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EXCITED GLUEBALL STATES IN 4 - d SU(3) LATTICE GAUGE THEORY

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ABSTRACT

For $SU(N)$ ($N \geq 2$) lattice gauge theories in four dimensions we construct all irreducible representations of the full cubic group on spacelike Wilson loop operators up to length 6. Relying on this set of operators preliminary Monte Carlo results for $SU(3)$ excited glueball states are reported.

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$$D_1^0 = E \oplus T_1 \quad (1.c)$$

$$D_3^0 = A_1 \oplus T_1 \oplus T_2 \quad (1.d)$$

We are now prepared to discuss the identification of Hamiltonian eigenstates of a definite spin. For brevity we do not include parity and C parity in this part of our discussion. The extension is trivial. We assume that the continuum limit of the SU(3) lattice gauge theory exists, rotation symmetry becomes restored, and that we have an energy spectrum of discrete levels (in physical units). The last assumption is only for simplicity (certainly unrealistic for the true theory), and can be relaxed. A general state $|\psi\rangle$ can be expanded in terms of Hamiltonian eigenstates $H|\psi_n\rangle = E_n|\psi_n\rangle$:

$$|\psi\rangle = \sum_n c_n |\psi_n\rangle \quad (2)$$

Here the label is $n = (J, m; \alpha)$. J, m specify a vector of an orthonormal basis of the spin J representation and α is an additional label which may account for excited states of identical spin. The eigenvalue E_n depends only on J and α . From the space of all states $\{|\psi\rangle\}$ a Hamiltonian eigenstate of lowest eigenvalue can be obtained by means of the Ritz variation principle. The next Hamiltonian eigenstate is obtained by applying the variational principle to the orthogonal complement. In that way all eigenstates can be successively obtained. If there is no accidental degeneracy the spin corresponding to a Hamiltonian eigenvalue can be deduced from the multiplicity of its eigenstates.

Let $R = A_1, A_2, E, T_1$ or T_2 be one of the irreducible representations of the restricted cubic group. If we restrict the above analysis to states $|\psi\rangle_R$, which transform under R , a state of spin J is only obtained if the subduced representation D_J^0 contains R . Thus the subduced representations provide us with additional information which would enable us (with the exception of some very artificial cases) to determine the spin even in case of accidental degeneracy. Up to a rather large spin J Eq. (2) will still make sense in some approximate way if we are (on a finite lattice) sufficiently close to the continuum limit. Thus there is in principle no problem. Within the power of the presently available MC variational method, however, the procedure described above is rather useless. We are able to get some approximation to the lowest lying eigenvalues (above the vacuum) in the sector of states $\{|\psi\rangle_R\}$, but we cannot calculate orthonormal complements and determine multiplicities.

About ten years ago the existence of glueballs was first suggested by Fritzsche and Gell-Mann¹⁾. Nowadays Monte Carlo (MC) calculations within lattice gauge theories²⁾ seem to be close to the point, that reliable predictions for the QCD glueball spectrum can be made. In a previous note³⁾ we have reported a self-consistent MC estimate of the $m(0^{++})$ glueball mass for SU(3) lattice gauge theory. In the present letter the extension of this calculation to excited SU(3) glueball states is considered. For strong coupling (SC) expansion within the Hamiltonian framework excited glueball states were first defined in the pioneering work of Kogut, Sinclair and Susskind⁴⁾. Here we are interested in performing an MC variational calculation^{*)}, where one tries a systematic approximation of Hamiltonian eigenstates by computing correlations between Wilson loops, which are local in time. Our systematic approach is to reduce the representations of the full cubic group on all Wilson loops (or on subclasses of Wilson loops) up to a certain length, and to perform an MC calculation for all correlations between these Wilson loops.

The full cubic group O_h (= symmetry group of the cube) is a direct product $O_h = O \times \{+1, -1\}$, where the restricted cubic group O is the subgroup of pure rotations of O_h and $\{+1, -1\}$ is the group of order two. The five irreducible representations of O are well known. In the standard notation for point groups [cf., e.g., Ref. 5)] A_1, A_2 are the 1-d representations, E is the 2-d representation and T_1, T_2 are the 3-d representations. A_1 is the trivial and T_1 is the vector representation. The irreducible representations of O_h are obtained as Kronecker products of irreducible representations of O and $\{+1, -1\}$. In an obvious notation, we denote the representations of O_h by $A_1^\pm, A_2^\pm, E^\pm, T_2^\pm$ and $T_1^\pm$, according to parity $P = \pm$.

