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WEAK CURRENT ALGEBRA, COMPOSITE W BOSONS
AND UNIVERSALITY OF THE WEAK INTERACTIONS

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A B S T R A C T

We postulate a local current algebra for the weak isovector currents of the sub-constituents forming leptons, quarks and intermediate bosons. Dominating these currents by the lowest lying boson yields universality of weak interactions as well as the standard mass formulas for the lowest charged and neutral bosons.

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Recently various authors have studied models in which leptons, quarks and weak bosons are composed of subconstituents [for a recent review, see Ref. 1)]. In some of these models the weak interactions can be interpreted as effective interactions much like the nuclear forces in QCD, e.g., in the haplon model of Ref. 2) [see also Refs. 3) and 4)]. In such composite models it is not a priori clear why universality of the weak interactions of leptons and quarks should be valid, while it is known to be empirically true. If there is a spectrum of bound state vector bosons and if the leptons and quarks are bound states, one might expect that the Fermi couplings of, e.g., the (u,d) doublet and the (c,s) doublet turn out to be different. This should especially be true in models in which the heavy lepton and quark flavours are considered to be dynamical excitations of the lightest flavours (ν_e, e^-) and (u,d) . In this respect, composite models are different from the (spontaneously broken) $SU(2) \times U(1)$ gauge theory, where universality of the weak interactions follows from the fact that different generations transform identically under the underlying gauge group.

It has recently been emphasized by two of us ⁵⁾ that not only the observed structure of the weak neutral current, but also universality of the weak interactions as well as the standard model mass formulas for the lowest mass weak bosons ($M_{W^\pm} \approx 79$ GeV, $M_{Z^0} \approx 89$ GeV), are obtained in composite models, provided weak interactions as well as (weak isovector) electromagnetic form factors of leptons and quarks are dominated by the lowest mass intermediate vector boson. This hypothesis has been denoted by "W dominance". It has been motivated by the analogy to hadron physics. The composite nature of hadrons leads to ρ^0 , ω , ϕ dominance of photon hadron interactions at low energies. The composite nature of leptons, quarks and weak bosons should lead to W dominance of photon lepton (quark) interactions. In Ref. 5) this conjecture has been formulated in terms of an effective Lagrangian abstracted from the underlying subconstituent dynamics and expected to be valid in the low energy (compared with weak boson masses) region.

In the present note, we will focus in particular on the universality aspect of weak interactions in composite models. Rather than postulating a phenomenological effective Lagrangian for the weak and electromagnetic interactions between composite leptons and quarks right in the beginning, we will start out from the weak isospin algebra and its possible origin in the structure of local subconstituent currents. Requiring W dominance of the matrix elements of the weak isospin current will then be shown to imply universality, as the weak isospin algebra fixes the normalization of the matrix elements of leptons and quarks at zero momentum transfer. We will only briefly comment on the interaction of photons

with composite leptons and quarks and the neutral current structure and refer to Ref. 5) for details. Before turning to the weak interactions, we will remind the reader of the rôle of ρ^0 dominance ⁶⁾ in QCD.

Let us start from the triplet of isovector currents $J_\mu^{(3)} = (1/2)(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)$, $J_\mu^+ = \bar{d}\gamma_\mu u$ and $J_\mu^- = \bar{u}\gamma_\mu d$, where u and d denote the interacting quark fields of QCD. The currents fulfil the local current algebra

$$[J_\mu^+(x), J_\nu^-(y)]_{x^0=y^0} = 2 J_\mu^{(3)}(x) \delta^3(\vec{x}-\vec{y}) \quad (1)$$

which implies the isospin charge algebra

$$[I^+, I^-] = 2 I^3. \quad (2)$$

The matrix element of $J^{(3)}$ between the vacuum and the ρ^0 state defines the decay constant f_ρ via

$$\langle 0 | \frac{1}{2} (\bar{u} \gamma_\mu u - \bar{d} \gamma_\mu d) | \rho^0 \rangle = \frac{m_\rho^2}{f_\rho} \epsilon_\mu. \quad (3)$$

