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DYNAMICAL SUPERSYMMETRY BREAKING IN A
FINITE FIELD THEORETICAL MODEL

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A B S T R A C T

We present a supersymmetric field theory in two or three space-time dimensions with an internal symmetry of the $O(N)$ type. In the large N limit the model is finite and supersymmetry is spontaneously broken. The fields representing the order parameters of the broken supersymmetry phase acquire dynamics through quantum corrections. In particular the Goldstone fermion is a zero-mass fermionic bound state.

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Supersymmetric field theories have many attractive features. For example, they may lead to the solution of the hierarchy problem or the non-renormalizability of quantum gravity. Indeed, convergence of the perturbation theoretical expressions for physical quantities, such as S-matrix elements, is usually much improved by the introduction of supersymmetry. It is even conjectured, that $N = 4$ supersymmetric Yang-Mills theory and $N = 8$ supergravity are completely finite to all orders in perturbation theory¹⁾.

However, the problem of obtaining a realistic particle spectrum in supersymmetric field theories has proved to be a very difficult one. In particular the spontaneous breaking of supersymmetry presents a major obstacle. In supersymmetric gauge theories, spontaneous supersymmetry breaking is not induced by radiative corrections²⁾. Therefore it has either to be effected at the tree level, or must result from non-perturbative effects, such as instantons³⁾, or large N behaviour⁴⁾. In supergravity, which is the gauge theory of local supersymmetry, the difficulties are even more complex and intriguing⁵⁾. Here certainly non-perturbative effects, such as the formation of bound states, must play a major rôle in the generation of the low energy spectrum, and one may expect that spontaneous supersymmetry breaking is closely connected to the solution of this problem.

In view of the above, it would be of interest to have a model which exhibits all these features simultaneously: supersymmetry, finiteness, generation of composite states and spontaneous supersymmetry breaking. It is the purpose of this letter to describe such a model. The model we present is a two-dimensional field theory with an internal $O(N_1) \times O(N_2)$ symmetry, which is an extension of the supersymmetric $O(N)$ σ model⁶⁾. It is constructed from the scalar supermultiplets

$$\vec{\Phi}_\nu = (\vec{\varphi}_\nu, \vec{\psi}_\nu, \vec{F}_\nu) \quad \text{and} \quad \vec{\Psi}_\nu = (\vec{\varphi}_\nu, \vec{\omega}_\nu, \vec{G}_\nu), \quad *)$$

in the vector representation of $O(N_\nu)$, $\nu = 1, 2$. The multiplets $\vec{\Phi}_\nu$ satisfy the constraints

$$\sum_\nu g_\nu \vec{\varphi}_\nu^2 = 0, \quad \sum_\nu g_\nu \vec{\varphi}_\nu \cdot \vec{\psi}_\nu = 0, \quad \sum_\nu g_\nu (\vec{F}_\nu \cdot \vec{\varphi}_\nu - \frac{1}{2} \vec{\psi}_\nu \cdot \vec{\psi}_\nu) = 0, \quad (1)$$

where the g_ν are constants.

*) The first component in each multiplet is a real scalar of dimension 0, the second a Majorana spinor of dimension 1/2, and the third an (auxiliary) scalar of dimension 1.

These constraints, which form a scalar supermultiplet by themselves, are imposed by introducing a scalar multiplet of Lagrange multipliers $S = (s, u, D)$, which are singlets under the internal symmetry. The Lagrangian reads:

$$\mathcal{L} = \sum_{\nu} \left\{ -\frac{1}{2}(\partial_{\mu} \vec{\Phi}_{\nu})^2 - \frac{1}{2}(\partial_{\mu} \vec{\Psi}_{\nu})^2 - \frac{1}{2} \vec{U}_{\nu} \cdot \delta \vec{U}_{\nu} - \frac{1}{2} \vec{W}_{\nu} \cdot \delta \vec{W}_{\nu} + \frac{1}{2} \vec{F}^2 + \frac{1}{2} \vec{G}^2 \right. \\ \left. - \vec{F}_{\nu} \cdot \frac{\partial \mathcal{V}}{\partial \vec{\Phi}_{\nu}} - \vec{G}_{\nu} \cdot \frac{\partial \mathcal{V}}{\partial \vec{\Psi}_{\nu}} - D \frac{\partial \mathcal{V}}{\partial s} + \frac{1}{2} (u, \vec{U}_{\nu}, \vec{W}_{\nu}) \mathcal{V}'' \left(\frac{u}{\vec{W}_{\nu}} \right) \right\}, \quad (2)$$