By trivially embedding the restricted cubic group O into the rotation group, any irreducible spin J representation D_J of the rotation group defines a "subduced" representation D_J^0 of O . We only consider integer spin $J = 0, 1, 2, \dots$. Up to $J = 12$ the subduced representations can be read off from the tables of Ref. 5) ^{**)} . For the convenience of the reader we list the first four subduced representations:

$$D_0^0 = A_1 \tag{1.a}$$

$$D_1^0 = T_1 \tag{1.b}$$

*) The reader is assumed to be familiar with Ref. 3), where more details and further references can be found. Our Hamiltonian is the log of the transfer matrix.

**) A recent discussion within lattice gauge theory is given in Ref. 6). We do not agree with the conceptual statements of this paper.

In the rest of this paper we rely on some assumptions which determine uniquely the spin up to $J = 3$ for energy minima, calculated on the sectors $\{|\psi_R\rangle\}$ for the irreducible representations of the cubic group. The assumptions are:

$$E_{J=0} < E_{J \geq 4} \quad \text{for using } A_1, \quad (3.a)$$

$$E_{J=1} < E_{J \geq 3} \quad \text{for using } T_1, \quad (3.b)$$

$$E_{J=2} < E_{J \geq 4} \quad \text{for using } E, \quad (3.c)$$

$$E_{J=2} < E_{J \geq 3} \quad \text{for using } T_2, \quad (3.d)$$

$$E_{J=3} < E_{J \geq 6} \quad \text{for using } A_2. \quad (3.e)$$

In part, the assumptions follow immediately from a Regge picture, and the reader will also recognize that consistency checks are provided.

Next we consider all Wilson loops up to length 6 which are local in time. As we are interested in zero momentum eigenstates, we first sum (with fixed orientations) over all spacelike positions. Then there are four different prototypes of Wilson loops, depicted in Fig. 1 of Ref. 3) and in Fig. 1 of this paper which cannot transform into one another under transformations of the full cubic group. The operators #1,2,3,4 come in 3,6,12,4 different spacelike orientations respectively, and for $SU(N)$ with $N \geq 3$ the Wilson loops also have an orientation, defined by the direction along which the trace is taken. On each set of operators $\{O\}_i$ ($i=1,\dots,4$) a representation R_i ($i=1,\dots,4$) of O_h is defined. For $N \geq 3$ these representations are 6,12,24,8-dimensional for $i = 1,2,3,4$ respectively. Wilson loops with opposite orientation can be combined to real and imaginary parts. C parity is + on real and - on imaginary parts. We have reduced all representations on Wilson loops up to length 6. In the notation $R(\#i) = \sum_j \oplus R_j^{PC}$ ($i=1,\dots,4$) we obtain the following results:

$$R(\#1) = A_1^{++} \oplus E^{++} \oplus T_1^{+-} \quad (4.a)$$

$$R(\#2) = A_1^{++} \oplus A_2^{++} \oplus 2 E^{++} \oplus T_1^{+-} \oplus T_2^{+-} \quad (4.b)$$

$$R(\#3) = A_1^{++} \oplus E^{++} \oplus T_2^{++} \oplus T_1^{-+} \oplus T_2^{-+} \\ \oplus T_1^{+-} \oplus T_2^{+-} \oplus A_1^{--} \oplus E^{--} \oplus T_2^{--} \quad (4.c)$$

$$R(\#4) = A_1^{++} \oplus T_2^{++} \oplus A_2^{+-} \oplus T_1^{+-} \quad (4.d)$$

Without C parity Eq. (4.a) is of course the result of Ref. 4) and (4.b) has been independently obtained by the author of Ref. 6) (unpublished). For SU(2) the corresponding representations are 3,6,12,4-dimensional and obtained by omitting the C = - contributions in Eqs. (4). For each of the irreducible representations we have explicitly calculated an orthonormal basis. The expressions are rather lengthy and will be published elsewhere⁷⁾. We have also reduced the representations of the full cubic group on all classes of length 8 Wilson loop operators^{*)}.

We now turn to our MC calculation. We work on a $4^3 \cdot 8$ lattice, details are given in Ref. 3). We have to add a remark concerning the detailed balance. As explained by Binder⁸⁾ detailed balance requires a random choice of the site (in our case link), where upgrading is performed. We call this r-upgrading. In nearly all of the recent MC investigations of lattice theories the random choice is replaced by going in a systematic way through the lattice^{**) ,} we call this s-upgrading. s-upgrading is argued¹⁰⁾ to give a reasonable good approximation to detailed balance. It seems to be considered so save that its use is in general not even mentioned. In the present investigation we have, however, found that (a special) s-upgrading may destroy the positive definite Hilbert space. A positive correlation (corresponding to the 0^{--} state) came out negative (confidence level >95%). Also the $m(0^{++})$ state is lowered by r-upgrading, as compared to previously used³⁾ s-upgrading, by about 7%, but this can hardly be distinguished from a statistical fluctuation.

*) Prior to publication⁷⁾ this result can be obtained from the authors by request.

