POLARIZED ELECTRON-HADRON SCATTERING
AND WEAK INTERACTION THEORY

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ABSTRACT

It is shown that the naive valence quark parton model assumptions used to calculate the parity violation in polarized e- d scattering are not needed in order to extract information from the experimental data about the neutral current couplings. Much more general assumptions which can be justified within the QCD approach are sufficient in order to arrive at essentially the same conclusions for the neutral current coupling parameters.

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1. INTRODUCTION

The experimental results on polarised electron scattering off deuteron targets obtained recently \(^1\) are in good agreement with the minimal SU(2) x U(1) theory with \(\sin^2 \theta_W = 0.20 \pm 0.03\). Although these experimental data show beyond any doubt that the neutral current interaction violates parity, the interpretation of these data within the SU(2) x U(1) theory, especially the estimate of \(\sin^2 \theta_W\), is not so clear, since it is assumed that the electron scattering off the deuteron can be described as an incoherent scattering off the valence quarks. Effects due to the \(\bar{q}q\) sea, scale breaking effects etc. are neglected. Therefore the question arises, how much the effects due to the scale breaking etc. can influence the result for \(\sin^2 \theta_W\). We note that such effects could indeed modify the result to a large extent, since the typical energy and momentum transfer in the experiment reported in ref. (1) is not large enough in order to justify the approximations described above.

This question has been addressed recently by Wolfenstein \(^2\), who showed that under certain assumptions the results of ref. (1) can be justified without using the parton model approximation. In this paper we present a more detailed analysis of this problem. We reach the same conclusion as Wolfenstein, as far as the \(y\)-independent term in the \(e - d\) asymmetry is concerned. This term is essentially independent of the valence quark parton model assumptions. However, this is not true for the \(y\)-dependent term. Here we expect sizeable deviations, and we discuss some numerical estimates based on the experimental results from inelastic electron and neutrino scattering.

2. PARITY VIOLATION IN ELECTRON-HADRON SCATTERING AND QUARK PARTONS

Below we present a short review of the basic formula used to calculate the asymmetry in the scattering of polarised electrons off hadrons. For most part of our analysis we base ourselves on the minimal SU(2) x U(1) theory \(^3\). To order \(g\) the effect of parity violation in electron-hadron scattering is due to the interference between the neutral current and the electromagnetic interaction. This is described by the tensor \(^4\)

\[
\mathcal{R}_{\mu \nu} = \frac{1}{4\pi} \int d^4x \ e^{-ixp} \langle p | \left[ j_{\mu}^{e}(x) j_{\nu}^{\nu}(0) + j_{\nu}^{\nu}(x) j_{\mu}^{e}(0) \right] | p \rangle
\]  

(2.1)
where \( j^i_\mu \) denotes the neutral current, \( j^\rho_\mu \) the electromagnetic current, and \( |p> \) denotes the target particle.

Since we do not take into account terms proportional to \( (m_e/M) \) (M: target mass) we can treat the neutral current as a conserved current, in which case the tensor \( R_{\mu\nu} \) is described by three structure functions \( R_1, R_2, \) and \( R_3 \). Those are defined analogously to \( W_1, W_2, W_3 \) in inelastic lepton production and depend on \( Q^2 = -q^2 \) and \( \nu = \frac{Q^2}{M} \):

\[
R_{\mu\nu} = -g_{\mu\nu} R_1 + \frac{p_\mu p_\nu}{M^4} R_L - \frac{i}{2 M^2} \epsilon_{\mu\nu\rho\sigma} p^\rho q^\sigma R_3. \tag{2.2}
\]

The asymmetry in polarised electron-hadron scattering defined by

\[
A = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} \tag{2.3}
\]

\((R:\text{ righthanded, } L:\text{ lefthanded})\) is given by

\[
A = -\frac{Q^2}{Q^2 + M_Z^2} \frac{g_A \left[ 2 R_2 Q^2 + R_L (4E E' - Q^2) \right] + g_V R_3 Q^2 (E + E') M^{-1}}{2 W_1 Q^2 + W_2 (4E E' - Q^2)}. \tag{2.4}
\]

where \( M_Z \) is the mass of the intermediate Z-Boson and \( E(E') \) are the energies of the incoming (outgoing) electron.

