Ref.TH.2728-CERN

TESTING QCD PREDICTIONS BEYOND THE LEADING ORDER FOR NON-SINGLET
STRUCTURE FUNCTIONS IN NEUTRINO SCATTERING

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ABSTRACT

Quantum chromodynamics predictions for the non-singlet pieces of the structure functions in neutrino inclusive scattering are tested against experiment. Next-to-leading QCD terms, and higher twist contributions are taken into account. After discussing the uncertainties (both experimental and theoretical) inherent to the untempered use of moments, pointwise reconstruction is effected. The ensuing fits to experimental data are excellent in all the x, Q^2 region explored ($2 \text{ GeV}^2 \lesssim Q^2 \lesssim 180 \text{ GeV}^2$). The preferred value of Λ (defined in the minimal renormalization scheme) obtained is $\Lambda = 0.17 \pm 0.08 \text{ GeV}$, perfectly compatible with one found previously from $ep, \mu p$ scattering, $\Lambda = 0.20 \pm 0.06$.

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Ref.TH.2728-CERN

22 August 1979

1. INTRODUCTION

While certainly less precise than electron- or muon-production data, data on deep inelastic neutrino ($\nu, \bar{\nu}$) scattering cover a very wide Q^2 range^{*}). Also, a structure function contributes here, viz., $xF_3(x, Q^2)$, which is comparatively easy to extract from experiment and which, being pure non-singlet, is particularly suited for comparison with theoretical predictions.

The structure functions of deep inelastic lepton scattering satisfy approximate Bjorken scaling. Theory (QCD) then predicts the deviations from exact scaling, that is to say, if $F(x, Q^2)$ is a generic structure function, theory allows us to calculate the difference

$$F(x, Q_0^2) - F(x, Q^2), \quad (1.1)$$

where Q_0^2 is a fixed reference momentum, and we allow Q^2 to vary.

The QCD predictions have been compared with experiment in a number of papers, some of which (for neutrino scattering) are listed in Refs 2) to 5). In these comparisons only the leading order QCD corrections are taken into account; next-to-leading contributions and higher twist effects are ignored. Furthermore, the comparison is made employing either the Buras-Gaemers parametrizations⁶⁾ of $F(x, Q^2)$ or the moments (or logarithms of the moments) of the structure functions.

In the present paper we use the analytical expressions^{7), 8)} for the next-to-leading QCD contributions⁷⁾⁻¹²⁾ as well as estimates of higher twist ones¹⁰⁾ to extend the comparison between theory and experiment beyond the leading order. What is more, since we perform a point-like reconstruction of the structure functions not only in the leading approximation¹³⁾ but also to second order⁷⁾, we are able to avoid the difficulties connected with the Buras-Gaemers, and, above all, the moment method.

The results of the fits are excellent, and substantiate what was already known from the corresponding⁸⁾ analysis for ep and μp scattering: the χ^2 of the fits decreases when including second order corrections (it cannot decrease much, since it was very good already at the leading order). Λ , as defined in the minimal renormalization scheme, diminishes from some 0.30 ± 0.20 GeV to 0.17 ± 0.08 GeV; its error also becomes smaller (to be expected; cf. the discussion in Ref. 8)). The new value is perfectly compatible with what one gets from ep, μp : $\Lambda = 0.20 \pm 0.06$, if including next-to-leading contributions⁸⁾. Finally, potential

^{*}) We use the standard notation where Q^2 is minus the momentum transfer, and x is Bjorken's variable. For notation, review of the theoretical background and references, cf. e.g., Ref. 1).

disagreements between theory and experiment for x near 1 are shown to be of the order of the estimated higher twist contributions.

2. REVIEW OF THEORETICAL PREDICTIONS, AND METHODS OF COMPARISON WITH EXPERIMENT

It is with regret that we include this section in the present paper. Unfortunately, some of the methods employed for comparing neutrino data with experiment are subject to so many hazards that we feel that space devoted to clarification may be well employed.

