QCD ANALYSIS OF THE STRUCTURE FUNCTION F_2 IN MUON NUCLEON SCATTERING

The European Muon Collaboration

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Abstract

A QCD analysis in leading and next to leading order of the structure function F_2 measured in deep inelastic muon nucleon scattering is presented. Taking into account several phenomenological uncertainties, including the gluon description and the charm contribution, fits to the data give values of the QCD scale parameter Λ_{LO} in the range 70-250 MeV. A consistent description of muon scattering and lower Q^2 electron scattering requires the presence of higher twist $1/Q^2$ contributions to the scale breaking which are then determined as a function of x from the data.

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Introduction

In previous publications [1] measurements of the structure function F_2 in deep inelastic muon nucleon scattering were presented as a function of the Bjorken variable x and Q^2 . Leading order QCD fits to the data for the range $x > 0.25$ resulted in a value for the scale breaking parameter Λ of 110 MeV for the hydrogen data and 122 MeV for the iron data. However, a comparison of the values of Λ obtained from different processes within deep inelastic scattering or from e^+e^- reactions is only possible when higher order corrections are taken into account [2]. This letter describes a non-singlet analysis of F_2 in next to leading order for the range $x > 0.25$ and a singlet analysis of F_2 in leading order for the full x -range. In addition the contributions from the following processes and their effects on the determination of Λ have been investigated:

- influence of assumptions on sea and gluon distributions,
- soft gluon emission for $x \rightarrow 1$,
- influence of the threshold for charm production,
- $1/Q^2$ contributions from target mass and higher twist corrections.

The analysis is based on direct fitting of the Altarelli-Parisi equations [3] to the measurements of $F_2(x, Q^2)$. These equations are strictly valid only well above threshold for a given number of flavours and since the data used here are in general above threshold for charm, 4 flavours have been used. For the whole analysis the ratio of longitudinal to transverse photon cross-sections $R = \sigma_L/\sigma_T$ has been taken as zero consistent with the preliminary EMC result for both targets [4]. The fits were performed with the program MINUIT [5] calculating the weight for each data point from only the statistical error. The statistical error quoted on the value of Λ corresponds to an increase from χ^2 to $\chi^2 + 1$ for the fit. Although the values of χ^2 per degree of freedom for the fits are generally high, systematic errors have not been included per data point as there is no unique way to do so. The systematic error on Λ was obtained by shifting the data points according to individual systematic errors and then repeating the fit. The final systematic error on Λ corresponds to the square root of the quadratic sum of the individual systematic errors on $\log \Lambda^2$. Details of the principal fits discussed below plus those already published [1]* are collected in Table 1 for the hydrogen data and in Table 2 for the iron data.

* Small numerical differences for the parameters are due to the change of Q_0^2 from 4 GeV² to 5 GeV².

Non-singlet analysis of F_2

In a first step the analysis was made in the region $x > 0.25$ where valence quarks are dominant. The methods of Abbott et al. [6] and Gonzales-Arroyo et al. [7] were used to solve the Altarelli-Parisi equation in next to leading order for a non-singlet structure function. $F_2(x, Q^2)$ at $Q_0^2 = 5 \text{ GeV}^2$ was parameterised as $F_2(x, Q_0^2) = Ax^\alpha(1-x)^\beta(1-\gamma x)$, and a simultaneous fit to A , α , β , γ and Λ was made. In the $\overline{\text{MS}}$ renormalisation scheme a value $\Lambda_{\overline{\text{MS}}} = 139^{+68}_{-56} {}^{+156}_{-87} \text{ MeV}$ was obtained for the hydrogen data with the same χ^2 as for the leading order fit, and a value of $173^{+29}_{-27} {}^{+158}_{-97} \text{ MeV}$ for the iron data with a better χ^2 than in leading order. If the expression for α_s with 3 flavours instead of 4 is used these values of Λ increase by about 30% and a corresponding decrease is obtained for 5 flavours. Both programs gave consistent results which are not very different from the values for Λ in leading order. These results on Λ are not sensitive to the particular parameterisation of F_2 or to the choice of Q_0^2 .