Let us consider the matrix element of $J^{(3)}$ between hadron states $|A\rangle$. To be specific, we assume the hadron A to consist of a u or d quark and one or two "heavy" quarks $s, c, b, \dots$. The dependence of the matrix element on the momentum transfer t is given by the (isovector) form factor $F_A(t)$ of the hadron A . Using ρ^0 dominance it can be written as

$$F_A(t) = \frac{m_\rho^2}{f_\rho} \frac{f_{\rho AA}}{m_\rho^2 - t}, \quad (4)$$

where m_ρ denotes the ρ^0 mass and $f_{\rho AA}$ the coupling constant to the isodoublet hadron A . The weak isospin algebra (2) implies that the matrix element $\langle A | J^{(3)}(0) | A \rangle$ at $t = 0$ is normalized to the third component of the isospin of A , and thus (upon factorizing out the isospin factor)

$$F_A(0) = 1. \quad (5)$$

As a consequence of this normalization, we obtain from (4) the well-known universality relation ⁶⁾ for the ρ^0 coupling,

$$f_\rho = f_{\rho AA}, \quad (6)$$

i.e., the ρ^0 couples with the same strength to all hadron states A , and this coupling strength is simply related to the ρ^0 decay constant. We have chosen $|A\rangle$ to be an isodoublet state composed of a u or d and a heavy quark. We thus obtain universal couplings to all hadrons of the composition $q\bar{s}$, $q\bar{c}$, qss , qsc , qcc , ..., where q stands for u or d . In particular, the ρ^0 couples universally to K mesons, D mesons and B mesons, and to their excited states.

With respect to QCD relation (6) and the universality of couplings is interesting. Equation (6) relates two numbers which at first sight seem to be independent from each other. In a non-relativistic approach the decay constant f_ρ may be calculated in terms of the ρ^0 wave function at the origin. According to Eq. (6) it is equal to the coupling constant $f_{\rho AA}$ describing the coupling of three complicated bound states to each other. Moreover, as $f_{\rho K^+K^-} = f_{\rho D^+D^-} = f_{\rho B^+B^-} = \dots$ (see Fig. 1), the coupling $f_{\rho AA}$ does not depend on the details of the wave function of the hadron A . [Within a quark pair creation model ⁷⁾, this behaviour is to be expected, if one believes that the creation probability for a spectator pair is flavour-independent.] We also note that the ρ^0 should couple with equal strength to the D^+ , which is the ground state of $\bar{d}c$, and to any excited D^+ state, e.g., the axial vector $\bar{d}c$ meson which, in a non-relativistic approach, is a p wave. These examples show that vector meson dominance imposes highly non-trivial constraints on coupling constants.

Let us note at this point that the universality of vector meson couplings was originally introduced as being due to a non-Abelian gauge principle within the framework of a gauge theory of the strong interactions ⁸⁾ based on vector mesons as gauge particles. From our present point of view, the (approximate) universality of vector meson couplings is recognized as a consequence of the fact that the matrix elements of the currents composed of light quarks u and d are dominated at low frequencies by the lowest lying pole. The currents obey the QCD current algebra, which fixes the normalization at zero momentum transfer $t = 0$. Thus one can say : the universality of vector meson couplings is a consequence of the underlying quark structure and pole dominance. No gauge principle is needed. We should like to argue that the universality of weak interactions may find a similar explanation.

In Nature we observe that the left-handed leptons and quarks form doublets of the weak isospin. The weak isospin charges F_i^W ($i=1,2,3$) obey the isospin charge algebra

$$[F_i^W, F_j^W] = i\epsilon_{ijk} F_k^W. \quad (7)$$

We shall assume that these charges can be constructed as integrals over local charge densities $\mathcal{F}_{oi}^w(x)$, i.e.,

$$F_i^w(x^0) = \int \mathcal{F}_{oi}^w(x) d^3x. \quad (8)$$

Furthermore we assume that the charge densities obey at equal times the local current algebra

$$[\mathcal{F}_{oi}^w(x), \mathcal{F}_{oj}^w(y)]_{x^0=y^0} = i \varepsilon_{ijk} \mathcal{F}_{ok}^w \delta^3(\vec{x}-\vec{y}). \quad (9)$$

The algebra (9) is trivially fulfilled in a model in which leptons and quarks are pointlike objects and the weak currents are simply bilinear in the lepton and quark fields. However, if leptons and quarks are extended objects, the situation changes drastically. Currents, which are bilinear in the (composite) lepton and quark fields would not obey the local algebra (9), just like the currents, which are bilinear in nucleon fields, do not obey the local current algebra of QCD. Thus the algebra (9) becomes a highly non-trivial constraint.