2.1 Theoretical predictions

It is sometimes stated that QCD only makes predictions for the moments of the structure functions. To leading order only, one has

$$\mu_n(Q^2) = A_n \left(\frac{1}{\log Q^2/\Lambda^2} \right)^{d_n}, \quad (2.1)$$

where the A_n are unknown constants, Λ a parameter to be obtained from experiment, and the d_n calculable parameters in QCD. You may get rid of the A_n by normalizing at a given Q_0^2 , obtaining a prediction for the ratio $\mu_n(Q^2)/\mu_n(Q_0^2)$. If you had exact Bjorken scaling, this ratio would be constantly equal to 1; therefore, the relevant quantities when comparing theory and experiment are the deviations from this, predicted by asymptotic freedom:

$$1 - \frac{\mu_n(Q^2)}{\mu_n(Q_0^2)} = \frac{(\log Q^2/\Lambda^2)^{d_n} - (\log Q_0^2/\Lambda^2)^{d_n}}{(\log Q^2/\Lambda^2)^{d_n}} \quad (2.2)$$

How accurate is (2.2)? Clearly, the answer depends on the corrections to Eq. (2.1). These can be classified into three sets: i) higher-order QCD corrections, independent of target mass or structure. They modify (2.1) to

$$\mu_n(Q^2) = A_n \left(\frac{1}{\log Q^2/\Lambda^2} \right)^{d_n} \left\{ 1 + C_n^{(2)} \frac{1}{\log Q^2/\Lambda^2} + \dots \right\}, \quad (2.3)$$

where the $C_n^{(2)}$ are calculable (and calculated^{9)-12), 7)} quantities, constant up to $\log \log Q^2/\Lambda^2$ terms. Because of the peculiar structure of the leading contribution, the relative size of the contribution of these second-order terms to (2.2)

is quite large and, indeed, for fixed Q^2 , dominates the leading one at large n ; however, we will not discuss these corrections for the moment.

ii) Corrections depending on the structure of the target, technically known as higher twists. Considering only the leading one, it alters (2.3) to

$$\mu_n(Q^2) = A_n \left(\frac{1}{\log Q^2/\Lambda^2} \right)^{d_n} \left\{ 1 + C_n^{(2)} \frac{1}{\log Q^2/\Lambda^2} + \dots \right\} + B_n/Q^2 + \dots \quad (2.4)$$

the B_n are unknown quantities. However, one can plausibly argue¹⁰⁾ that

$$B_n \sim n \mu^2 \left(\frac{1}{\log Q^2/\Lambda^2} \right)^{d_n} \quad (2.5)$$

μ^2 is related to the transverse momentum of partons in the target or its radius, and one would expect

$$\mu^2 \sim \langle p_t^2 \rangle \sim R_t \sim 0.1 \text{ to } 0.2 \text{ GeV}^2$$

Hence, higher twists are small for small n , but potentially large for large n . Finally, we have (iii) target mass corrections. These are $\sim x m_p^2/Q^2$, where m_p is the proton mass; they are usually taken into account by using Nachtmann moments. For the range of x, Q^2 values we will be considering for neutrino scattering, these corrections can be neglected, except in one case, discussed at the end of this paper in connection with the fit for BEBC data.

The relative importance of the leading QCD prediction, and the expected twist-four correction are shown in Fig. 1. It may easily be seen that a calculation based only on leading QCD, Eq. (2.2), will suffer from severe errors (20% to 40%) already for $n = 4$ and will become totally warped (38% to 75% error) for $n = 6$. We took $Q_0^2 = 2 \text{ GeV}^2$; had we chosen $Q_0^2 = 4 \text{ GeV}^2$, the twist-four contribution would have been halved; but the leading one fares not much better. Indeed, provided $2 \leq Q_0^2 \leq 8 \text{ GeV}^2$, the relative contribution of higher twists is in the range quoted. The moral is simple: none but the very few first moments are reliably given by the theoretical calculation.

This sounds very discouraging but in fact is only teaching you a lesson: moments are not the best tool to study QCD predictions. [Of course, this does not mean they are utterly useless: for example, one can use the first three or four moments to find the experimental value of the first few coefficients d_n (cf., e.g., Ref. 4).] Fortunately, we have an alternative, inasmuch as theory predicts not only moments, but actually $F(x, Q^2)$ itself. This may be seen by inverting the moment formula, which has been done to leading order a long time ago¹³⁾ and to next-to-leading order in Ref. 7) for electroproduction and in the present paper for neutrino production. (Another way to see this, at least to leading order, is to use the Altarelli-Parisi formalism¹⁴⁾.) The result is that you can write

$$F^{(2)}(x, Q^2) = \int_x^1 dy \, b(x, y; Q^2, Q_0^2) F^{(2)}(y, Q_0^2). \quad (2.6)$$

