CERN-EP 80-47

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EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

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CERN-EP/80-47
16 April 1980

CM-P00059331

THE SEQUENTIAL DECAY OF A FIRE SAUSAGE:

THE ORIGIN OF JETS IN HADRONIC FINAL STATES

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(Submitted to Nuclear Physics)

1. INTRODUCTION

The detailed structure of hadronic final states has recently attracted considerable attention both from the experimental and the theoretical side. From the experimental side, the analysis of the detailed structure of hadronic final states is the logical development of the extensive effort devoted in the early seventies to the study of the physics of inclusive phenomena. Feynman scaling, p_T cut-offs, (quasi-)constancy and the universal structure of the central plateau, are the important notions which have derived their experimental basis from inclusive analyses of inelastic reactions, both hadron- and lepton-initiated. Theoretically these notions have been neatly embodied in the Mueller-Regge [1] theory of many-point functions at high energy, and as a result we can confidently state that our description and understanding of inclusive inelastic phenomena is altogether very satisfactory.

As far as the detailed structure of final states is concerned, the situation is, generally speaking, not so comfortable. Here we encounter two types of difficulties: the high mean multiplicities, whence the great complexity, of the high-energy inelastic events, and the lack of specific and precise mechanisms of multiparticle production.

Some progress has been made on both problems in recent years, even though we are still far from a decent, generally accepted description of this important aspect of high-energy physics -- an aspect which has vigorously come to the forefront also for practical reasons. In fact while planning new high-energy accelerators, such as LEP, it has become particularly clear that it is important to have an idea of what type of events we are likely to observe in the very high energy régime. Thus machines and detectors must be designed so as to optimize their performance in the particular physical conditions which are expected to be prevailing in the energy domain that will be explored; and very high energy necessarily means multiparticle states. Thus it is not an accident that the work we are going to present here has been carried out within the framework of the ECFA-LEP Working Group.

Let us now, before proceeding to our analysis, briefly discuss what kind of progress has recently been made with the problems of describing the structure and the production mechanisms of multiparticle states. When dealing with 20 or so particles in the final states, it is out of the question to attempt an analytical description of such a complicated state of affairs. Here the fast computers must come to our aid, and keep track of the complexities of the kinematics; recently several different types of Monte Carlo programs have in fact been written down for generating multiparticle events. The dynamical inputs (the weights in the Monte Carlo generations) have been chosen in a variety of ways, all of increasing intricacy and ingenuity. From the simple longitudinal phase structure of the uncorrelated jet model, increasingly realistic descriptions of the dynamics of multiparticle states have been obtained by following through the ideas embodied in the quark-parton model. The most famous and the most sophisticated of these approaches is the Feynman-Field model [2], which today is certainly the one most widely employed for generating multiparticle events in all kinds of deep inelastic phenomena, ranging from e^+e^- annihilation to large- p_T multiparticle production.

But, as everybody who has been working with the Feynman-Field approach knows very well, that model, besides having a high degree of arbitrariness (and of flexibility), presents several problems of interpretation, all of which are connected with the fact that quark confinement is totally extraneous to its description. This is the state of the art today; or, perhaps it would be better to say, this is what most high-energy physicists *believe* is the state of the art. For back in 1976 [3,4], a complete strategy was laid down for describing the structure and the production mechanisms of high-energy multiparticle states in a way that would be both predictive and realistic. As this suggested line of research has generally been ignored, in this paper we would like to again call the attention to its validity, based on a Monte Carlo analysis of final states produced in high-energy e^+e^- annihilation.

2. WHAT IS A FIRE SAUSAGE?

In the Quark-Geometrodynamical (QGD) approach, which should be considered as a fairly accurate [5] approximation to a fundamental theory of confined coloured quarks (such as Quantum Chromodynamics), a spectrum of mesonic ($q\bar{q}$) states is obtained, which to a good approximation comprises an infinite set of straight and parallel Regge trajectories without odd daughters. Asymptotically (i.e. for large masses) such a spectrum is given [6], neglecting spin, by the equation:

$$M_{n,\ell}^2 \cong \frac{\pi}{R^2} (2n + \ell) , \quad (2.1)$$

where ℓ is the orbital angular momentum of the $q\bar{q}$ system, and n is the "radial" quantum number; R^2 is a constant with the dimension of a (length)² whose value has been determined as $R^2 \cong 2 \text{ GeV}^{-2}$.

In addition to the spectrum, the QGD approach produces a set of normalized wave functions $\psi_{n,\ell}(p,x)$, describing the quantum amplitude of the $q\bar{q}$ system as a function of the total four-momentum p and the relative variable $x = x_q - x_{\bar{q}}$. The Fourier transform of $\psi_{n,\ell}$, describing the momentum space vertex function of fig. 1, is given by the simple expression:

$$\psi_{n,\ell}(p,k) \cong (2\pi)^2 N(M_{n,\ell}^2) \delta_{R^2}(p_1^2 - p_2^2) \delta_{R^2} \left(\frac{p_1^2 + p_2^2}{2} \right) Y_{\ell}^m(\Omega_k) , \quad (2.2)$$