where $\mathcal{V}$, the superpotential, is the following function of the physical scalar fields:

$$\mathcal{V}(s, \vec{\Phi}_{\nu}, \vec{\Psi}_{\nu}) = \sum_{\nu} g_{\nu} \left[\frac{1}{2} s \vec{\Phi}_{\nu}^2 + m \vec{\Phi}_{\nu} \cdot \vec{\Psi}_{\nu} \right]. \quad (3)$$

The matrix of its second derivatives, denoted by $\mathcal{V}''$, is:

$$\mathcal{V}'' = \begin{pmatrix} 0 & g_{\nu} \vec{\Phi}_{\nu} & 0 \\ g_{\nu} \vec{\Phi}_{\nu} & g_{\nu} s \vec{1} & g_{\nu} m \vec{1} \\ 0 & g_{\nu} m \vec{1} & 0 \end{pmatrix}. \quad (4)$$

Furthermore we impose the restriction

$$N = N n_{\nu}, \quad \sum_{\nu} n_{\nu} g_{\nu} = 0. \quad (5)$$

This makes the theory finite in the large N limit, keeping the ratio $N_1/N_2 = -g_2/g_1$ fixed. The constraints (1) clearly make this model a generalization of the supersymmetric $O(N)$ σ model. As is the case there, the constraint fields (s, u) in (2) will be shown to become dynamical through quantum corrections. At the same time the auxiliary field D acquires a vacuum expectation value that breaks supersymmetry, resulting in mass splittings in the various supermultiplets.

In particular the spinor u is massless and becomes the Goldstone fermion of the spontaneously broken supersymmetry. Thus supersymmetry is broken dynamically in this model.

We now analyze the model in detail. An effective action for the singlet multiplet S can be obtained by integrating out the $O(N_V)$ vector fields in the path integral. This leads to a graded determinant which can be written as the exponent of the following expression:

$$S_{\text{eff}} = -\frac{N}{2} \text{tr} \left[\sum_{\nu} n_{\nu} (\ln B_{\nu} - \ln F_{\nu}) \right], \quad (6)$$

with

$$B_{\nu} = \begin{pmatrix} -\square + m^2 g_{\nu}^2 + g_{\nu} D + g_{\nu}^2 (S^2 + \bar{u} (\not{\partial} - g_{\nu} S - m^2 g_{\nu}^2 \frac{\not{\partial}}{\square})^{-1} u) & m g_{\nu}^2 S \\ m g_{\nu}^2 S & -\square + m^2 g_{\nu}^2 \end{pmatrix},$$

$$F_{\nu} = \begin{pmatrix} -\not{\partial} + g_{\nu} S & m g_{\nu} \\ m g_{\nu} & -\not{\partial} \end{pmatrix}. \quad (7)$$

The trace in (6) includes integrations over space-time indices. We emphasize that owing to the condition (5) S_{eff} is completely finite for arbitrary N ⁷⁾. As indicated below in the limit $N \rightarrow \infty$ only quadratic terms in S_{eff} have to be retained; hence no new divergences appear and, in this limit, the theory is fully finite. A general discussion of finiteness in two-dimensional scalar models will be presented elsewhere⁷⁾.