To investigate possible effects of sea quark contributions in this x domain, a leading order fit was performed using the Q^2 -evolution equations for $F_2(x, Q^2)$ including the gluon distribution. The value of Λ extracted from the hydrogen data in the region $x > 0.25$ is shown in fig. 1 for different gluon distributions. Compared to the result in leading order from the non-singlet analysis, where the glue is neglected, the increase of Λ for both targets is less than 100 MeV as long as the gluon distribution is softer than $(1-x)^3$ at $Q_0^2 = 5 \text{ GeV}^2$. Harder gluon distributions, however, are unlikely following a recent measurement of the sea quark content in the nucleon [8]. A non-singlet fit of F_2 after directly subtracting this measured Q^2 -dependent sea contribution leads to a value for Λ of 146 MeV for the hydrogen data and of 199 MeV for the iron data in leading order.

Recently it has been shown [9] that for $x \rightarrow 1$ soft gluon emission could considerably change the Q^2 -evolution of the structure function. For large x these effects are only partly taken into account in the next to leading order calculation. A parameterisation by J.L. Meunier [10] sums up these corrections to all orders. This procedure reduces the value of Λ to $(40 \pm 17) \text{ MeV}$ with no change in χ^2 for the hydrogen data and to $(62 \pm 18) \text{ MeV}$ for the iron with χ^2 slightly improved, however the upper limit in x of the data is 0.65. A

significant decrease of Λ was also observed in the analysis of ν -data [11] and is mainly due to the rescaling of the argument of α_s from Q^2 to $Q^2(1-x)$.

Singlet analysis of F_2

To extend the analysis into the region $x < 0.25$ where effects from sea quarks and gluons are important, the singlet evolution equations have been applied using the procedures of Abbott et al. [5] and of Gonzales-Arroyo et al. [6] in leading order. The singlet term $F_2^S(x, Q^2)$ at $Q_0^2 = 5 \text{ GeV}^2$ was parameterised as $F_2^S(x, Q_0^2) = Ax^\alpha(1-x)^\beta + B(1-x)^\gamma$ according to the contributions from valence and sea quarks. For the hydrogen data a non-singlet term $F_2^{NS} = 1/2(F_2^P - F_2^n)$ has been added, for which a form $F_2^{NS}(x, Q_0^2) = C \cdot \sqrt{x} (1-x)^\delta (1-\epsilon x)$ was assumed. The parameters C, δ, ϵ were determined from a fit to $F_2^P - F_2^n$ as deduced from the EMC [4] and the SLAC [12] results from hydrogen and deuterium targets.

For the gluon distribution the form $xG(x, Q_0^2) = D(1-x)^{5.9} (1+3.5x)$ was chosen according to a recent result from CDHS [8]. D was determined by imposing the momentum sum rule in the fit. To avoid effects from the charm threshold which are mainly concentrated at small x , the analysis was restricted to the range $x > 0.08$, from the region $x < 0.08$ only the points at the lowest Q^2 were used to constrain $F_2(x, Q^2)$ towards $x = 0$. In leading order from both programs a value of $\Lambda = 81_{-30}^{+36} {}_{-32}^{+44} \text{ MeV}$ for the hydrogen data was obtained and a value of $\Lambda = 163_{-22}^{+22} {}_{-64}^{+99} \text{ MeV}$ for the iron data. The results for Λ changed by less than 30 MeV using different parameterisations for $F_2(x, Q_0^2)$ or changing Q_0^2 . Assuming a much softer gluon distribution like $(1-x)^7$ or a much harder one as proposed by Glück et al. [13] Λ varied between 70 and 110 MeV for the hydrogen data and between 125 and 245 MeV for the iron data. The slopes $dF_2/d\ln Q^2$ obtained from the fits are compared with those of the data in fig. 2. For both targets the behaviour of the slopes is seen to be reproduced by the fits, especially in the case of the iron data. The hydrogen data points for $0.1 < x < 0.3$ are systematically low indicating a slight preference for a gluon distribution which is steeper than that of CDHS.