where $\delta_{R^2}(z)$ is a "fat δ -function" well approximated by the simple entire function $(1/\pi)(\sin R^2 z/z)$; $N(M_{n,\ell}^2)$ is a (spin-dependent) normalization factor, depending on the mass of the mesonic state; and Ω_k are the angular variables of the space-part of the vector $\vec{k}$ in the rest frame $p \equiv (M, \vec{0})$. If we now fix $2n + \ell$ at a given integer N , we see that there exist $[N/2]$ mesonic states which are approximately degenerate, with a mass $M_{n,\ell}^2 = (\pi/R^2)(N)$ and with angular momenta $\ell = 0, 2, \dots, N$ for N even, or $\ell = 1, 3, \dots, N$ for N odd. But not all of these states are strongly coupled. In fact, in order to ensure the finite range of strong interactions, it is necessary that only those states couple which have $\ell < \ell_0$, where

$$\ell_0 = \frac{M}{2} R_T(M) , \quad (2.3)$$

and R_T is at most logarithmically increasing with the mass of the mesonic state.

Thus at a given large mass M , we can form coherent states of the following type:

$$\psi(p, x; \Omega_0) = \sum_{n, \ell=0,1}^{\ell_0} \sum_{m=-\ell}^{+\ell} Y_{\ell}^{-m}(\Omega_0) \psi_{n, \ell}(p, x) \quad (2.4)$$

with a well-defined $p^2 = M^2$, and a $q\bar{q}$ space-distribution in the rest frame which looks like a cigar or a sausage (fig. 2). In contrast with the old concept of "fireball", which is totally inapplicable in high-energy interactions, it appeared natural to call such a coherent state a "fire sausage", with axis Ω_0 .

We may now ask: Why introduce this notion of a fire sausage which adds nothing to the description of mesonic states? The answer is simpler than we might have expected. When we look at the quark degrees of freedom of mesonic matter we would like to develop a "partonic" description of the quark distributions inside these physical states. This means that we would like to be able to associate, say, to a $q\bar{q}$ pair of total energy M in their centre of mass, two plane-waves of momenta (neglecting quark masses)

$$\begin{aligned} p_1 &= (|\vec{k}|, \vec{k}) \\ p_2 &= (|\vec{k}|, -\vec{k}) \end{aligned} \quad |\vec{k}| = \frac{M}{2} .$$

But such a system, being unconfined, cannot be described as a superposition of physical states, which are necessarily confined. The fire sausage with axis $\Omega_{\vec{k}}$ is the physical (confined) $q\bar{q}$ state with mass M which resembles most closely a free $q\bar{q}$ pair of momenta p_1 and p_2 . This is the real and simple reason for the great relevance of the notion of fire sausage.

3. THE DECAY OF THE FIRE SAUSAGE

In QGD [5] the main interaction mechanism between mesonic states is the three-meson vertex (fig. 3). This amplitude should be computed by taking the space-time overlap of the three mesons' wave functions multiplied by appropriate quark-tunnelling amplitudes, which we shall assume to be as simple as possible. Before getting to a quantitative analysis, from the very structure of the overlap integral we can readily conclude [3,4] that the decay configurations of a meson M_1 of high mass:

$$\begin{aligned} M_1 &\rightarrow M_2 \text{ (low mass)} + M_3 \text{ (high mass)} \\ M_2 &\text{ (high mass)} + M_3 \text{ (low mass)} \end{aligned} \quad (3.1)$$

is highly enhanced over the configuration where both M_2 and M_3 have a large mass.

Thus given a fire sausage (FS), its main decay mode will be

$$FS \rightarrow FS' + \text{low-mass meson } (\mu) . \quad (3.2)$$

This leads us immediately to the sequential decay picture of the FS reported in fig. 4. Let us call A_n the amplitude for the FS disintegration in $n + 1$ low-mass mesons. The total decay width $M\Gamma$ of the FS is thus given by

$$(M\Gamma) = \frac{1}{2} \sum_{n=1}^{\infty} |A_n|^2 . \quad (3.3)$$

Two important facts again follow from the simple structure of the mesonic wave functions:

- i) the particles $\mu_1, \mu_2, \dots, \mu_{n+1}$ are strongly ordered in rapidity; i.e. in $|A_n|^2$, configurations with crossed legs are strongly (exponentially) suppressed for large mass M ;
 - ii) in the diagrammatic expansion for $|A_n|^2$ the FS lines appear dominantly in a symmetrical way, i.e. the dominant diagram is the "rainbow diagram" of fig. 5.
- From this we can easily conclude that in the decay process quantum (interference) effects are strongly suppressed so that we can visualize the FS decay in a classical way; that is to say, in terms of probabilities rather than probability amplitudes.

4. THE EVOLUTION EQUATIONS

Let us consider a FS in the initial state with mass M and axis $\Omega \equiv (\theta, \phi) = (0, 0)$. We define the width operator $[M\Gamma(M)]_a^b$, whose diagonal matrix elements give the $(M\Gamma)$ of the FS of mass M *) (see fig. 6).

It follows directly from the "rainbow approximation" that $(M\Gamma)$ obeys the following integral equation (the dashed lines represent low-mass mesons, while the continuous lines represent quarks):

*) Note that the physical mesonic states are so chosen as to diagonalize $(M\Gamma)$.

$$(4.1)$$

where a sum over different low-mass mesons is implied.