**) We are only aware of one exception⁹⁾, but there may be others.

To our previously considered set of operators we have added operator #4, and we treat now all different spacelike orientations separately. Altogether this amounts to calculating $2 \cdot (26 \cdot 25/2) = 650$ complex correlations between 25 different Wilson loops at distance $t = 0, 1, \dots$. We distinguish glueball masses by a label $r = (i, R^{PC})$ which determines the considered set of operators $\{O\}_i$ ($i=1, 2, 3$ or 4) and the used irreducible representation R^{PC} . For each label r we define a glueball mass $m_r(t)$, $t = 1, \dots$ by means of

$$m_r(t) = -\frac{1}{t} \log \left(\frac{\sum_{i=1}^{N_r} \langle O_i^r | O_i^r(t) \rangle}{\sum_{i=1}^{N_r} \langle O_i^r | O_i^r \rangle} \right). \quad (5)$$

Here $N_r = 1, 2, 3$ is the dimension of the representation R^{PC} and the operators O_i^r are an orthonormal basis for this representation (i.e., an orthonormal linear combination of operators from the set $\{O\}_i$).

We have first repeated with a similar statistics our previous MC calculation, measuring the new large set of Wilson loops. The indication for the excited glueball states was that they are all much higher, i.e., larger by a factor two or more than $m(0^{++})$. In most of the cases no definite conclusion could be drawn because of large noise. We therefore decided to concentrate our investigation on the value $\beta = 5.6$, where we expect a relatively strong signal, because we observe maxima in the correlation functions.

Our extended investigation lowers our previous result for $m(0^{++})$. Including only data with r upgrading we obtain from the distance $t=1$ after minimization

$$m(0^{++}) = (325 \pm 40) \Lambda^L \quad (6)$$

From results at the distance $t = 2$ we obtain a 11% lower value. This may in part be due to the bias brought by the minimization procedure.

For excited glueball states only our results from distance $t = 1$ are significant. For all considered irreducible representations we summarize in the table our results for glueball masses $m_r(1)$ which rely on 8400 sweeps at $\beta = 5.6$ (r -upgrading). Additionally we had spent 2200 sweeps without measurements for equilibrium first. The error bars are relative to samples of 400 consecutive events and calculated directly with respect to the quotient of the involved correlations. The error bars of the masses $m_r(1)$ are obtained by

applying the log to the minimum, the mean and the maximum value for the quotient. Large error bars in the correlations lead to asymmetric error bars for $m_p(1)$. If the correlation at distance $t = 1$ is (within one error bar) compatible with zero, we only give the lower error bar for $m_p(1)$ as lower bound ($\geq$) for the corresponding glueball mass.

Our aim is to calculate mass ratios between $m(0^{++})$ (obtained from representations of A_1^{++}) and excited glueball states $m(J^{PC})$. For most of the considered glueball states an application of the minimization procedure is at present not suitable. We therefore extract mass ratios by the following method:

- 1) If only one operator contributes to $m(J^{PC})$, we compare the value with the average value which is obtained for $m(0^{++})$ from single operators (1.18 a^{-1} at $\beta = 5.6$).
- 2) If several operators contribute to $m(J^{PC})$, we take the minimum value which can be obtained from a single operator and compare with the corresponding value for $m(0^{++})$ (1.14 a^{-1} at $\beta = 5.6$).

Then the results are:

$$m(0^{--}) = (4.6 \pm 1.7) m(0^{++}) \quad (7)$$

$$m(1^{+-}) = (4.2 \pm 0.3) m(0^{++}) \quad (8.a)$$

$$m(1^{-+}) \geq 4.8 m(0^{++}) \quad (8.b)$$

$$m(2^{++}) = (2.93 \pm 0.07) m(0^{++}) \quad (9.a)$$

$$m(2^{+-}) \geq 4.9 m(0^{++}) \quad (9.b)$$

$$m(2^{-+}) = (3.47 \pm 0.17) m(0^{++}) \quad (9.c)$$

$$m(2^{--}) = (4.8 \pm_{-0.5}^{+1.0}) m(0^{++}) \quad (9.d)$$

$$m(3^{++}) = (3.14 \pm 0.09) m(0^{++}) \quad (10.a)$$

$$m(3^{+-}) = (3.7 \pm 0.3) m(0^{++}) \quad (10.b)$$

Discussion: Eqs. (7)-(10) summarize first MC results for excited SU(3) glueball states. The results are preliminary and involve many uncertainties. Equations (7)-(10) are - in contrary to Eq. (6) - not self-consistent in the sense that the expected scaling law is really seen. Instead they rely on measurements at a single value β , which is argued to be in the scaling region. The error bars in Eqs. (7)-(10) are statistical error bars from this single point and should by no means be confused with the error bar in Eq. (6). The danger of using only one point is that we might have missed the scaling region for excited glueball states. One could imagine that the onset of scaling for excited glueball states is, as compared with $m(0^{++})$, shifted to higher β , and larger Wilson loops could then become very important. Another possibility could be that due to subtle cancellations the spin wave region is reached for $\beta < 5.6$ for some of the considered operators. We have (weak) indications that both possibilities are not purely academic. For instance, Eqs. (8a) and (10.b) are inconsistent with assumption (3.b). If we are deep enough in the scaling region Eqs. (1.b) and (1.d) imply $m(T_1^{PC}) \leq m(A_2^{PC})$ for minimized values of all contributing operators. To be really conclusive this requires a better statistics and the inclusion of more operators in the analysis. The mentioned uncertainties are conjectured to lower the values of Eqs. (7)-(10).

Summary and conclusions: Our results for excited SU(3) glueball states are depicted in Fig. 2. Here we have omitted the inconsistent value for $m(1^{+-})$ and the rather inconclusive lower bounds. Our results for $m(1^{+-})$ (8.a) and $m(2^{++})$ (9.a) are quite different from earlier (rather low order) SC extrapolations⁴⁾. Only the qualitative feature that $m(2^{++}) < m(1^{+-})$ is in agreement. As explained, our results are afflicted by several uncertainties. This suggests to extend our present analysis to a larger lattice (in space directions) and to include at least those length 8 Wilson loop operators which contribute to interesting states. This we promise for a future publication⁷⁾.

To the extent that quark corrections are small (i.e., the "quenched" approximation to QCD works) SU(3) glueball results are predictions for QCD. A more detailed investigation of quark corrections seems to be useful when the uncertainties within the SU(3) investigation are settled first. Our qualitative result is that we obtain a rather low glueball $m(0^{++})$. Present difficulties in estimating the MeV value are discussed in Ref. 3). A possible candidate for the $m(0^{++})$ glueball could be the broad ϵ enhancement in the 0^+ wave at ~ 600 MeV (if it still exists). Candidates with a reasonably small width would, however, be more favourable. All our excited glueballs have a much larger mass than $m(0^{++})$. If this persists for the 0^{-+} state (which shows up for the first time in five different classes of length 8 Wilson loop operators) a small $m(0^{++})$ mass would be consistent with $\rho(1440)$ being a 0^{-+} glueball¹¹⁾. Assuming again $m(0^{++}) \approx 600$ MeV our 2^{++} tensor mass value (9.a) agrees well with $\theta(1640)$ [cf. Ref. 11)].

We like to emphasize that the 0^{-+} state is naturally defined within the lattice approach. The experimental verification of such a state would be a strong test in favour of glueballs because it cannot mix with quark model states (even not with exotic diquark-antidiquark bound states). Similar considerations apply to the 1^{-+} and 2^{+-} glueball states (excluding exotic mesons in case of the 1^{-+}). Within a conventional picture of glueballs as bound states of gluon constituents, the existence of these states has been previously stressed in Ref. 12). In further agreement with this reference we find from our analysis of length 8 Wilson loop operators that all possible J^{PC} combinations appear [$J \leq 3$ by means of our assumptions (3)]. We like to remark, however, that gluons make no sense on the lattice for sufficiently strong coupling. This includes the whole scaling region which we can consider within the present limitations of the MC method.

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FIGURE CAPTIONS

Fig. 1 : Operator 4.

Fig. 2 : Preliminary estimates for some SU(3) glueball states. The scale is set by $m(0^{++}) = 1$.

Table

Operator	P	$A_1: 0^{PC}$	C	$T_1: 1^{PC}$	C	$E: 2^{PC}$	C	$T_2: 2^{P+}$	$T_2: 2^{P-}$	$A_2: 3^{PC}$	C
#1	+	1.28 ± 0.05	+	$5.8^{+2.2}_{-0.6}$	-	3.34 ± 0.08	+	—	—	—	
#2	+	1.14 ± 0.05	+	4.8 ± 0.3	-	3.45 ± 0.07	+	—	≥ 5.6	3.71 ± 0.16	+
#3	+	1.14 ± 0.05	+	$4.8^{+0.4}_{-0.3}$	-	3.47 ± 0.09	+	3.74 ± 0.11	≥ 5.8	—	
#3	-	$5.4^{+2.0}_{-0.6}$	-	≥ 5.7	+	≥ 5.2	-	4.1 ± 0.2	$5.7^{+1.1}_{-0.6}$	—	
#4	+	1.17 ± 0.04	+	5.0 ± 0.3	-	—		3.73 ± 0.10	—	4.4 ± 0.3	-

Glueball estimates for irreducible representations of the full cubic group. All values rely on 8 400 sweeps at $\beta = 5.6$ and are given in units [a^{-1}] (a lattice spacing).

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Operator # 4

Figure 1

Figure 2