The kernel b is given in Ref. 7) for ep , μp , and in the Appendix here for neutrino scattering. $F^{(2)}$ is supposed to be only the twist-two part of the structure function; the whole function we write as $F = F^{(2)} + F^{(4)} + \dots$, where the twist-four piece is¹⁰⁾

$$F^{(4)}(x, Q^2) \simeq \frac{\mu^2}{Q^2} \left\{ F^{(2)}(x, Q^2) - x \frac{\partial}{\partial x} F^{(2)}(x, Q^2) \right\}. \quad (2.7)$$

Hence, if we are given experimentally $F^{(2)}(x, Q^2)$ (or just a theoretic ansatz for it) we can obtain the scaling violations^{*)}

$$F(x, Q_0^2) - F(x, Q^2)$$

quite easily. This is plotted for a few values of x in Fig. 2, where the next-to-leading and higher twist corrections are shown: it will be seen that, except when $x \gtrsim 0.7$, the last are very small which is not surprising since, from Eq. (2.7), we may estimate higher twist corrections to be

$$\sim \frac{\mu^2}{Q^2} \left(0.5 + \frac{2x}{1-x} \right),$$

i.e., small except for x near unity. This is in dramatic contrast with what occurs if one uses moments (cf. in Fig. 1). Figure 2 also shows that next-to-leading corrections are quite large: something $\gtrsim 30\%$ of the leading ones for $x \gtrsim 0.6$; actually, they dominate the leading ones as $x \rightarrow 1$, which indicates the

*) Twists higher than four are $\sim \mu^4/Q^4$, and we neglect them.

expected breakdown of perturbation theory near the elastic region. A similar result is also obtained in $e/\mu p$ scattering.

Another method used to compare theory and experiment is the Buras-Gaemers method. Inspired in the Feynman-Field parametrizations, Buras and Gaemers write

$$F(x, Q^2) = \sum_i F_i(x, Q^2), \quad (2.8)$$

where i ranges over various contributions (valence quarks, sea quarks, gluons) and for each of them

$$F_i(x, Q^2) = C_i x^{\alpha_i} (1-x)^{\beta_i} \quad (2.9)$$

One can then easily show that, to leading order in QCD, α_i, β_i are linear functions of

$$s = \log \frac{\log Q^2/\Lambda^2}{\log Q_0^2/\Lambda^2}$$

with

$$\beta_i = \beta_i(0) + \frac{16}{3\beta_0} s; \quad (2.10)$$

β_0 is the renormalization group coefficient (for four quark flavours, $\beta_0 = 3/25$). This method possesses obvious phenomenological advantages: the same functions contribute to different processes (ν, e, μ deep inelastic scattering; Drell-Yan; high p_t scattering...) so it is useful to have a handy parametrization compatible with QCD. The disadvantage is that (2.8-2.10) force a specific functional form at all Q^2 which A) may (and does) lead to biases, e.g., in the determination of the basic parameter Λ ; and B) does not include higher order corrections. Indeed⁷⁾, these modify (2.9); for example, near $x = 1$, the behaviour

$$(1-x)^{\beta_i(0) + 16s/3\beta_0} \quad (2.11)$$

is modified by a factor

$$1 + \frac{\alpha_c(Q_0^2) - \alpha_c(Q^2)}{4\pi} \cdot \frac{8}{3} [\log(1-x)]^2, \quad (2.12)$$

besides an alteration in the value of s which is to be calculated to second order now. Therefore, at large Q^2 , the distortion will be as much as 50% if $x > 0.9$, and 30% if $x \gtrsim 0.65$.

2.2 Comparison with experiment

We will start by considering the moments. One may wonder if one could not turn around the pessimistic conclusions of Sect. 2.1 above and decide that, since higher moments are sensitive to higher twists, why not use the moments precisely to get a handle on these higher twists? Well, maybe this is possible, but certainly not with present neutrino experiments, which reach with any precision only low values of x ($x \lesssim 0.6$) whereas high moments are only sensitive to the large x ($1 - x \lesssim 1/n$) behaviour of the structure functions. Therefore, for large n , what are taken for "experimental values" of the moments actually have little to do with experiment: they just reflect the foibles of the extrapolation used in going from the experimentally accessible region, $x \rightarrow 0.6$, to $x \rightarrow 1$. This is seen quantitatively as follows.