The coupling constants \( g_A \) and \( g_V \) denote the axial vector and vector couplings of the neutral current to electrons. Within the minimal SU(2) x U(1) theory they are given by

\[
g_A = \left( -2 \sin 2 \theta_W \right)^{-1} \tag{2.5}
\]

\[
g_V = \frac{1 - 2 \cos 2 \theta_W}{2 \sin 2 \theta_W}.
\]

In the scaling limit \( \nu \rightarrow \infty, Q^2 \rightarrow \infty, x = Q^2/2M^2 \) fixed eq. (2.4) reduces to
\[ A \rightarrow \frac{Q^2}{M_Z^2} \left[ g_A \frac{v \cdot R_2(x)}{v \cdot W_2(x)} + g_V \frac{v \cdot R_3(x)}{v \cdot W_2(x)} \cdot K(y) \right] \quad (2.6) \]

where \( K(y) = \frac{[1 - (1-y)^2]}{[1 + (1-y)^2]} \) and \( R_2(x), \ R_3(x) \) denote the scaling limits of \( R_2(q^2, v) \) and \( R_3(q^2, v) \). We have used the relation \( v \cdot R_2(x) = 2 M x R_1(x) \) which is obtained in case of pointlike spin 1/2 constituents.

These functions can be expressed in terms of the quark and antiquark distribution functions as follows:

\[ \begin{align*}
\frac{v}{M} R_2(x) &= 2 x \sum_q e_q^V G_q^V (q(x) + \bar{q}(x)) \\
\frac{v}{M} R_3(x) &= 2 \sum_q e_q^A G_q^A (q(x) - \bar{q}(x))
\end{align*} \quad (2.7) \]

where \( e_q^V, e_q^A \) denote the vector and axialvector coupling constants of the neutral current referring to the quark flavor q. Within the minimal SU(2) \( \times U(1) \) theory one has

\[ \begin{align*}
G_\nu^V &= \frac{1 - \frac{1}{2} \sin^2 \theta_W}{\sin 2 \theta_W} \\
G_\nu^A &= \frac{1}{2 \sin 2 \theta_W} \\
G_\nu^D &= \frac{1 + \frac{1}{2} \sin^2 \theta_W}{\sin 2 \theta_W} \\
G_\nu^A &= \frac{-1}{2 \sin 2 \theta_W}
\end{align*} \quad (2.8) \]

In terms of the quark distribution functions the asymmetry is given by

\[ A = \frac{Q^2}{M_Z^2} \left[ g_A \frac{\sum_q e_q^V G_q^V (q + \bar{q})}{\sum_q e_q^V (q + \bar{q})} + g_V \frac{\sum_q e_q^A G_q^A (q - \bar{q})}{\sum_q e_q^A (q - \bar{q})} \cdot K(y) \right] \quad (2.9) \]

If we apply eq. (2.9) for deuteron targets (where \( u = d \)) and neglect the antiquark distributions as well as s and c quarks, we obtain
\[ A = - \frac{Q^2}{M_Z^2} \frac{g}{2 \alpha \sin^2 2\theta_W} \left[ \left( 1 - \frac{2Q}{g} x_W \right) + \left( 1 - 4x_W \right) K(\gamma) \right] \]

\[ = -\frac{G F Q^2}{2 \pi^2 \alpha} \frac{g}{10} \left[ \left( 1 - \frac{2Q}{g} x_W \right) + \left( 1 - 4x_W \right) K(\gamma) \right] \quad (2.10) \]

\[ = -1.5 \times 10^{-4} \left[ \left( 1 - \frac{2Q}{g} x_W \right) + \left( 1 - 4x_W \right) K(\gamma) \right] \cdot Q^2 \cdot [GeV] \]

