ref. 13)]}$$

and

$$\Lambda^L = (7 \pm 2) \cdot 10^{-3} \sqrt{K} \quad [\text{Ref. 12)]}$$

As these values are less accurate than our estimate of the glueball $m(0^+)$, we take the mean value of Eq. (5) for the glueball mass, and $\Lambda^L = (6 \pm 2) \cdot 10^{-3} \sqrt{K}$ for the string tension. With $\sqrt{K} = 440$ MeV we thus obtain

$$m(0^+) = (320 \pm 310) \text{ Mev.} \quad (6)$$

Summary and outlook: we have found scaling behaviour for the $SU(3)$ glueball mass $m(0^+)$ in a certain region of the coupling β . With Eq. (5) we have obtained an accurate value for the glueball mass in units of Λ^L . In a forthcoming paper we will present results for the excited $SU(3)$ glueball states. In the more distant future one may include quark corrections.

β	$C_{11}(0)$	$C_{22}(0)$	$C_{33}(0)$	$C_{11}(1)$	$C_{22}(1)$	$C_{33}(1)$	Sweeps
5.0	0.095 ± 0.002	0.178 ± 0.005	0.408 ± 0.011	0.004 ± 0.001	0.009 ± 0.003	0.015 ± 0.007	800
5.1	0.093 ± 0.002	0.186 ± 0.004	0.444 ± 0.006	0.008 ± 0.002	0.015 ± 0.004	0.040 ± 0.007	800
5.2	0.099 ± 0.002	0.210 ± 0.003	0.495 ± 0.011	0.011 ± 0.001	0.024 ± 0.002	0.058 ± 0.003	2,000
5.3	0.100 ± 0.001	0.228 ± 0.004	0.530 ± 0.011	0.014 ± 0.001	0.033 ± 0.003	0.081 ± 0.009	2,000
5.4	0.102 ± 0.001	0.257 ± 0.003	0.583 ± 0.009	0.017 ± 0.001	0.045 ± 0.002	0.106 ± 0.006	2,000
5.5	0.104 ± 0.002	0.298 ± 0.007	0.640 ± 0.013	0.021 ± 0.002	0.070 ± 0.005	0.150 ± 0.010	2,000
5.6	0.106 ± 0.004	0.339 ± 0.013	0.693 ± 0.027	0.029 ± 0.003	0.107 ± 0.012	0.216 ± 0.025	2,000
5.8	0.069 ± 0.001	0.247 ± 0.007	0.464 ± 0.010	0.008 ± 0.001	0.037 ± 0.005	0.062 ± 0.008	400
6.0	0.061 ± 0.001	0.222 ± 0.006	0.406 ± 0.013	0.004 ± 0.002	0.022 ± 0.006	0.030 ± 0.012	200

Table 1

Correlations of the operations O_i ($i = 1, \dots, 3$) of Fig. 1 with themselves and the number of sweeps for measurement.

β	$C_{12}(0)$	$C_{13}(0)$	$C_{23}(0)$	$C_{12}(1)$	$C_{13}(1)$	$C_{23}(1)$
5.0	0.110 ± 0.003	0.178 ± 0.005	0.214 ± 0.006	0.007 ± 0.002	0.007 ± 0.003	0.010 ± 0.005
5.1	0.112 ± 0.002	0.186 ± 0.003	0.231 ± 0.005	0.012 ± 0.002	0.019 ± 0.003	0.024 ± 0.005
5.2	0.125 ± 0.002	0.205 ± 0.004	0.268 ± 0.006	0.016 ± 0.001	0.025 ± 0.002	0.038 ± 0.003
5.3	0.133 ± 0.002	0.215 ± 0.004	0.297 ± 0.006	0.022 ± 0.002	0.033 ± 0.003	0.050 ± 0.004
5.4	0.145 ± 0.002	0.230 ± 0.003	0.339 ± 0.005	0.027 ± 0.001	0.042 ± 0.002	0.068 ± 0.004
5.5	0.160 ± 0.003	0.245 ± 0.005	0.391 ± 0.009	0.039 ± 0.003	0.057 ± 0.004	0.101 ± 0.006
5.6	0.175 ± 0.007	0.260 ± 0.010	0.444 ± 0.019	0.055 ± 0.006	0.078 ± 0.010	0.151 ± 0.018
5.8	0.118 ± 0.003	0.171 ± 0.003	0.302 ± 0.008	0.016 ± 0.003	0.022 ± 0.003	0.049 ± 0.007
6.0	0.104 ± 0.003	0.151 ± 0.004	0.266 ± 0.008	0.009 ± 0.003	0.011 ± 0.005	0.027 ± 0.008

Table 2

Mixed correlations of the operators O_i of Fig. 1

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FIGURE CAPTIONS

- Fig. 1 : Operators considered in our calculation.
- Fig. 2 : Glueball results from plaquette-plaquette correlations.
- Fig. 3 : Glueball results after minimization.

Operator # 1

Operator # 2

Operator # 3

Fig. 1

Fig. 2

Fig. 3