The function one measures experimentally is, say, $xF_3(x, Q^2)$. This is shown in Fig. 3(a), for $Q^2 = 12.5 \text{ GeV}^2$. One may fit it with a function

$$xF_3 = C_1 x^\alpha (1-x)^\beta; \quad (2.13)$$

here the maximum is necessarily tied to the endpoint behaviour, which will cause some biases. So we may generalize (2.13), writing

$$xF_3 = (a+bx) x^\alpha (1-x)^\beta \quad (2.14)$$

In Fig. 3(a) we show the fit with (2.13) and $\alpha = \frac{1}{2}$, $\beta = 3$ (solid line); or with (2.14) and $\alpha = \frac{1}{2}$, $\beta = 2$ (broken line; this is closer to what one actually obtains with an unbiased calculation) or with (2.14) and $\alpha = \frac{1}{2}$, $\beta = 1$ (dotted line). In selecting the last parametrization we have admittedly been somewhat nasty. All three fit the experimental data equally well; and they do not give much variation for the lower moments. The higher ones are a different kettle of fish: the variation of $xF_3 \cdot x^4$ is sizeable [Fig. 3(b)] already for the broken line; for $xF_3 \cdot x^6$ [Fig. 3(c)] the variation of the broken line is ~40%, and that of the dotted line is three times bigger than all the area of the original fit. This is not surprising. When calculating μ_n one is calculating the area under $xF_3 \cdot x^n$. For $n = 4$ the "experimentally known" part of the area (shaded region in Figs 3) is smaller than the extrapolated area; for $n = 6$, the experimental part of μ is pitifully small.

This phenomenon does not occur if one uses the pointlike reconstruction, Eqs (2.6), (2.7). This is because the kernel is very peaked around $x = y$ and therefore the predicted value of $xF_3(x, Q^2)$ depends mostly on the original value at $xF_3(x, Q_0^2)$. This is shown in Fig. 4, where, for purposes of comparison with Fig. 3, we have assumed $Q_0^2 = 12.5 \text{ GeV}^2$, $Q^2 = 200 \text{ GeV}^2$: the variation from the solid line to the broken or dotted line is almost nil. This has an important implication: the "experimental" values are reliable now, and, as we will see, this allows us to see some of the twist four contributions (at large x , small Q^2).

3. POINTWISE RECONSTRUCTION: COMPARISON WITH EXPERIMENT BEYOND THE LEADING ORDER

As stated, we write, for the twist two part of a structure function,

$$F^{(2)}(x, Q^2) = \int_x^1 dy \, b(x, y; Q^2, Q_0^2) F^{(2)}(y, Q_0^2), \quad (3.1a)$$

the difference with the full structure function being¹⁰⁾

$$F(x, Q^2) = F^{(2)}(x, Q^2) + \frac{\mu^2}{Q^2} \left\{ F^{(2)}(x, Q^2) - x \frac{\partial}{\partial x} F^{(2)}(x, Q^2) \right\}. \quad (3.1b)$$

The kernel b in (3.1a) has been calculated, to leading order, in Ref. 13), and, to next to leading order, in Refs 9)-12), Ref. 7) and^{*)} is given in the Appendix here. Unfortunately, only non-singlet structure functions may be treated in this way. The relevant calculations for the singlet piece¹⁵⁾ had, at the time these calculations were made^{**)}, not yet been cast in a form suitable for pointlike reconstruction method¹⁶⁾, or some interpolative method¹⁷⁾. We will take into account leading and next to leading QCD corrections, and twist form contributions [Eq. (3.1b)]. The errors are expected to be $O[\alpha_c^2(Q_0^2)]$, $O[(\mu^2/Q^2)^2]$, i.e., the squares of the second order and twist four ones^{***)}. This is very small up to very low values of Q_0^2 , for $x < 0.9$; for example, the fourth order QCD corrections are expected to contribute ~ 0.015 at $Q_0^2 = 2 \text{ GeV}^2$. Therefore we fix our reference momentum at,

*) Actually, such calculations were made for electroproduction; some work has been done to extend them for neutrino scattering.