3. PARITY VIOLATION IN ELECTRON-DEUTERON SCATTERING

The asymmetry in electron-hadron scattering is determined by the commutator of the electromagnetic current and the neutral current (see eq. (2.1)). Both the electromagnetic and the neutral current are superpositions of the various quark currents, e.g. the electromagnetic current is given by \[ \sum_q e_q \bar{q} \gamma_\mu q. \] The commutator of the electromagnetic current and the neutral current consists of two different types of terms:

a) terms of the type \[ [\bar{q} \gamma_\mu q, \bar{q} \gamma_\nu q] \]

where q is the same quark flavor.

b) terms of the type \[ [\bar{q}_1 \gamma_\mu q_1, \bar{q}_2 \gamma_\nu q_2] \]

where q₁, q₂ are different quark flavors.

In the free quark model (and in the naive quark parton model) all terms of type b) vanish. Of course, in reality the terms of type b) do not vanish, but they may be small and may vanish in the scaling limit. In order to pursue this matter further, we study the current commutators of type b) within QCD. In lowest order of QCD perturbation theory the terms of type b) vanish, since there exists no communication between different flavor currents. Such a communication can only be provided via gluons. The lowest nonvanishing contributions to the connected parts of the current commutators of type b) arise in sixth order of the quark-gluon coupling constant. Either the two currents can act on an external gluon line (see Fig. (1)) or on an external quark line (see Fig. (2)).
For this reason the terms of type b) (interference terms) are expected to be very small. In the deep inelastic limit where QCD perturbation theory can be applied they are proportional to \((q^2/\pi)^3\) which is of the order of 1/1000. But even for relatively small \(q^2\) (\(q\): current four-momentum) we expect that the terms of type b) are very small. A measure of how large these terms could be can be obtained e.g. by considering the quasi-elastic process \(\omega N \rightarrow \phi N\). This process is forbidden by the Zweig rule; the violation of the latter is of the order of a few %. For this reason we feel confident to set all terms of type b) to zero. This should be a good approximation not only in the scaling limit, but also for relatively small current momenta; we expect that the terms of type b) contribute less than 1 % to the matrix element eq. (2.1).

We define the structure functions of the pure quark currents as follows:

\[
\frac{1}{4\pi} \int d^4 x \, e^{i q x} \left< \rho \left[ \bar{q}(x) (q_o - q) \gamma^\mu q(x), \bar{q}(x) \gamma^\mu q(x) \right] \right> = - g_{\gamma\rho} W_1^q + \frac{P^\rho P^\sigma}{M^2} W_2^q \\
\frac{1}{4\pi} \int d^4 x \, e^{i q x} \left< \rho \left[ \bar{q}(x) (q_o - q) \gamma^\mu q(x), \bar{q}(x) \gamma^\mu q(x) \right] \right> = - \frac{i}{2M^2} \epsilon_{\rho\sigma} P^\rho q^\sigma W_3^q
\]

(3.1)

In the scaling limit the structure functions defined above approach the quark distribution functions:

\[
2 M \times W_1^q \rightarrow \frac{2}{M} W_1^q \rightarrow x (q + \bar{q})
\]

\[
\frac{2}{M} W_3^q \rightarrow (q - \bar{q}).
\]

(3.2)

Neglecting the term of type b) we can express the asymmetry in terms of \(w_1^q\):

\[
A = \frac{Q^2}{M^2} \left[ g_A \sum q \frac{e_q G_1^V}{(2 W_1^q Q + W_2^q (4 EE' - Q^2))} \sum q \frac{e_q}{(2 W_1^q Q + W_2^q (4 EE' - Q^2))} \right]
\]

\[
+ g_V \sum q \frac{e_q G_1^A W_3^q (Q^2 (E + E') M^{-1})}{\sum q \frac{e_q}{(2 W_1^q Q + W_2^q (4 EE' - Q^2))}}.
\]