This equation can immediately be brought to quadratures. We have, in fact, using the diagonality of the $M\Gamma$ operator:

$$(4.2)$$

We now notice that in (4.2) we have terms of the type:

$$+ \frac{i}{s - M_n^2 + i(M\Gamma)_n} (M\Gamma)_n \frac{i}{s - M_n^2 - i(M\Gamma)_n},$$

which are equivalent to $\text{Im} (1/s - M_n^2 + iM\Gamma_n)$. On the other hand, as we shall check later, asymptotically we have

$$M\Gamma_n = \text{const} = M\Gamma_0 \quad (4.3)$$

so that in the r.h.s. of (4.2) we can replace $\text{Im} (1/s - M_n^2 + iM\Gamma_n)$ by $-\pi\delta(s - M_n^2)$. Thus we can compute the width operator by performing a simple integration over meson wave functions, according to the equation:

$$(4.4)$$

5. THE WIDTH OPERATOR AND THE PARTICLE KERNELS

According to (4.4), the width operator can be computed from diagrams of the type (for the time being we neglect spin-complications):

$$\sum_i \sum_n \text{Diagram} = M^2 \int \frac{d^3 k}{(2\pi)^3} \frac{1}{2E_k} \sum_{i,n} K_i(x, \vec{k}_T)_a^n, \quad (5.1)$$

where $x = 2E/M$, and $\vec{k}_T$ is the transverse momentum of the low-mass meson i with respect to the incident quark direction in the $q\bar{q}$ centre of mass.

The kernel K_a^n can be explicitly computed from the structure of the meson wave functions and of the three-bag overlap [5]; we obtain the low-mass meson i :

$$\begin{aligned} [K_i(x, k_T)]_a^n &\simeq \lambda^3 + (M_i)_a^n (M_i^+)_n^a \frac{(1-x)}{x^2} N_i^2 M_i^4 \left\{ M_i^2 + \frac{\pi}{R^2} + \frac{2}{x^2} [k_T^2 + M_i^2(1-x)] + \frac{2-x}{x} a \right\} \\ &\times \delta_{R^2}^2 \left(\frac{-a}{2} + \frac{m_1^2 - m_2^2}{2} \right) \delta_{R^2}^2 \left[\frac{k_T^2}{x^2} + \frac{M_i^2}{x^2} (1-x) + m_3^2 + \frac{a}{x} \right] \\ &\times \delta_{R^2}^2 \left[\frac{k_T^2}{x^2} + \frac{M_i^2}{x^2} (1-x) + \frac{m_1^2 + m_3^2}{2} + \frac{a}{x} \left(\frac{1-x}{2} \right) \right] \\ &\times \exp \left\{ -\frac{2R^2}{\pi} \left[\frac{k_T^2}{x^2} + \frac{M_i^2}{x^2} \left(\frac{1-x}{2} \right)^2 + \frac{a}{x} \left(\frac{1-x}{2} \right) \right] \right\}, \end{aligned} \quad (5.2)$$

where m_1, m_2, m_3 are the masses of the quarks a, n, b , respectively [see formula (5.1)]; $a = M_i(E_a - E_n - \omega_i)$, where E_a and E_n are the quark energies and ω_i is the binding energy of the meson i [5]; N_i is the normalization factor of the meson w.f.; λ is a coupling constant; and M is the meson quark matrix. We should notice several interesting features of this equation for the particle-kernels:

- i) the $(1-x)$ behaviour for $x \rightarrow 1$;
- ii) the strong k_T cut-off;
- iii) the strong vanishing for $x \rightarrow 0$;
- iv) the M_i^4 dependence; M_i being the mass of the produced meson i .

From (5.1) and (5.2) we can easily ascertain that the operator $(M\Gamma)$ behaves for large M as M^2 ; and projecting on a given partial wave to pick out $(M\Gamma)_n$ we obtain the result (4.3), which tells us that the lifetime of hadron states increases linearly with their mass. A value $M\Gamma \simeq 4 \text{ GeV}^2$ seems compatible with all we know at present.

5.1 Light quarks

We evaluate (5.2) for light quarks u, d, and s, also taking care of the spin structure. We arrive at three important conclusions:

- i) π , K, and η production is suppressed because of their low mass and the spin-structure of the coupling;
- ii) vector mesons substantially dominate the single-particle production;
- iii) vector mesons are predominantly produced in the transversely polarized configuration.

Thus all pseudoscalar mesons are essentially second-generation particles (their direct production rate is less than 10%).