**) Very recently, A. Gonzalez-Arroyo and C. Lopez have succeeded in solving the inversion problem for the singlet part.

***) The relatively large importance of next-to-leading QCD corrections is a peculiarity of deep inelastic scattering. It is discussed in detail in Ref. 8).

precisely,

$$Q_0^2 = 2 \text{ GeV}^2$$

For this fixed Q_0^2 , and for $x < 0.9$, we assume that one can fit

$$y F_3^{(2)}(y, Q_0^2) = \frac{3 \Gamma(1+\alpha+\beta)}{\Gamma(\alpha) \Gamma(1+\beta)} y^\alpha (1-y)^\beta \left(1 - \frac{\alpha_c(Q_0^2)}{\pi}\right),$$

$$F_2^{(2)N.S.}(y, Q_0^2) = \frac{3 \Gamma(1+\alpha'+\beta')}{\Gamma(\alpha') \Gamma(1+\beta')} y^{\alpha'} (1-y)^{\beta'} \quad (3.2)$$

Note that, unlike in the method of Buras and Gaemers we do not require a fit like (3.2) to hold for all Q^2 . The N.S. in F_2 means that only the non-singlet piece of it is considered; this implies that we have to select our data points for $F_2(x, Q^2)$ at values of x such that its singlet piece be negligible; this will probably be the case if we only consider $x > 0.3$. (One estimates that here the singlet piece is $\leq 10\%$ of the non-singlet.) Because it is only assumed at a fixed Q , the specific form (3.2) has little influence on the predictions; this has already been discussed (cf. Sec. 2.2, and Fig. 4).

3.1 CDHS data

Let us now discuss the experimental data. We have chosen to fit with the CDHS data²⁾ in the region $Q^2 > 3.75$, $x < 0.7$; they are compatible within their common x, Q^2 range^{*)} with other data (e.g., BEBC⁴⁾), and are reasonably accurate. xF_3 may be deduced unambiguously from the data. Fitting it with Eqs (3.1), (3.2) gives the parameters of Table 1, and the curves of Fig. 5. Inclusion of the second order QCD contributions improves the fit, but not much since already the fit with leading contributions only was very good. This just reflects the fact that the kernel b has a very similar shape in the leading order, and with second order corrections included; it does not mean that second order corrections are small (they are not: cf. Fig. 2) or unimportant; for example, they alter the value of Λ from 0.24 to 0.16, and diminish its error from a huge +0.29, -0.19 to a more reasonable +0.13, -0.12. Moreover, they make the calculation meaningful until low $Q^2 \sim 2 \text{ GeV}$, which will be important later.

We may think of including the function F_2 into our fit. This presents the following problem: the sum of $\nu, \bar{\nu}$ cross-sections depends on two structure functions, F_1, F_2 . If gluon corrections were zero, one could write $F_1 = F_2$ (Callan-Gross relation). With gluon corrections one instead has

^{*)} With some exceptions; see below.

$$F_2(x, Q^2) - F_1(x, Q^2) = R(x, Q^2) F_2(x, Q^2)$$

$$R(x, Q^2) = \frac{4\alpha_c(Q^2)}{3\pi F_2(x, Q^2)} \int_x^1 dy \frac{x^2}{y^3} F_2(y, Q^2). \quad (3.3)$$

In Ref. 2) the integrated quantity R is found to be small (~ 0.03) and hence data are extracted with $F_1 = F_2$. This is all right if one stays at leading order in QCD, but clearly we have lost sensitivity to higher order gluon corrections. What we have done to obviate the problem is to fit xF_3 and F_2 including systematic errors (that take account of violations of Callan-Gross) in the last. The result is presented in Table 2 and Fig. 6. Again Λ , and its error, diminish when including second order QCD corrections. The fit is so good it is hard to decide whether the wiggles at $x = 0.3 \sim 0.5$, $Q = 20 \sim 60$ GeV mean anything.