What are the weak local currents $\mathcal{F}_{oi}^w$ made of? The simplest models which lead to the algebra (6) are models in which the currents are bilinear in some elementary fields, which could either be fermions or bosons. One realization of the algebra (9) is to write the currents $\mathcal{F}_{oi}^w$ as bilinears of two fundamental left-handed Weyl fields α and β ,

$$\mathcal{F}_{\mu i}^w(x) = \bar{\psi}(x) \gamma_{\mu L} \frac{\tau_i}{2} \psi(x) \quad (10)$$

with

$$\psi = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}, \quad \gamma_{\mu L} \equiv \gamma_{\mu} \frac{(1-\gamma_5)}{2}. \quad (11)$$

This situation is, e.g., realized in the haplon model of Ref. 2).

It is not known what the spectral functions of the weak isospin currents look like. It could be that they are dominated at low energies by a single pole (like the spectral functions of the $\bar{u}u$ or $\bar{d}d$ currents in hadron physics), by several poles (like the spectral functions of heavy quark currents, e.g., $\bar{c}c$), or by a continuum of states. As in Ref. 5), we shall suppose that the first case is realized, and that the weak spectral functions at low frequencies are dominated by the lowest-lying pole (W dominance). Of course, at higher energies, higher excited states as well as the continuum will become relevant.

Let us look at the form factors of the left-handed weak neutral current $\mathcal{F}_{\mu 3}^L(x)$, i.e., the matrix elements of this current between different left-handed lepton or quark states, e.g., $\langle e_L^- | \mathcal{F}_{\mu 3}(0) | e_L^- \rangle$. Denoting these form factors by $F_e(t)$, $F_\mu(t)$, $F_\tau(t)$, etc., the weak isospin algebra requires a universal normalization at $t = 0$, i.e., $F_e(0) = F_{\nu_e}(0) = F_\mu(0) = F_u(0) = \dots = 1$. Assuming W dominance to be a reasonably good approximation, we may write for the dependence on the four-momentum transfer t ,

$$F_f(t) = \frac{m_W^2}{f_W} \frac{f_{Wff}}{m_W^2 - t}, \quad (12)$$

where f denotes any one of the fermions e^- , ν_e , μ^- , ν_μ , etc. From $F_f(0) = 1$, we obtain in complete analogy to (6) the universality relation

$$f_W = f_{Wff} \equiv g. \quad (13)$$

The neutral W and, because of the weak isospin algebra, also the charged W bosons couple universally to leptons and quarks, $f_{Wff} \equiv g$.

So far we have dealt with purely weak interactions. The inclusion of electromagnetic interactions and the emergence of the observed neutral current structure from W dominance in composite models has been treated in detail in Ref. 5). So let us only mention the main conclusions.

As the subconstituents of leptons, quarks and weak bosons carry charge, the photon makes transitions to weak bosons, which are quite analogous to the $\gamma \rightarrow \rho^0$ transitions in QCD. The γW transition strength, where W denotes the neutral member of the weak boson isotriplet, is then given by $\lambda_{\gamma W} = e/f_W$, where $e = \sqrt{4\pi\alpha}$ and f_W is the W decay constant used previously. W dominance of the weak isovector current at low energies requires that the (weak isovector) photon interacts with leptons and quarks via the form factor $F(t)$, which is dominated by the W . In terms of diagrams, weak and electromagnetic interactions at low energies are described according to Fig. 2. [To fully account for electromagnetic interactions the diagrams corresponding to a weak hypercharge contribution have to be added in Fig. 2. For details and the effective Lagrangian corresponding to the diagrams of Fig. 2, we refer to Ref. 5).] W dominance of the photon lepton (quark) interaction according to the universality relation (13) then yields

$$\lambda_{\gamma W} = \frac{e}{g}. \quad (14)$$

The calculation of the $t \rightarrow 0$ limit of the interaction according to the diagrams of Fig. 2 (supplemented by a weak hypercharge contribution) then yields the photon pole contribution (i.e., the electromagnetic interaction) as well as the observed neutral current structure, provided one identifies

$$\lambda_{\gamma W}^2 = \sin^2 \theta_W, \quad (15)$$

where $\sin^2 \theta_W \approx 0.23$ is the weak angle measured at low energies. From e and $\sin^2 \theta_W$ one computes $g = 0.64$, and $M_{W^\pm} \approx 78$ GeV from g and the Fermi coupling as in the standard $SU(2) \times U(1)$ gauge theory. The neutral boson mass is shifted relative to the charged one by mixing with the photon (W boson insertions in the photon propagator) and is also identical to the canonical value, $M_{Z_0} \approx 89$ GeV.