(3.3)
Let us consider eq. (3.3) in case of deuteron targets. Here isospin symmetry requires \( W_i^U = W_i^d \). We shall neglect the strange and charmed quarks in the nucleon, which means \( W_i^S = W_i^C = 0 \). In this case the first term in the parenthesis of eq. (33) reduces to

\[
g_A \frac{e_u G_u^A + e_d G_d^A}{e_u^2 + e_d^2}
\]

i.e. one obtains the same result as using the naive parton model. This result is independent of the amount of antiquarks in the nucleon as well as of the value \( \sigma_L/\sigma_T \) in the electron-deuteron scattering. The only approximations we made in order to arrive at this result is to neglect \( W_i^S \) as well as \( W_i^C \) and the terms of type \( b \).

In the minimal SU(2) \( \times U(1) \) theory the leptonic axialvector coupling constant \( g_A \) vanishes for \( \sin^2 \theta_W = 1/4 \). Hence in this case the asymmetry \( R \) for isoscalar targets does not depend on the details of the quark structure of the target. It is independent of the amount of antiquarks as well as of \( \sigma_L/\sigma_T \) (see also ref. (2)).

Let us consider the second term in eq. (3.3). Again we set \( W_i^S = W_i^C = 0 \). Furthermore we use the definition \( W_i^U = W_i^d =: W_i \). The second term in eq. (3.3) can be rewritten as follows:

\[
C \frac{W_i Q^L (E + E') M^{-1}}{2 Q^L W_i + W_L (4 E E' - Q^2)}
\]

where \( C \) stands for the coefficient \( g_v(e_u G_u^A + e_d G_d^A) (e_u^2 + e_d^2)^{-1} \).

We introduce the ratio

\[
R = \frac{W^2 \frac{Q^L - Q^L}{Q^L} - W_i}{W_i} \quad (3.4)
\]

and eliminate \( W_i \). The term eq. (3.3) can be rewritten as
\[
C \frac{W_3 Q^2 (E+E') M^{-4}}{2 Q^2 W_L (1 + \frac{v}{2M_K}) (1+R)^{-1} + W_L (4EE'-Q^2)}
= C \frac{(\nu \cdot W_3)}{(\nu \cdot W_L)} \frac{2 (E+E')}{\frac{4M_K + 2\nu}{1+R} + \frac{4EE'-Q^2}{\nu}}
= C \frac{(\nu \cdot W_3)}{(\nu \cdot W_L)} \frac{\gamma (1+\gamma-1)}{\gamma - 2\gamma + 2 + \left(\gamma^2 + \frac{2M_K \gamma}{E}\right) \left(\frac{1}{1+R} - 1\right)}
= C \frac{(\nu \cdot W_3)}{(\nu \cdot W_L)} \frac{1 - (1-\gamma)^2}{1 + (1-\gamma)^2 + \left(\gamma^2 + \frac{2M_K \gamma}{E}\right) \left(\frac{1}{1+R} - 1\right)}.
\]

We define the function \(B(q^2, \nu)\) as

\[
B(q^2, \nu) = \frac{(\nu \cdot W_3(q^2, \nu))}{(\nu \cdot W_L(q^2, \nu))}. \tag{3.6}
\]

In the minimal SU(2) x U(1) theory we obtain

\[
A = - \frac{Q^2}{M_Z^2} \frac{3}{2C \times w} \left[ (1 - \frac{2}{3} x_w) + (1 - 4 x_w) B \frac{1 - (1-\gamma)^2}{1 + (1-\gamma)^2 + \left(\gamma^2 + \frac{2M_K \gamma}{E}\right) \left(\frac{1}{1+R} - 1\right)} \right]. \tag{3.7}
\]

Of course, this expression for \(R\) coincides with our previous expression (eq. (2.10)) in the special case \(B = 1, R = 0\). We emphasise that the asymmetry \(A\) depends on \(x\), since the functions \(B\) and \(R\) are in general functions of \(x\).