In order to be able to perform a $1/N$ expansion of S_{eff} , we make the following rescalings:

$$g_{\nu} \rightarrow \frac{g_{\nu}}{\sqrt{N}}, \quad (8)$$

$$m \rightarrow m\sqrt{N},$$

and introduce vacuum expectation values for S and D :

$$S = \bar{S} \sqrt{N} + \sigma \quad (9)$$

$$D = \bar{D} \sqrt{N} + \Delta$$

The actual values of $\bar{s}$ and $\bar{D}$ will be determined from the saddle point condition for (6), which implies the vanishing of terms of $O(\sqrt{N})$, i.e., those linear in the quantum fluctuations σ, Δ . Finally we introduce the propagators:

$$\Pi_B^\nu = \frac{1}{(-\square + g_\nu^2 m^2)(-\square + g_\nu^2 m^2 + g_\nu \bar{D} + g_\nu^2 \bar{s}^2) - m^2 g_\nu^4 \bar{s}^2} \begin{pmatrix} -\square + m^2 g_\nu^2 & -m g_\nu^2 \bar{s} \\ -m g_\nu^2 \bar{s} & -\square + m^2 g_\nu^2 + g_\nu \bar{D} + g_\nu^2 \bar{s}^2 \end{pmatrix}$$

$$\Pi_F^\nu = \frac{1}{\delta(-\delta + g_\nu \bar{s}) + m^2 g_\nu^2} \begin{pmatrix} \delta & m g_\nu \\ m g_\nu & \delta - g_\nu \bar{s} \end{pmatrix}. \quad (10)$$

With these notations, the effective action can be written:

$$S_{\text{eff}} = -U_0 - \frac{N}{2} \text{tr} \sum_\nu n_\nu \left\{ -\ln \left(1 + \Pi_F^\nu \begin{pmatrix} \frac{g_\nu^2}{N} \sigma & 0 \\ 0 & 0 \end{pmatrix} \right) + \right.$$

$$\left. + \ln \left(1 + \Pi_B^\nu \begin{pmatrix} \frac{1}{N} (g_\nu \Delta + 2 g_\nu^2 \bar{s} \sigma) + \frac{g_\nu^2}{N} (\sigma^2 + \bar{u} (\delta - g_\nu \bar{s} - g_\nu^2 m^2 \frac{\delta}{\square} - g_\nu \sigma) u) & \frac{m g_\nu^2 \sigma}{N} \\ \frac{m g_\nu^2 \sigma}{N} & 0 \end{pmatrix} \right) \right\} \quad (11)$$

where U_0 denotes the vacuum energy:

$$U_0 = -\frac{N}{2} \text{tr} \sum_\nu n_\nu (\ln \Pi_B^\nu - \ln \Pi_F^\nu). \quad (12)$$

U_0 represents in fact the complete contribution of terms of $O(N)$ to the action. As explained above the terms of $O(\sqrt{N})$ must vanish. This leads to the determination of the values of $\bar{s}$ and $\bar{D}$. First, requiring that the term linear in σ vanishes, gives:

$$-\bar{s} \bar{D} \sum_\nu n_\nu g_\nu^3 \ln \left\{ \frac{\square (-\square + m^2 g_\nu^2)}{[(-\square + m^2 g_\nu^2)^2 - g_\nu^2 \bar{s}^2 \square + g_\nu \bar{D} (-\square + m^2 g_\nu^2)] [(-\square + m^2 g_\nu^2)^2 - g_\nu^2 \bar{s}^2 \square]} \right\} = 0 \quad (13)$$

implying either $\bar{s}$ or $\bar{D}$ to be zero. The other condition, that terms linear in Δ are absent, reads after performing the trace:

$$0 = \frac{-1}{16\pi} \sum_{\nu} n_{\nu} g_{\nu} \left\{ \ln(m^2 g_{\nu}^4 + g_{\nu}^2 \bar{D}) + (-1)^{\nu} \frac{\bar{D} + g_{\nu} \bar{s}^2}{\sqrt{\bar{D}^2 + \bar{s}^2 (4m^2 g_{\nu}^2 + 2g_{\nu} \bar{D} + g_{\nu}^2 \bar{s}^2)}} \cdot \ln \left(\frac{\bar{D} + g_{\nu} (\bar{s}^2 + 2m^2) + (-1)^{\nu} \sqrt{\bar{D}^2 + \bar{s}^2 (4m^2 g_{\nu}^2 + 2g_{\nu} \bar{D} + g_{\nu}^2 \bar{s}^2)}}{\bar{D} + g_{\nu} (\bar{s}^2 + 2m^2) - (-1)^{\nu} \sqrt{\bar{D}^2 + \bar{s}^2 (4m^2 g_{\nu}^2 + 2g_{\nu} \bar{D} + g_{\nu}^2 \bar{s}^2)}} \right) \right\}. \quad (14)$$