The results of the calculations are not very impressive by themselves: the decreases in χ^2 , Λ , $\delta\Lambda$ are not spectacular. They become more so if we add what you get⁸⁾ from ep, μp working there also to second order (Table 3). As emphasized in Ref. 8), unambiguous measurement of Λ requires incorporation of second order terms and, sure enough, the dispersed values of Λ obtained with first order only (0.24 to 0.45 GeV) contrast with the impressive regularity found when second order terms are included ($\Lambda = 0.15$ to 0.17 GeV). We here compare without target mass corrections (t.m.c.). For ep, μp , they alter Λ to⁸⁾

$$\Lambda_{t.m.c.} = 0.20 \pm 0.06 \text{ GeV}.$$

Including singlet contributions to second order for ep, μp Ross has obtained¹⁷⁾

$$\Lambda_R = 0.2 \pm 0.1$$

All these values are compatible. Another interesting result is that, systematically, the values of Λ obtained with point-like reconstruction are smaller than those obtained with, e.g., the Buras-Gaemers method. This is probably due to a bias of the last, even to leading order (where, nevertheless, our values are compatible with those of Ref. 2) within errors).

3.2 BEBC data and higher twists

There is a region of x, Q^2 where there exist both CDHS²⁾ and BEBC⁴⁾ data, and they agree. Hence, our fit to the first is bound to fit also the second. This is so, as shown in Figs 7,8. The corresponding χ^2 is of 25 for 21 experimental

points^{*)}. However, there is a region where BEBC-Gargamelle data⁴⁾, but not CDHS data²⁾, exist. Here we can use our fit to the CDHS data for $Q^2 \geq 3.75 \text{ GeV}^2$ to predict the structure functions in the two regions: $Q^2 \sim 2 \text{ GeV}^2$, x varying, and $x = 0.6 \sim 1$, Q^2 varying. This we do including second order QCD corrections; the agreement is as good as the quality of the data allows for $Q^2 \sim 2 \text{ GeV}^2$, $x \lesssim 0.65$, as shown in Figs 9,10. In the bin $x = 0.6 \sim 1.0$, and for small $Q^2 = 1.75 \sim 7 \text{ GeV}^2$, our prediction is systematically below the data, if neglecting target mass and the twist four contribution. Adding the last with $\mu^2 \approx 0.1 \text{ GeV}^2$ restores perfect agreement^{**)}. We regard this as some evidence that, using our method, higher twist effects may be extracted. It should, however, be clear that we need more and more precise data points in this region (large x , small Q^2) before a more reliable theoretical investigation, that should include target mass corrections, can be fruitfully implemented. In this sense, the improvement of the fit by introducing twist four effects with $\mu^2 \sim 0.1$ is to be taken as no more than a hint that such effects are there, and can be studied.

4. CONCLUSIONS

Summing up: we have shown that point-like reconstruction allows simple incorporation of higher order QCD and higher twist effects. This permits use of theoretical analysis at low Q^2 (~ 1.7 to 2 GeV^2) with good accuracy. In particular, this leads to a fairly precise and unambiguous determination of the parameter Λ ; if we define it in the minimal renormalization scheme, we find

$$\Lambda = 0.17 \pm 0.08 \text{ GeV},$$

in excellent agreement with what one gets from e, μ scattering⁸⁾⁻¹⁷⁾.

ACKNOWLEDGEMENTS

Useful discussions with A. de Rújula, A. Para and K. Kleinknecht are gratefully acknowledged.

*) A good part of the error comes for the points at 60 GeV , $x = 0.3 \sim 0.4$, and $x = 0.4 \sim 0.6$, which disagree with the results for Ref. 2) and probably suffer from large systematic errors.

***) Considering only the points at $Q^2 = 1.75, 2.5, 4, 7 \text{ GeV}^2$, and combining $x F_3$ and F_2 to improve experimental errors we find $\chi^2 \approx 7.0$, without twist four, and $\chi^2 \approx 3.5$ when twist four is included. See the indent. in Fig. 10. In principle, target mass corrections are larger than higher twist ones because $m_p > \mu$; however, they tend to depress, rather than enhance, the low Q^2 region.