In the following we shall only keep the vector meson contribution, neglecting both pseudoscalars and $\ell = 1$ mesons such as A_2 , f_0 , etc., the treatment of these being postponed for a future analysis. An adequate and simple representation of the kernel for vector mesons is

$$\begin{aligned} (K_i^V)_a^b &\simeq (M_i^\dagger M)_a^b (2.34) \frac{(1-x)}{x^2} (2x-1) \frac{\lambda}{\pi} \exp[-\lambda p_T^2] \theta\left(x - \frac{1}{2}\right), \\ &= (M_i^\dagger M)_a^b (2.34) V(x, p_T), \end{aligned} \quad (5.3)$$

where $\lambda^{-1} \simeq 0.20 \text{ GeV}^2$, implying $\langle p_T \rangle \simeq 0.45 \text{ GeV}$. Note that all quantities have been evaluated from the theory! For the X^0 , which is the only pseudoscalar meson that is produced directly with an appreciable rate, we compute

$$(K^{X^0})_a^b = (X^\dagger X)_a^b (0.5) V(x, p_T). \quad (5.4)$$

5.2 Charmed quarks

The structure of (5.2) is such that the emission of a charmed meson from a non-charmed quark is strongly suppressed (by about 10^{-3}). The physical reason for this suppression is very simple: the emission of a heavy meson from a light quark a forces the outgoing quark n , which is thus charmed, at values of its four-momentum squared, which are large and negative; but the w.f. is strongly peaked at $p_n^2 = m_c^2$. This means w.f. mismatch and hence suppression. On the other hand, when the incoming quark is a c-quark ($m_c \simeq 1.8 \text{ GeV}$) we have kinematically

$p_n^2 \approx 0$, which is quite close to the light quark "mass-shell" $p_n^2 = m_{u,d,s}^2$; thus no suppression arises.

A simple and adequate representation of the kernel in this case is:

$$K_i(x, p_T) \Big|_c^{\text{light}} \approx (M_i^\dagger M)_c^{\text{light}} F_i \frac{(1-x)}{x^2} f(x) \exp \left[-\frac{2R^2}{\pi} \frac{p_T^2}{x} \right], \quad (5.5)$$

where

$$f(x) = \exp \left[-\frac{16}{\pi} \left(\frac{1-x}{x} \right) \left(1 - \frac{5}{4}x - \frac{1}{4}x^2 \right) \right],$$

and the F_i 's are given for different charmed mesons as

	D(1850)	D(2000)	F(1950)	F*(2100)
$F_i \text{ (GeV}^{-2}\text{)}$	1.5	12.6	1.2	6.8

(5.6)

Note that the average transverse momentum for charmed particles is $\langle p_T \rangle \approx 750 \text{ MeV}$, to be compared with 450 MeV for vector mesons composed of light quarks.

6. ONE-PARTICLE INCLUSIVE DISTRIBUTIONS

The integral equation for the one-particle inclusive distributions in the "rainbow" approximation is

where more explicitly

Using the observations at the end of Section 4, we readily see that we can write

$$\text{Diagram 1} = \text{Diagram 2} \times \text{Diagram 3} \quad (6.3)$$

We are therefore led to the integral equations:

$$\begin{aligned} [V_j(x, p_T)^\mu]_a^b = & \sum_m \int \frac{d^3 q}{2E_q (2\pi)^3} K_n(\xi, \vec{q}) (\lambda_i)_a^n V_j \left[\frac{x}{1-\xi}, p_T + \frac{q_T}{2} \frac{x}{(1-\xi)^{1/2}} \right] \\ & \times (\lambda_j^\dagger)_n^a \frac{1}{(M\Gamma)_n^b} + K_j(x, p_T) (\lambda_1^\dagger \lambda_1)_a^b + (\text{antiquark term}) . \end{aligned} \quad (6.4)$$

Here we shall not pursue a full analysis of this integral equation which can be carried out along the lines of ref. 3; we would like to list only the main features of its asymptotic solutions:

- i) $V_j(x, p_T; M)$ is independent of M , i.e. it scales à la Feynman;
- ii) V_j is cut off in p_T , and its behaviour for large p_T follows dimensional scaling, i.e. it goes like $1/p_T^4$;
- iii) $V_j \xrightarrow{x \rightarrow 1} (1-x)$;
- iv) $V_j \xrightarrow{x \rightarrow 0} V_j(0) \neq 0$; we have a plateau in rapidity which asymptotically tends to a constant.

From these results it should be clear that the picture which emerges resembles very closely what has long been observed in multiparticle high-energy states.

7. THE ITERATIVE SOLUTION

From the sequential structure of the FS decay it is clear that we can solve our dynamical problem in an iterative fashion. The decay will proceed in steps. At each step we must specify the momenta of the $q\bar{q}$ pair comprising the incoming fire sausage, i.e. the total four-momentum and its axis, and using the geometrodynamical amplitudes we can calculate the probability

$$P_j(M_{in}, \Omega_{in}, M_{out}, \Omega_{out}; x, \vec{p}_T) \Big|_a^b$$

of producing a particle j , with $x = 2E/M_{in}$ and momentum $\vec{p}_T$ transverse to the initial FS direction Ω_{in} , starting from a quark a and in the FS (M_{in}, Ω_{in}) and ending in the quark b in the FS (M_{out}, Ω_{out}) .

It is clear that this probability can be factorized as

$$P_j(M_{in}, \Omega_{in}; M_{out}, \Omega_{out}; x, \vec{p}_T) \Big|_a^b = K_j(x, \vec{p}_T)_a^b \frac{dN}{d \cos \theta} (M_{out}, \cos \theta), \quad (7.1)$$

where K_j is the kernel discussed in Section 5, and $dN/d \cos \theta$ is the probability for "tilting" the FS by an angle θ ; θ is the angle between the axes Ω_{in} and Ω_{out} .