To investigate the influence of charm production the analysis was also made with 3 flavours in the Altarelli-Parisi equations instead of 4 after subtracting a charm contribution using a parameterisation of the EMC results [14] on $F_2^{e\bar{e}}$ from the semileptonic decays of charmed particles. A value of $\Lambda = 108$ MeV was obtained for the hydrogen data and $\Lambda = 199$ MeV for the iron data. If also the number of flavours in the expression for α_s is reduced to 3, these values increase to 139 MeV and 248 MeV respectively. The result of the fit for $x = 0.08$ and the extrapolation to $x < 0.08$ is shown in fig. 3 for the hydrogen data. The scaling violations in the data are seen to be well described by this addition of the measured F_2^{cc} contribution and the QCD evolution with 3 flavours.

Contributions from $1/Q^2$ terms

Besides the leading twist effects which are calculated in perturbative QCD, scaling violation can also be induced by $1/Q^2$ contributions like target mass effects [15] and higher twist corrections [16]. Target mass corrections can be explicitly taken into account by using the variable $\xi = 2x/(1 + \sqrt{1 + 4M_p^2 x^2/Q^2})$ instead of x . Due to the kinematical range covered by the EMC experiment these effects are small. Including target mass corrections increases Λ by 15 MeV for the hydrogen data and by 5 MeV for the iron data.

Within the given Q^2 range of a single experiment it is difficult to distinguish the logarithmic QCD behaviour from a scaling violation with $1/Q^2$. The EMC iron data for instance can be described for $x > 0.25$ by the functional form $F_2(x, Q^2) = A(x) + B(x)/Q^2$ with the same x^2 as the second order QCD fit. However, x^2 decreases by more than 25% by adding to the leading twist term a $1/Q^2$ contribution of the form $F_2(x, Q^2) = F_2^{QCD}(x, Q^2) + Cx^\alpha (1-x)^\beta/Q^2$, indicating the presence of higher twist. For a separation of the higher twist contributions from the logarithmic QCD behaviour a range of Q^2 as wide as possible is required. Therefore the EMC data were combined with the SLAC data [14] at low Q^2 in the range $x > 0.25$. To avoid uncertainties due to nuclear effects which may not be properly taken into account by the standard Fermi motion correction [17], no combination was made between data from different targets such as SLAC results from deuterium and EMC measurements from iron. Therefore this analysis of higher twist terms was restricted to the proton data. For the

determination of F_2 from the measured cross sections of the SLAC experiments a value of $R = 0.21$ was assumed [18]. Cuts were made at $Q^2 > 1.5 \text{ GeV}^2$ and $W > 2 \text{ GeV}$, to exclude resonance contributions. The relative normalisation was determined from the overlap region of both experiments and resulted in a -10% shift for the SLAC data independently of x .

A simultaneous fit to leading twist contributions and $1/Q^2$ corrections was made according to the form $F_2(x, Q^2) = F_2^{\text{QCD}}(1 + \frac{h_4(x)}{Q^2})$, where $F_2^{\text{QCD}}(x, Q^2)$ is considered as a non-singlet structure function. Since the x dependence of h_4 is a priori unknown it was fitted for each of the x bins individually. From the fit a value of $\Lambda_{\overline{\text{MS}}} = 124_{-51}^{+66} \text{ MeV}$ was obtained. The resulting values of $h_4(x)$ are shown in fig. 4a). Also indicated is the range of $h_4(x)$ fixing $\Lambda_{\overline{\text{MS}}}$ to 50 MeV and 250 MeV. Taking into account target mass effects explicitly in the Altarelli-Parisi equation results in a value of $\Lambda_{\overline{\text{MS}}} = 100_{-49}^{+65} \text{ MeV}$ and an x dependence of $h_4(x)$ as shown in fig. 4b). Since the SLAC data extend to rather low Q^2 the analysis was also done with 3 flavours in the expression for α_s instead of 4. $\Lambda_{\overline{\text{MS}}}$ increased to 180 MeV and 150 MeV respectively after applying target mass corrections whereas $h_4(x)$ hardly changed at all.