Let us note that naturally in composite models the full photon interaction proceeds via interaction of the photon with subconstituents according to Fig. 2. The dynamical assumption of W dominance then necessarily implies relations (14) and (15), i.e., $\lambda_{\gamma W}$ as well as g are determined from the known values of e and $\sin^2 \theta_W$, and $M_{W^\pm} \approx 78$ GeV and $M_{Z_0} \approx 89$ GeV are uniquely determined. These restrictions are stronger than the ones obtained in the general γW mixing formalism⁹⁾ not based on composite fermions, in which one introduces a pointlike γ lepton (quark) interaction, and in addition, and quite independently, a γW mixing term. Instead of (15), one obtains the weaker relation⁹⁾

$$\lambda_{\gamma W} \frac{e}{g} = \sin^2 \theta_W, \quad (16)$$

and (14) does not hold in general. As in this case $\sin^2 \theta_W$ only determines the ratio $\lambda_{\gamma W}/g$, the value of the $W^\pm$ mass is not fixed, while a relation between the Z_0 and $W^\pm$ masses is still derivable⁹⁾. If the hypothesis of compositeness makes sense and a single weak boson is dominating at low energies (i.e., higher mass excited boson states are of sufficiently high mass), the stronger relations (14) and (15) should hold, which imply the canonical values for the lowest boson masses. This is at variance with some earlier conjectures in composite models based on simply assuming $g \approx 1$ and arriving at substantially higher masses for the W and Z bosons²⁾⁻⁴⁾.

Let us finally stress that W dominance is a dynamical approximation of the underlying hypercolour dynamics, which is expected to be valid in the low energy region. At high energies (~ 1 TeV) lepton lepton (quark) scattering is expected to show completely new phenomena, like multiple lepton (quark) production,

similar to hadron hadron interactions at high energies. Since universality is valid, we expect W dominance to be a good approximation in the low energy region. As a consequence, deviations from the predicted charged and neutral boson masses may not be too large. Nevertheless, deviations of the order of $\pm 10\%$ to $\pm 20\%$, which cannot be accounted for by radiative corrections within the $SU(2) \times U(1)$ gauge theory, are rather expected than excluded. If the lowest neutral boson mass is close to the $SU(2) \times U(1)$ value of $37.3 \text{ GeV}/\sin\theta_W \cos\theta_W \approx 89 \text{ GeV}$, it would mean that the first excited state is rather heavy. If the lowest mass deviates substantially from the $SU(2) \times U(1)$ value, the first excited state may well be below 200 GeV .

In summary, we postulate a local current algebra for subconstituents of leptons and quarks which is fulfilled, e.g., by currents bilinear in subconstituent fields. Assuming W dominance of the weak spectral functions of the weak isospin current to be valid as a dynamical low energy approximation, universality of weak interactions of the different generations of leptons and quarks emerges, as well as the empirical neutral current structure and the standard values of charged and neutral boson masses. Precise experimental values will be of great interest for composite models.

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FIGURE CAPTIONS

Figure 1 The ρ^0 meson couples universally to states containing a u or d quark and a heavy quark. Likewise the W boson couples universally to the various leptons and quarks. The dashed line denotes additional constituents [see, e.g., Ref. 2)].

Figure 2 In Fig. 2a the weak interactions between composite leptons and quarks due to composite W exchange (in the low energy approximation) are shown. The dashed lines denote further lepton (quark) constituents which are not constituents of the W. In Fig. 2b the electromagnetic interaction (weak isovector part) is shown. The weak isovector part of the γ fermion vertex is supposed to be dominated by the lowest lying W boson [see Ref. 5) for details and a treatment of the isoscalar contribution].

Fig. 1

Fig. 2