Let us discuss the result eq. (3.7). The ratio \(R\) has been measured in electroproduction at relatively large values of \(Q^2 \) and \(x\), where it is quite small: \(R = 0.25 \pm 0.10 \) \(^5\), as expected for spin \(1/2\) constituents. For the values of \(x\) and \(y\) tested in the experiment of ref. (1) the value of \(R\) is about 0.3 (with large errors). The correction of the asymmetry eq. (3.7) due to \(R \neq 0\) is
very small for small values of \( y \) (\( y \leq 0.3 \)). It is typically of the order of 1%. For small values of \( y \) the largest correction to the naive valence quark parton result comes from the fact that \( B \neq 0 \). The \( B \)-function has been measured in neutrino scattering (see e.g. ref. (6)); for \( x \approx 0.2 \) and \( Q^2 \approx \) few GeV\(^2\) one has \( B \approx 0.7 \ldots 0.8 \). We display the asymmetry \( A \) for various assumptions about \( B \) and \( R \). Of course, in reality \( B \) and \( R \) are functions of \( x \) and \( y \). Especially we expect \( B + 1 \) in the scaling limit for \( x \to 1 \). For simplicity we shall assume that \( B \) and \( R \) are constants. In Fig. (3) we show the asymmetry \( A \) (\( y = 0.21 \)) as a function of \( \sin^2 \theta_W \), as predicted by the minimal SU(2) \times U(1) theory, for various values of \( B \). For \( y = 0.21 \), which is the \( y \)-value relevant for the experiment described in ref. (1), the dependence of the asymmetry on the ratio \( R \) can be neglected. For this reason only the \( B \)-dependence is shown. The case \( B = 1 \) corresponds to the naive valence quark result. The case \( B = 0.8 \) is the one expected to be close to reality. As one can see from Fig. (3), the deviation from the naive valence quark result is very small. The tendency is to decrease the value of \( \sin^2 \theta_W \). If one infers from the experiment a particular value of \( \sin^2 \theta_W \) based on an analysis using the naive valence quark approximation, this value should be replaced by \( \sin^2 \theta_W - c \), where \( c \) is 0.005 for \( B = 0.8 \) and 0.01 for \( B = 0.5 \). Thus the deviation of the result for \( \sin^2 \theta_W \) from the naive parton model result is very small and can be neglected, if the experimental error for \( \sin^2 \theta_W \) is larger than 0.01. Of course, the reason for this insensitivity is simply the fact that one is close to the case \( \sin^2 \theta_W = 0.25 \), where the second term in the asymmetry et. (3.3) vanishes, and that the first term is essentially independent of the strong interactions.

How do scale breaking terms etc. influence the \( y \)-distribution of the asymmetry? In eq. (3.7) exists a term \((2 \, M \, y / E)\) which is \( x \)-dependent and is multiplied by \( 1/(1 + R) \). We shall neglect this term since its contribution to the asymmetry is very small (of the order 1%). In Fig. (4) we display in case of \( \sin^2 \theta_W = 0.20 \) the \( y \)-dependence of \( A \) for various values of \( B \) and \( R \). The \( y \)-dependence does not change much if \( R \) varies from zero to 0.4 and \( B \) from 0.6 to 1. The total change of \( A \) is less than 10% if \( R \) and \( B \) vary in the range denoted above which corresponds to the range allowed by the neutrino and electroproduction experiments.