with $(-1)^{\nu} \equiv g_{\nu}/|g_{\nu}|$ and excluding $\bar{D} = \bar{s} = 0$. In this last case, or whenever $\bar{D} = 0$, Eq. (14) is replaced by

$$\sum_{\nu} n_{\nu} g_{\nu} \ln g_{\nu}^2 = 0. \quad (15)$$

This equation is not fulfilled unless $g_1^2 = g_2^2$. Hence in general we must have

$$\begin{aligned} \bar{s} &= 0, \\ \bar{D} &= -m^2(g_1 + g_2). \end{aligned} \quad (16)$$

We conclude that supersymmetry is broken for $g_1^2 \neq g_2^2$. Notice, that when $g_1^2 = g_2^2$ supersymmetry is unbroken and the Lagrangian possesses the extra invariance

$$\begin{aligned} \vec{\Phi}_1 &\leftrightarrow \pm \vec{\Phi}_2, \\ \vec{\Psi}_1 &\leftrightarrow \pm \vec{\Psi}_2, \\ S &\rightarrow -S. \end{aligned} \quad (17)$$

However, in this case the saddle point approximation does not determine the vacuum expectation value $\bar{s}$.

It remains to calculate the two-point functions and establish the mass splittings in the supermultiplets, as well as the existence of a Goldstone fermion. Expanding Eq. (11) to $O(N^0)$, we obtain

$$\begin{aligned} S_{\text{eff}} = & -U_0 - \frac{1}{2} \hbar \sum_{\nu} n_{\nu} g_{\nu}^2 \left\{ \frac{1}{-\square + M^2} \sigma \frac{-\square}{-\square + \mu_{\nu}^2} \sigma + \left(\frac{\partial_{\mu} \sigma}{-\square + \mu_{\nu}^2} \right)^2 \right. \\ & \left. - \frac{1}{-\square + M^2} \bar{u} \frac{\delta}{-\square + \mu_{\nu}^2} u - \frac{1}{2} \left(\frac{1}{-\square + M^2} \Delta \right)^2 \right\}. \end{aligned} \quad (18)$$

with

$$\begin{aligned} M^2 &= -g_1 g_2 m^2, \\ \mu_v^2 &= g_v^2 m^2. \end{aligned} \quad (19)$$

Notice that for $g_1 = -g_2$ we obtain $\bar{D} = 0$ and all the masses defined in (19) are equal as expected.

From Eq. (18) we infer that σ and u become indeed propagating quantum fields, while Δ appears only quadratically (in the limit of large N) and can be integrated away. The spinor u is massless, but requires a finite wave function renormalization. Namely, its two point function reads

$$\frac{1}{2} \sum_v \frac{g_v^2}{16\pi^3} \int d^4 q \bar{u}(-q) \not{A}_v(q^2) u(q), \quad (20)$$

where

$$A_v(q^2) = \frac{1}{M^2 - \mu_v^2} \left(1 + \frac{\mu_v^2}{M^2 - \mu_v^2} \ln \frac{M^2}{\mu_v^2} \right) + O(q^2).$$

Thus $\not{A}_v(q^2)$ has a zero at $q^2 = 0$, and u has indeed become the Goldstone spinor of broken supersymmetry, as expected from the non-zero vacuum expectation value of its associated auxiliary field D .