$h_4(x)$ is found to be practically zero up to $x = 0.4$ and then steeply rising with x . From this behaviour $h_4(x) \propto x^2/(1-x)^2$ is a better parameterisation than the usually assumed forms $h_4(x) \propto x/(1-x)$ or $1/(1-x)$. These results do not significantly change by changing the assumption on R . Also a subtraction of the sea contributions of F_2 as evaluated from the CDHS measurements [8] does not alter the results. It should be noted that an additive ansatz $F_2(x, Q^2) = F_2^{\text{QCD}}(x, Q^2) + \frac{h'_4(x)}{Q^2}$ fits the data equally well.

Conclusions

It has been shown that the scaling violations observed in inelastic muon nucleon scattering can be described by QCD. Next to leading order fits to the data at $x > 0.25$, neglecting gluon and sea contributions, give values of $\Lambda_{\overline{\text{MS}}} = 139_{-93}^{+187} \text{ MeV}$ for hydrogen and $173_{-98}^{+164} \text{ MeV}$ for iron, where the statistical and systematic errors on $\log \Lambda$ have been combined in quadrature. The corresponding

values of $\alpha_s^{\overline{MS}}$ at $Q^2 = 25 \text{ GeV}^2$, in the centre of the range of the data, are $0.168^{+0.044}_{-0.036}$ and $0.177^{+0.038}_{-0.031}$. Values of $\Lambda_{\overline{MS}}$ are found to be $\sim 30\%$ higher than the Λ_{LO} values for the corresponding fits. The sensitivity of Λ to theoretical and phenomenological uncertainties in its extraction has been tested in a series of leading order fits which gave values of Λ_{LO} in the range 70 to 146 MeV for hydrogen and 125 to 250 MeV for iron. The dominant uncertainty is due to the imperfect knowledge of the gluon distribution. A consistent description of both muon and electron data can be obtained with the inclusion of a $\frac{1}{Q^2}$ dependent higher twist term whose form in x is determined by the experimental data; the value of Λ obtained is not significantly changed.

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Table 1: Fit Parameters for F_2 on Hydrogen

F_2	Order	A	α	β	γ at $Q_0^2 = 5 \text{ GeV}^2$	B	C	δ	ϵ	D	Λ [MeV]	Error stat. syst. χ^2 (stat.)	Degrees of Freedom
N.S. *) ($x > 0.25$)	L.O.	1.52	0.55	2.28	0.75	-	-	-	-	-	$\Lambda = 110$	+58 +124 -46 -69	97
"	N.L.O.	1.44	0.53	2.21	0.76	-	-	-	-	-	$\Lambda_{MS} = 139$	+68 +156 -56 -87	99
"	+T.M.C.	1.60	0.58	2.22	0.78	-	-	-	-	-	$\Lambda_{MS} = 154$	+70 +173 -60 -97	98
S.+N.S. **)	L.O.	1.54	0.66	3.23	13.4	0.24	0.077	2.81	-5.4	2.55	$\Lambda = 81$	+36 +44 -30 -32	198
" (F_2^{cc} subtr.)	"	1.54	0.66	3.20	13.0	0.23	0.077	2.81	-5.4	2.55	$\Lambda = 108$	+48 +59 -40 -42	210