A similar analysis as the one carried out here for isoscalar targets can be made in case of proton targets. Here the results depend on the \( u \) and \( d \) quark distribution functions explicitly. Detailed knowledge of the quark distribution
functions is necessary in order to draw conclusions from the experimental data about the neutral current coupling constants. As in the case of isoscalar targets the effects due to $R \neq 0$ can be neglected for $y \ll 0.5$. Applying eq. (33) and setting $R = 0$ we arrive at the following expression for $A$:

$$A = -\frac{Q^2}{2 M^2} \left[ g_A \frac{\sum q^2 G^V_q [q(x, Q^2) - \bar{q}(x, Q^2)]}{\sum q^2 [q(x, Q^2) + \bar{q}(x, Q^2)]} + g^V \frac{\sum q^2 G^A_q [q(x, Q^2) - \bar{q}(x, Q^2)]}{\sum q^2 [q(x, Q^2) + \bar{q}(x, Q^2)]} \right] K(y) \right]$$

(3.8)

where $q(x, Q^2)$ are the quark distribution functions including the scale breaking effects, e.g. logarithmic violations of scaling, via the dependence on the virtual current mass $Q^2$: $q + \bar{q} = \frac{Q^2}{2M} W^2_3$, $q - \bar{q} = \frac{Q^2}{M} W^2_3$. Within the minimal $SU(2) \times U(1)$ theory one finds:

$$A = -\frac{G Q^2}{2 \pi^2 \alpha^2} \left[ \frac{\frac{}{}}{} \right] K(y)$$

(3.9)

where we have used the definition

$$\sigma = u + d$$

$$\Delta = u - d$$

4. CONCLUSIONS

In this paper we have demonstrated that only very weak assumptions are needed in the analysis of polarised $e - d$ scattering in order to obtain information about the neutral current coupling constants. Within the framework of QCD we conclude that the large $y$-independent term in the asymmetry $A$ is independent of scale breaking terms etc.; it depends only on the coupling constants of the neutral current. The $y$-dependent term of the asymmetry does depend on scaling violations, antiquark densities, $R = \sigma_L / \sigma_T$. For relatively small values of $y (y \ll 0.5)$ one
can neglect the effects due to $R \neq 0$, and all effects due to scaling violations, presence of antiquarks etc. can be taken into account by multiplying the $y$-dependent term by $B = x_{W}^{3}$. For $0.1 \leq \sin^{2}\theta_{W} \leq 0.3$ the deviation of the actual result from the naïve valence quark parton result is very small. In the minimal SU(2) × U(1) theory the overall effect is a decrease of $\sin^{2}\theta_{W}$ depending on the magnitude of $B$: for $B \approx 0.7 \sin^{2}\theta_{W}$ decreases by about 0.005. In case of $e^{-}p$ scattering one can calculate the asymmetry $A$ in terms of the structure functions $W_{2}^{0} = x[G(x,Q^{2}) + \bar{q}(x,Q^{2})]$ and $W_{3}^{0} = q(x,Q^{2}) - \bar{q}(x,Q^{2})$ taking into account the violations of scaling expected within QCD.

Acknowledgement:

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References


5) For reviews see e.g.:

6) For a discussion see e.g.:
Figure captions:

Fig. 1: Contribution to the current commutator of type b) (connected part) which is proportional to the gluon part of the hadronic wave function.

Fig. 2: Contribution to the current commutator of type b) (connected part) which is proportional to the quark part of the hadronic wave function.

Fig. 3: The asymmetry A at y = 0.21 in units of $10^5/Q^2 \text{ GeV}^2$ as a function of $\sin^2 \theta_W$ for various assumptions about B. The shaded area corresponds to the experimental result reported in ref. (1).

Fig. 4: The asymmetry A in units of $10^5/Q^2 \text{ GeV}^2$ as a function of y for $\sin^2 \theta_W = 0.20$ and various assumptions about R and B.
$A$

$\sin^2 \Theta_w = 0.20$

- $R = 0.4, B = 0.6$
- $R = 0.4, B = 0.8$
- $R = 0.2, B = 0.8$
- $R = 0, B = 1$

**Fig. 4**