How can $dN/d \cos \theta$ be determined? The answer in QGD is again fairly simple. We have seen that [see, for instance, eqs. (6.2) and (6.3)] after having emitted a meson; the quark a turns into the quark n and propagates inside the produced FS m to emerge as the quark n' ready to decay again (and analogously for the anti-quark). During such a propagation, n changes in general direction so that the Ω_{out} axis does not coincide with Ω_{in} . Thus we have

$$\begin{aligned} \frac{dN}{d \cos \theta} (M, \cos \theta) &= \sum_m \left| \begin{array}{c} \text{Diagram: A circle labeled } m \text{ with two vertices. The left vertex has two incoming lines labeled } n \text{ and } \bar{n}. \text{ The right vertex has two outgoing lines labeled } n' \text{ and } \bar{n}'. \end{array} \right|^2 \\ &= \sum_N \frac{1}{M_N^4} \frac{(\mathcal{M}\Gamma)_0}{(M^2 - M_N^2)^2 + (\mathcal{M}\Gamma)_0^2} \\ &\quad \times \left| \sum_{\ell=0}^{R_T(M_N/2)} (2\ell + 1) P_\ell(\cos \theta) \right|^2, \end{aligned} \quad (7.2)$$

and passing to the continuum $\sum_N \rightarrow (R^2/\pi) \int_{m_0^2}^{\infty} dm^2$, we have (c is a normalization constant):

$$\frac{dN}{d \cos \theta} (M, \cos \theta) = c \frac{R^2}{\pi} \int_{m_0^2}^{\infty} \frac{dm^2}{m^4} \frac{(\mathcal{M}\Gamma)_0}{(M^2 - m^2)^2 + (\mathcal{M}\Gamma)_0^2} \left| \sum_{\ell_0}^{R_T(m/2)} (2\ell + 1) P_\ell(\cos \theta) \right|^2. \quad (7.2')$$

Using the approximate equality [7]: $[\ell_0 = R_T(m/2)]$,

$$\sum_{\ell=0}^{\ell_0} (2\ell + 1) P_\ell(\cos \theta) \approx \frac{1}{\ell_0} \frac{1}{\left[1 - \cos \theta + \frac{1}{\ell_0^2} (2 - \cos \theta)\right]^{3/2}},$$

we obtain

$$\frac{dN}{d \cos \theta} (M, \cos \theta) = c \int_{m_0^2}^{\infty} \frac{dm^2}{m^4} \frac{(\overline{M}\Gamma)_0}{(m^2 - M^2)^2 + (\overline{M}\Gamma)^2} \frac{1}{\ell_0^2} \frac{1}{\left[1 - \cos \theta + \frac{1}{\ell_0^2} (2 - \cos \theta)\right]^3}, \quad (7.3)$$

which for $M^2 \rightarrow \infty$ is given by $[(4/R_T^2) \simeq m_\rho^2]$ [5]:

$$\begin{aligned} \frac{dN}{d \cos \theta} (M, \cos \theta) \simeq & \frac{2m_\rho^4}{M^4} \frac{1}{\left[1 - \cos \theta + \frac{m_\rho^2}{M^2} (2 - \cos \theta)\right]^3} + \frac{(\overline{M}\Gamma)}{\pi M^2} \frac{1}{(3 - 2 \cos \theta)^2} \\ & \times \frac{1}{\left[1 - \cos \theta + \frac{m_\rho^2}{M^2} (2 - \cos \theta)\right]}. \end{aligned} \quad (7.4)$$

Whereas the first term dominates for $\cos \theta = 1$, the second dominates for large angles, being $O(1/M^2)$ instead of $O(1/M^4)$. Such a term is responsible for the large p_T tails in one-particle inclusive distribution. In fact, once we have tilted the FS by a finite angle θ , the particles coming from its successive disintegration will cluster along this direction, with a density d^2N/dp_T^2 decreasing like $1/p_T^4$, as can immediately be obtained from (7.4). The high p_T events will thus be characterized by *the formation of secondary jets along the direction of large p_T particles* -- a structure which has recently been revealed, and in our opinion, theoretically misinterpreted, in several experiments at PETRA.

Our results (7.1) and (7.4) are the key dynamical inputs for the Monte Carlo program, details of which will be discussed in the next section.

8. THE MONTE CARLO STRATEGY

As we have pointed out, the complexity of the final states emerging in the e^+e^- annihilation at high energy and the sequential decay picture just described make a Monte Carlo approach extremely suitable for solving the problem of comparison between experiments and theory.

The multiparticle final states are obtained through the following simulation steps:

- i) generation of a $q\bar{q}$ pair ($q \equiv u, d, s, c$);
- ii) iterative decay $FS \rightarrow FS' + \mu$;

- iii) tilt at each decay step of the FS;
- iv) final FS decay when $M_{FS} < M_{FS}^0$;
- v) decay of all unstable particles.

Let us now describe these steps in detail.