APPENDIX

The theoretical formulas, and even the definition of Λ depend on the renormalization scheme used. Here we choose the minimal subtraction scheme in dimensional regularization^{9),12)}. Let

$$F(x, Q^2) = \int_x^1 dy \, b(x, y; Q^2, Q_0^2) F(y, Q_0^2) \quad (A.1)$$

we may write

$$b = b^{(0)} + \frac{\alpha_c(Q^2) - \alpha_c(Q_0^2)}{4\pi} b^{(1)} \quad (A.2)$$

for the leading and next to leading contribution, one has,

$$b^{(0)}(x, y; Q^2, Q_0^2) = \sum_{j=0}^{\infty} G_j(\epsilon) b_0(x, y; r+j) \quad (A.3)$$

$$b^{(1)}(x, y; Q^2, Q_0^2) = \sum_{p=0}^2 \sum_{j=0}^{\infty} a_p(r) b_p(x, y; r+j). \quad (A.4)$$

Here,

$$b_0(x, y; r+j) = \frac{x}{y^2} \cdot \frac{1}{\Gamma(r+j)} \left(\log \frac{y}{x} \right)^{r+j} e^{(3/4 - \mathbb{C})\epsilon} \quad (A.5)$$

$$b_1(x, y; r+j) = \left\{ \psi(r+j) - \log \log \frac{y}{x} \right\} b_0(x, y; r+j) \quad (A.6)$$

$$b_2(x, y; r+j) = \left\{ \left[\psi(r+j) - \log \log \frac{y}{x} \right]^2 - \psi'(r+j) \right\} \times b_0(x, y; r+j). \quad (A.7)$$

Here $\Gamma(z)$ is Euler's function, and $\psi(z) \equiv d \log \Gamma(z) / dz$, $\psi'(z) = d\psi(z) / dz$. $\mathbb{C}$ is Euler's constant, $\mathbb{C} = 0.5772$. The first functions $G_j(r)$ are given by (Gross¹³⁾)

$$\begin{aligned} G_0(r) &= 1, \quad G_1(r) = -r/2, \quad G_2(r) = r \frac{3r+14}{24}, \\ G_3(r) &= -r \frac{r^2+14r+24}{48}, \\ G_4(r) &= r \frac{15r^3+420r^2+2420r+2832}{5760}, \dots \end{aligned} \quad (A.8)$$

The functions a_{pj} depend upon the structure function under consideration. Labelling these by indices 1,2,3 with 1 corresponding to F_1 , 2 to F_2 , 3 to xF_3 , we find that they may be written as

$$a_{pj}(r) = \sum_{l=0}^j G_{j-l}(r) H_{pl}^{(i)}, \quad (A.9)$$

where the G are those in (A.8) and

$$H_{2l}^{(i)} = \frac{8}{3} \delta_{l0}, \quad (A.10)$$

and, for four flavours,

$$\left. \begin{aligned} H_{10}^{(i)} &= 18.689, H_{11}^{(i)} = 6.080, H_{12}^{(i)} = -4.818, \\ H_{13}^{(i)} &= 6.649, H_{14}^{(i)} = -7.742, \end{aligned} \right\} \quad (A.11)$$

$$\left. \begin{aligned} H_{00}^{(1)} &= -16.605, H_{01}^{(1)} = 20.755, H_{02}^{(1)} = -15.034, \\ H_{03}^{(1)} &= 16.390, H_{04}^{(1)} = -16.250, \end{aligned} \right\} \quad (A.12)$$

$$\left. \begin{aligned} H_{00}^{(2)} &= -16.605, H_{01}^{(2)} = 26.088, H_{02}^{(2)} = -20.367, \\ H_{03}^{(2)} &= 21.723, H_{04}^{(2)} = -21.583, \end{aligned} \right\} \quad (A.13)$$

$$\left. \begin{aligned} H_{00}^{(3)} &= -16.605, H_{01}^{(3)} = 20.755, H_{02}^{(3)} = -17.700, \\ H_{03}^{(3)} &= 19.056, H_{04}^{(3)} = -18.917. \end{aligned} \right\} \quad (A.14)$$

The quantity r is

$$r = r(Q^2, Q_0^2) = \frac{16}{3\beta_0} \log \frac{\alpha_c(Q_0^2)}{\alpha_c(Q^2)}, \quad (A.15)$$

and the colour coupling constant is to be taken to second order:

$$\alpha_c(Q^2) = \frac{4\pi}{\beta_0 \log Q^2/\Lambda^2} \left\{ 1 - \frac{\beta_1}{\beta_0^2} \cdot \frac{\log \log Q^2/\Lambda^2}{\log Q^2/\Lambda^2} \right\}. \quad (A.16)$$

Here, for n_f flavours,

$$\beta_0 = (33 - 2n_f)/3, \quad \beta_1 = 2(153 - 19n_f)/3.$$

The series (A.3), (A.4) converge very fast (roughly, like $1/j$!)