The scalar field σ on the other hand acquires a non-zero mass, as expected in two dimensions. This confirms the supersymmetry breaking. Its two-point function reads:

$$\frac{1}{2} \sum_v \frac{g_v^2}{16\pi^3} \int d^4 q \sigma(-q) B_v(q^2) \sigma(q), \quad (21)$$

with

$$\begin{aligned} B_v(q^2) &= \ln \frac{M^2}{\mu_v^2} - \frac{1}{\sqrt{(q^2 + M^2 - \mu_v^2)^2 + 4q^2 \mu_v^2}} \ln \left(\frac{q^2 + M^2 + \mu_v^2 - \sqrt{(q^2 + M^2 - \mu_v^2)^2 + 4q^2 \mu_v^2}}{q^2 + M^2 + \mu_v^2 + \sqrt{(q^2 + M^2 - \mu_v^2)^2 + 4q^2 \mu_v^2}} \right) \\ &\quad + \frac{1}{2} \frac{q^2 + 2\mu_v^2}{\sqrt{q^2(q^2 + 4\mu_v^2)}} \ln \left(\frac{q^2 + 2\mu_v^2 - \sqrt{q^2(q^2 + 4\mu_v^2)}}{q^2 + 2\mu_v^2 + \sqrt{q^2(q^2 + 4\mu_v^2)}} \right) \\ &= -1 - \frac{M^2}{M^2 - \mu_v^2} \ln \frac{\mu_v^2}{M^2} + q^2 \left(\frac{-1}{3\mu_v^2} + \frac{2\mu_v^2}{(M^2 - \mu_v^2)^2} + \frac{\mu_v^2 (M^2 + \mu_v^2)}{(M^2 - \mu_v^2)^3} \ln \frac{\mu_v^2}{M^2} \right) + O(q^4). \end{aligned} \quad (22)$$

To extract the exact zero point of $\sum_{\nu} n_{\nu} g_{\nu}^2 B_{\nu}(q^2)$ is not an easy task. However, it may be checked that it occurs at q^2 non-zero and negative.

Next the masses of the $O(N_{\nu})$ vector fields can be read off from Eq. (10), namely

$$m_{\nu}^2 = m^2 g_{\nu}^2 = \mu_{\nu}^2 \quad (23)$$

for the spinors v_{ν} , w_{ν} and

$$\begin{aligned} m^2(\vec{\varphi}_{\nu}) &= M^2, \\ m^2(\vec{\psi}_{\nu}) &= \mu_{\nu}^2, \end{aligned} \quad (24)$$

for the scalars $\vec{\phi}_{\nu}$ and $\vec{\chi}_{\nu}$ respectively. There is a mass splitting proportional to $\bar{D}$ for the multiplets $\vec{\phi}_{\nu}$, but none for the $\vec{\psi}_{\nu}$.

Finally we wish to point out that the breaking of supersymmetry in this model is not a peculiarity of its two-dimensional nature. Namely, in three space-time dimensions Eq. (13) remains valid, and Eq. (15), supposing $\bar{D} = 0$, now becomes

$$\sum_{\nu} n_{\nu} g_{\nu} \sqrt{g_{\nu}^2} = 0 \quad (25)$$

Again for $g_1^2 \neq g_2^2$ this is not satisfied, whence $\bar{D} \neq 0$ and supersymmetry is broken. Also the finiteness of the model persists to leading order in $1/N$.

We can summarize our results as follows. We have presented a generalization of the supersymmetric σ model in two or three space-time dimensions. The model is finite in the large N limit. Furthermore supersymmetry is broken dynamically in the $1/N$ -expansion. Indeed, to a large extent the breaking of supersymmetry depends on the finiteness of the theory. This observation, on which we will elaborate elsewhere⁷⁾, seems to us rather surprising. However, it is an encouraging result, giving extra support and plausibility to some unification schemes based upon $N = 8$ supergravity as the theory of fundamental interactions at the Planck energy.

As with the supersymmetric non-linear σ model the supermultiplet of Lagrange multipliers, imposing the constraints (1), acquires dynamics in the quantum theory. The generation of dynamical bound states is another desirable feature, hoped to occur in $N = 8$ supergravity. Moreover, in our model the fermionic member of the multiplet is massless, being the Goldstone fermion of supersymmetry breaking. Thus our model has a massless, composite fermion - an

essential feature for composite models of quarks and leptons. In the context of composite models it has already been suggested that quarks and leptons could be the Goldstone fermions of broken supersymmetry⁸⁾.

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