8.1 Generation of a $q\bar{q}$ pair

For a given total centre-of-mass energy E , we generate at rest a FS with mass $M_{FS} = E$, i.e. with four-momentum $p \equiv (E, \vec{p})$. The pair of quarks comprised in a FS of mass E have the four-momenta given by

$$\begin{aligned} p_q &= (E/2, \vec{p}_q) \\ p_{\bar{q}} &= (E/2, -\vec{p}_q) , \end{aligned} \quad (8.1)$$

where $E/2 = \sqrt{\vec{p}_q^2 + m_q^2}$, and m_q is the mass (effective) of the given type of quark. As determined in QGD [5], we have chosen

$$\begin{aligned} m_u &= m_d = 0.1 \text{ GeV} \\ m_s &= 0.47 \text{ GeV} \\ m_c &= 1.8 \text{ GeV} . \end{aligned} \quad (8.2)$$

The rates for the different flavours are taken proportionally to their charge squared, i.e. to their coupling to the photon. According to the spin $\frac{1}{2}$ nature of quarks [5], we have also generated the quark pairs with the θ -distribution

$$\frac{dN}{d \cos \theta} \propto 1 + \cos^2 \theta , \quad (8.3)$$

where θ is the angle between the direction of the quark pair and that of the e^+e^- annihilation.

8.2 Iterative decay $FS \rightarrow FS' + \mu$

The initial FS is now allowed to decay iteratively according to the iterative chain:

$$FS \rightarrow FS' + \mu_i , \quad (8.4)$$

where μ_i is one of the nine vector mesons, which according to our previous discussion dominate the process of FS decay. Let us call $p_i \equiv (E_i, \vec{p}_{iT}, p_{iL})$ the four-momentum of μ_i in the rest frame of the FS. We will thus generate the energy

fraction $x_i = 2E_i/M_{FS}$ and the momentum $\vec{p}_{iT}$ transverse to the FS direction according to the kernels (5.3) and (5.5). For a given M_{FS} and m_i (mass of the generated vector meson), the energy fraction x_i and the transverse momentum p_{iT} must satisfy:

$$\frac{M_{FS}^2 x_i^2}{4} - p_{iT}^2 - m_i^2 \geq 0 . \quad (8.5)$$

The relative weights of production of different mesons are determined by the Clebsch-Gordan coefficients appearing in the expressions for the kernels [eqs. (5.3) and (5.5)]. A table of allowed vector meson decays and their relative rates are given in the Appendix. The vector mesons are produced transversely polarized, as implied by our previous discussion.

8.3 The tilt of the FS

After each decay process $FS \rightarrow FS' + \mu_i$, a tilt of FS' is generated in its rest frame, according to the tilting probability (7.4).

8.4 The decay of the final FS

When, at a given step in the decay chain, the mass of the $q\bar{q}$ system (FS) is lower than $2m_\mu$ (m_μ is the mass of the low-lying vector meson to be produced in the next step), the final decay step occurs.

In this step, the energy, momentum, and flavour are conserved. A table of assumed decays can be found in the Appendix.

8.5 The decay of all unstable particles

As a result of the four previous steps, the generated event consists of a collection of vector mesons (with or without charm), with well-defined momenta. In order to obtain the final state, it is necessary to let these unstable particles decay into their decay channels according to their experimental (or expected) branching ratios. This is done as the final Monte Carlo step, where the information on the transverse polarization of vector mesons is used to provide the proper angular distribution of the decay products.

9. DISCUSSION OF THE RESULTS

In this section we shall concentrate on the analysis of a few items, postponing a more detailed and complete analysis to a future paper. The aspects we shall discuss are, in turn:

- i) the energy dependence of the total charged multiplicity;
- ii) the distribution $s(d\sigma/dx)$;
- iii) the energy dependence of $\langle p_T \rangle$, the average momentum, transverse to the sphericity axis;
- iv) the p_T^2 distribution.

We recall that in our analysis, b quarks have not been considered and K's have been treated under all circumstances as stable particles.

9.1 The energy dependence of the total charged multiplicity

In fig. 7 we have compared our prediction, under the stated assumptions, for $\langle n_{ch} \rangle$ as a function of E_{cm} , with the existing data from ADONE and PETRA [8-13]. We note that in the region below $E_{cm} = 10$ GeV, where our input assumptions are obviously correct, the agreement with data is really outstanding. Above this region our predictions show some discrepancies, which at the highest PETRA energies are of about two charged particles.

These discrepancies can be understood in the following way. Above $E_{cm} = 10$ GeV, b quarks start being produced with a relative rate which asymptotically reaches $1/11 = 9\%$. Assuming that the average charged multiplicity of a B-type meson is around 5 (as we can read off from our predictions at $E_{cm} = 5$ GeV), we can estimate an extra contribution from this source of about

$$\delta n_{ch} \approx 0.7 . \quad (9.1)$$

Increasing the energy, there is another source of low-momentum particles which we have neglected in our analysis: the star K_S 's which, depending on the experimental resolution, cannot be resolved from their decay products.

Low-momentum K_S 's have several origins, the most important being charmed particles, as is well known. From the particle structure of our final states, to obtain an extra $\delta n_{ch} \approx 1$ from this source does not seem to pose any serious problem. Thus we may conclude that, as far as the multiplicity is concerned, our approach is indeed very successful.

We shall see that our assessment of the origin of the slight discrepancies we have encountered here will receive further support as we go along with our analysis.