Table 1

Parameters of fits to $x\mathcal{F}_3$. (1) leading order
(2) including 2nd order QCD corrections. 44
experimental points.

	α	β	Λ	χ^2
(1)	0.24	2.60	0.24 ^{+0.29} _{-0.19}	30.0
(2)	0.60	2.35	0.16 ± 0.13	24.2

Table 2

Fits including $x\mathcal{F}_3$ and $\mathcal{F}_2$. 73 experimental points.

	α	β	α'	β'	Λ	χ^2
(1)	0.58	2.25	0.70	2.92	0.30 ^{+0.20} _{-0.16}	38.5
(2)	0.60	2.35	0.63	2.51	0.17 ± 0.08	32.6

Table 3

χ^2 and Λ for leptonproduction. (1) Leading QCD fit. (2) Including second order corrections. (a) This is obtained combining the second and third columns except, (b) data on F_2 for ν , and F_1 for e, μ scattering have also been included.

	$F_{2,ep}$	$F_{2,ep} \text{ \& } \mu p$	$x F_{3,\nu}$	All (a)
$\chi^2(1)$	12.2	28.7	30.0	58.7
$\chi^2(2)$	9.3	26.6	24.2	50.8
Λ (1)	0.36	0.45	0.24	0.39 ± 0.14 (b)
Λ (2)	0.15	0.17	0.16	0.17 ± 0.05
No. exp. points	18	31	44	75

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FIGURE CAPTIONS

- Fig. 1 : Leading (solid lines) and twist four (broken lines) contributing to scaling violations for the 4th and 6th moments of xF_3 . $Q_0^2 = 2\text{GeV}^2$.
- Fig. 2 : Scaling violations for the whole structure function, xF_3 . $xF_3^{(2)}$ ($x, Q^2 = 2\text{GeV}^2$) is assumed to be the twist two part of xF_3 . Broken line: only leading QCD contribution. Solid line: leading and next-to-leading contribution. Dotted line: Twist four contributions for $\mu^2 = 0.1$ or 0.2GeV^2 . Experiment points from Ref. 2).
- Fig. 3 : The functions $xF_3(x, Q^2 = 12.5\text{GeV}^2)$ for various n : (a) $n = 0$; (b) $n = 4$; (c) $n = 6$. The three fits (solid, broken and dotted lines) are discussed in the text. Data from Ref. 2) (solid dots) and Ref. 4) (open dot). Shaded areas: "experimental parts" of the areas.
- Fig. 4 : The function $b(x, y; Q^2, Q_0^2)yF_3(y, Q_0^2)$, with $Q_0^2 = 12.5$, $Q^2 = 300\text{GeV}^2$, for two typical x values. Experimental points from Ref. 2). Shaded areas: "experimental part" of the areas.
- Fig. 5 : Fit to $xF_3(x, Q^2)$. Experimental data from Ref. 12). Broken line: only leading QCD contributions. Solid line: second order corrections included.
- Fig. 6 : Fit to $F_2(x, Q^2)$. Experimental data from Ref. 2). Second order corrections included.
- Fig. 7 : Fit to $xF_3(x, Q^2)$ with data from Ref. 4).
- Fig. 8 : Fit to $F_2(x, Q^2)$ with data from Ref. 4).
- Fig. 9 : Predicted $xF_3(x, Q^2)$ at $Q^2 = 2\text{GeV}^2$. Experimental data from Ref. 4) except the star at $Q^2 = 2.25$, $x = 0.06 \sim 0.1$ from Ref. 2).
- Fig. 10 : Predicted $xF_3(x, Q^2)$ at $x = 0.6 \sim 1.0$. Data from Ref. 4). Indent: blow up of the $1.75 \leq Q^2 \leq 8\text{GeV}^2$ region. Data are averages of F_2, xF_3 from Ref. 4). Solid lines: prediction with leading and second order QCD contributions, but twist two only. Broken line: twist four incorporated.

Fig. 1

Fig. 2

Fig. 3a

Fig. 3b

Fig. 3c

$$Q_0^2 = 12.5 \text{ GeV}^2, \quad Q^2 = 300 \text{ GeV}^2$$

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10