*) $F_2(x, Q_0^2) = Ax^\alpha(1-x)^\beta(1-\gamma x)$

***) $F_2^S(x, Q_0^2) = Ax^\alpha(1-x)^\beta + B(1-x)^\gamma$; $xG(x, Q_0^2) = D(1-x)^{5.9}(1+3.5x)$;

$F_2^{NS}(x, Q^2) = 1/2(F_2^P - F_2^n)$; $F_2^{NS}(x, Q_0^2) = C \cdot \sqrt{x} (1-x)^\delta (1-ex)$

Table 2: Fit Parameters for F_2 on Iron

F_2	Order	A	α	β	γ	B	D	Λ [MeV]	Error stat. syst.	χ^2 (stat.)	Degrees of Freedom
N.S. ($x > 0.25$)	L.O.	0.60	0.08	3.0	-0.61	-	-	$\Lambda = 125$	+22 +114 -20 -70	209	102
"	N.L.O.	0.50	0.01	2.9	-0.90	-	-	$\Lambda_{MS} = 173$	+29 +158 -27 -97	197	102
"	+T.M.C.	0.49	0.01	3.0	-1.18	-	-	$\Lambda_{MS} = 178$	+29 +162 -27 -100	187	102
S. (**)	L.O.	1.33	0.61	3.2	8.2	0.23	2.53	$\Lambda = 163$	+22 +99 -22 -64	318	159
" (F_2^{cc} subtr.)	"	1.33	0.61	3.2	8.0	0.23	2.53	$\Lambda = 199$	+29 +120 -29 -78	309	159

*) $F_2(x, Q_0^2) = Ax^\alpha(1-x)^\beta(1-\gamma x)$

**) $F_2^S(x, Q_0^2) = Ax^\alpha(1-x)^\beta + B(1-x)^\gamma$; $xG(x, Q_0^2) = D(1-x)^{5.9}(1+3.5x)$

Figure Captions

Fig. 1 Dependence of Λ extracted from the region $x > 0.25$ on the gluon distribution for the hydrogen data.

Fig. 2 a) $dF_2/d\ln Q^2$ for the hydrogen data.

Curve A: non-singlet analysis

$$\Lambda_{LO} = 110 \text{ MeV}$$

$$\Lambda_{\overline{MS}} = 139 \text{ MeV.}$$

Curve B: singlet + non-singlet analysis

$$\Lambda_{LO} = 81 \text{ MeV, } xG(x, Q_0^2) = D(1-x)^{5.9}(1+3.5x) \text{ at } Q_0^2 = 5 \text{ GeV}^2, [8].$$

Curve C: singlet + non-singlet analysis

$$\Lambda_{LO} = 78 \text{ MeV, } xG(x) \sim (1-x)^7 \text{ at } Q_0^2 = 5 \text{ GeV}^2.$$

b) $dF_2/d\ln Q^2$ for the iron data.

Curve A: non-singlet analysis

$$\Lambda_{LO} = 125 \text{ MeV}$$

$$\Lambda_{\overline{MS}} = 173 \text{ MeV.}$$

Curve B: singlet analysis.

$$\Lambda_{LO} = 163 \text{ MeV, } xG(x, Q_0^2) = D(1-x)^{5.9}(1+3.5x) \text{ at } Q_0^2 = 5 \text{ GeV}^2, [8].$$

Fig. 3 $F_2^{\mu p}$ vs. Q^2 at small x . The complete curve is a result of a fit using QCD + $F_2^{c\bar{c}}$. The dashed curve is the same with $F_2^{c\bar{c}}$ removed.

Fig. 4 a) $1/Q^2$ coefficient h_4 as a function of x .

b) $1/Q^2$ coefficient h_4 as a function of x with target mass effects explicitly taken into account.

The solid curve is the function $x^2/(1-x)^2$ and dashed lines indicate the range allowed by varying $\Lambda_{\overline{MS}}$ between 50 MeV and 250 MeV.

FIG. 1

FIG. 2a

FIG. 2b

FIG. 3

FIG. 4a

FIG. 4b