9.2 The distribution $s(d\sigma/dx)$

In figs. 8 we compare our Monte Carlo calculation for $s(d\sigma/dx)$ with the results of DASP at 5 GeV (fig. 8a) [10], of SLAC-LBL at 7.4 GeV (fig. 8b) [9], and of TASSO at 13 GeV (fig. 8c), 17-22 GeV (fig. 8d), 27.4-31.6 GeV (fig. 8e) [12, 14-16].

The "low-energy" data, figs. 8a and 8b, are in striking agreement with our expectations; the "high-energy" data, on the other hand, show discrepancy at values of $x \leq 0.1$. Following our previous discussion this is perfectly understandable in terms of low-momentum particles generated by B-mesons and the decay of slow K_S 's. In view of this, it is indeed a success of our approach to have predicted with accuracy the high- x shape of $s(d\sigma/dx)$ at all known energies.

9.3 The average transverse momentum

Our prediction of $\langle p_T \rangle$ versus E_{cm} is plotted in fig. 9 where, for comparison, TASSO [15] and PLUTO [11] data for such a quantity are exhibited. Again we notice perfect agreement in the "low-energy" region ($E_{cm} \leq 10$ GeV), while some 50 MeV discrepancy is observed in the high-energy region. Needless to say, this discrepancy can be attributed to the low-momentum particle sources which in our calculation we have, for the time being, neglected.

9.4 The distribution $(1/\sigma)(d\sigma/dp_T^2)$

Our predictions for $E_{cm} = 15$ and 30 GeV are compared with the TASSO data at PETRA [16] in figs. 10a, 10b. Apart from the region of very small p_T^2 , the agreement is very good. In particular, the strong energy dependence of the high- p_T^2 tail is to be noted -- a fact that was long ago discovered in hadronic interactions, and which was later predicted to be a general feature of hadronic final states [4], independently of their production mechanism. It is in the high- p_T tail that one should observe events with the structure of multijets, according to the discussion of Section 7. We shall come to this fascinating subject in a future publication.

REFERENCES

- [1] A.H. Mueller, Phys. Rev. D 2, 2963 (1970).
- [2] R.D. Field and R.P. Feynman, Nucl. Phys. B136, 1 (1978).
- [3] N. Craigie and G. Preparata, Nucl. Phys. B102, 497 (1976).
- [4] G. Preparata, Proc. 8th Internat. Symposium on Multiparticle Dynamics, Kaysersberg, France, 1977 (C.R.N., Strasbourg, 1977), p. B-65.
- [5] For a recent general discussion, see G. Preparata, Proc. 1977 Internat. School of Subnuclear Physics, Erice, Italy, in the "Whys of Subnuclear Physics" (ed. A. Zichichi) (Plenum Publ. Co., New York, 1979), p. 727.
- [6] G. Preparata and N. Craigie, Nucl. Phys. B102, 478 (1976).
- [7] G. Preparata, Nucl. Phys. B89, 445 (1975).
- [8] C. Bacci et al., Phys. Lett. 86B, 234 (1979).
- [9] G.G. Hanson et al. (SLAC-LBL Collaboration), 13th Rencontre de Moriond, Les Arcs, 1978 (ed. J. Tran Thanh Van) (Editions Frontières, Dreux, 1978), vol. II, p. 15.
- [10] R. Brandelik et al. (DASP Collaboration, Nucl. Phys. B148, 189 (1979).
- [11] Ch. Berger et al. (PLUTO Collaboration), Phys. Lett. 81B, 410 (1979).
- [12] R. Brandelik et al. (TASSO Collaboration), Phys. Lett. 86B, 243 (1979).
- [13] W. Bartel et al. (JADE Collaboration), DESY Report 79/64 (1979).
- [14] R. Brandelik et al. (TASSO Collaboration), Phys. Lett. 89B, 418 (1980).
- [15] R. Brandelik et al. (TASSO Collaboration), DESY Report 79/61 (1979).
- [16] R. Brandelik et al. (TASSO Collaboration), Phys. Lett. 83B, 261 (1979).

APPENDIX

The canonical decay of a fire sausage $q_a \rightarrow p_i + q_b$, where p_i is a vector meson, is governed by the Clebsch-Gordan coefficients summarized in table 1.

Table 2 gives the decay channels considered in the final decay of the fire sausage.

Table 3 gives the relative decay channels and branching ratios for all the unstable vector mesons.

Table 1

Allowed fire sausage decays, $q_{FS} \rightarrow \text{vector meson} + q'_{FS}$, and relative rates

[illegible]

Table 2

Final fire sausage decay

Quark content	Decay products	Branching ratios
1) $u\bar{d}$	$\pi^+\pi^0$	1
2) $d\bar{u}$	$\pi^-\pi^0$	1
3) $d\bar{d}$	$\pi^0\pi^0$ $\pi^+\pi^-$	$\frac{1}{3}$ $\frac{2}{3}$
4) $d\bar{s}$	$K^+\pi^-$ $K^0\pi^0$	$\frac{2}{3}$ $\frac{1}{3}$
5) $u\bar{s}$	$K^+\pi^0$ $K^0\pi^+$	$\frac{1}{3}$ $\frac{2}{3}$
6) $s\bar{d}$	$K^-\pi^+$ $\bar{K}^0\pi^0$	$\frac{2}{3}$ $\frac{1}{3}$
7) $s\bar{u}$	$K^0\pi^-$ $K^-\pi^0$	$\frac{2}{3}$ $\frac{1}{3}$
8) $s\bar{s}$	K^+K^- $K^0\bar{K}^0$	$\frac{1}{2}$ $\frac{1}{2}$
9) $c\bar{u}$	$D^0\gamma$ $D^0\pi^0$	$\frac{1}{2}$ $\frac{1}{2}$
10) $c\bar{d}$	$D^+\gamma$ $D^0\pi^+$ $D^+\pi^0$	$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{6}$
11) $u\bar{c}$	$\bar{D}^0\gamma$ $\bar{D}^0\pi^0$	$\frac{1}{2}$ $\frac{1}{2}$
12) $\bar{c}d$	$D^-\gamma$ $D^0\pi^-$ $D^-\pi^0$	$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{6}$

Table 3

Unstable particles decays

Unstable particle	Decay products	Branching ratio
ρ^+	$\pi^+\pi^0$	1
ρ^0	$\pi^+\pi^-$	1
ρ^-	$\pi^-\pi^0$	1
K^{*0}	$K^+\pi^-$ $K^0\pi^0$	$\frac{2}{3}$ $\frac{1}{3}$
K^{*+}	$K^0\pi^+$ $K^+\pi^0$	$\frac{2}{3}$ $\frac{1}{3}$
$\bar{K}^{*0}$	$K^-\pi^+$ $\bar{K}^0\pi^0$	$\frac{2}{3}$ $\frac{1}{3}$
K^{*-}	$\bar{K}^0\pi^-$ $K^-\pi^0$	$\frac{2}{3}$ $\frac{1}{3}$
ω	$\pi^+\pi^-\pi^0$ $\gamma\pi^0$ $\pi^+\pi^-$	0.899 0.088 0.013
ϕ	K^+K^- $K^0\bar{K}^0$ $\pi^+\pi^-\pi^0$ $\gamma\eta$	0.486 0.351 0.147 0.016
π^0	$\gamma\gamma$	1
$\eta'(\chi^0)$	$\pi^+\pi^-\eta$ $\pi^0\pi^0\eta$ $\gamma\rho^0$ $\gamma\omega$ $\gamma\gamma$	0.426 0.236 0.298 0.020 0.02
η	$\gamma\gamma$ $\pi^0\pi^0\pi^0$ $\pi^+\pi^-\pi^0$ $\gamma\pi^+\pi^-$ $\gamma\gamma\pi^0$	0.381 0.300 0.236 0.052 0.031
D^{*0}	$D^0\gamma$ $D^0\pi^0$	$\frac{1}{2}$ $\frac{1}{2}$
D^{*+}	$D^+\gamma$ $D^0\pi^+$ $D^+\pi^0$	$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{6}$

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Table 3 (contd.)

Unstable particle	Decay products	Branching ratio
$\bar{D}^{*0}$	$\bar{D}^0 \gamma$ $\bar{D}^0 \pi^0$	$\frac{1}{2}$ $\frac{1}{2}$
D^{*-}	$D^- \gamma$ $D^0 \pi^-$ $D^- \pi^0$	$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{6}$
D^0	$K^{*-} \rho^+$ $\bar{K}^{*0} \rho^0$	$\frac{2}{3}$ $\frac{1}{3}$
D^+	$\bar{K}^{*0} \rho^+$	1
$\bar{D}^0$	$K^{*+} \rho^-$ $K^{*0} \rho^0$	$\frac{2}{3}$ $\frac{1}{3}$
D^-	$K^{*0} \rho^-$	1

Figure captions

- Fig. 1 : The $q\bar{q}$ particle vertex in momentum space.
- Fig. 2 : The spatial shape of a fire sausage of axis Ω_0 .
- Fig. 3 : The three-meson vertex; the dots denote the inclusion of the "quark-tunnelling" amplitude.
- Fig. 4 : The diagram corresponding to the amplitude A_n .
- Fig. 5 : The diagram corresponding to the square $|A_n|^2$.
- Fig. 6 : The width operator $(M\Gamma)_a^b$ is asymptotically given by a sum of rainbow diagrams.
- Fig. 7 : Average charged multiplicity $\langle n_{ch} \rangle$ as a function of the total centre-of-mass energy E . The solid line is the Monte Carlo result.
- Fig. 8 : Single-particle inclusive differential cross-section as a function of $x = 2p_L/\sqrt{s}$ for centre-of-mass energies:
- a) 5 GeV (ref. 10);
 - b) 7.4 GeV (ref. 9);
 - c) 13 GeV (ref. 12);
 - d) 17-22 GeV (ref. 15);
 - e) 27.4-31.6 GeV (ref. 14).
- Fig. 9 : Average $\langle p_T \rangle$ as a function of the total centre-of-mass energy E . The p_T is with respect to the sphericity axis, the solid line is the Monte Carlo result.
- Fig. 10 : Single-particle normalized inclusive differential cross-section as a function of p_T^2 for total centre-of-mass energies:
- a) 13-17 GeV (ref. 16);
 - b) 27.4-31.6 GeV (ref. 16).
- The solid line is the Monte Carlo result.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 8 (cont.)

Fig. 9

Fig. 10