Ref.TH.3272-CERN

CP VIOLATION AND SUPERSYMMETRY

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ABSTRACT

We re-examine the phenomenology of CP violation in the context of broken supersymmetric theories. We argue that since it is related to the mass counter-terms for fermion fields which vanish in a supersymmetric theory, the non-perturbative vacuum parameter θ is not renormalized and is hence "naturally" small in a theory with spontaneously broken supersymmetry. We argue that the usual estimates of CP violating phenomena in the Kobayashi-Maskawa model are likely to remain valid in such a theory, with the possible exception of the neutronelectric dipole moment which may be large if supersymmetry is broken on a scale $\gg O(100)$ GeV.

Ref.TH.3272-CERN

1 April 1982

There has recently been a resurgence of interest in supersymmetry (susy) because of the promise it holds of solving the gauge hierarchy problem¹⁾. Susy cannot be exact in the present world, but if it is broken at a scale $O(1)$ TeV - or larger if one is devious^{2),3)} - susy can be used to "protect" the masses of the Higgses responsible for the breaking of $SU(2) \times U(1)$ and keep the weak interaction scale small. The principal ingredient in this solution to the gauge hierarchy problem is provided by no-renormalization theorems⁴⁾ applying to the non-gauge couplings between chiral supermultiplets. These and other miraculous cancellations can easily be understood in the superspace formalism. They are due to the fact that superloop diagrams only give $\int d^4\theta$ quantities, while superfield interactions can only be expressed as $\int d^2\theta$ quantities⁵⁾. Even more, other quantities such as the anomalous magnetic moment of the muon vanish⁶⁾ because they have no supersymmetric extension. Such an argument rules out any effective electric dipole moment interaction $\epsilon^{\mu\nu\alpha\beta} \bar{\psi}_{\mu\nu} \psi_{\alpha\beta}$ in a supersymmetric theory. As part of the programme to construct a viable phenomenological model based on broken susy, it is natural to reconsider hoary old mysteries such as CP violation⁷⁾. While CP violation in the weak interactions can be incorporated naturally if inelegantly in the standard model⁸⁾, CP violation in the strong interactions presents a problem. Early analyses of perturbation theory in QCD showed⁹⁾ that potential violations of CP and other discrete symmetries could be removed during the process of renormalization by chiral rotations on the quark fields. However, it was subsequently realized¹⁰⁾ that CP violating effects could be reintroduced non-perturbatively through the QCD vacuum parameter θ . Indeed, the same chiral transformations on the quark fields which were previously used to make the quark mass matrix real and diagonal were now seen to induce via the Adler-Bell-Jackiw $U(1)$ anomaly a renormalization of the QCD θ parameter and hence CP violation. This renormalization is logarithmically divergent in any weak interaction model which has "hard" CP violation in dimension four terms in the Lagrangian, such as the popular Kobayashi-Maskawa (KM) model⁸⁾, whereas the experimental upper limit¹¹⁾ on the neutron electric dipole moment tells⁷⁾ us that $\theta < O(2 \times 10^{-9})$. This sounds like a "CP hierarchy" problem analogous to the conventional gauge hierarchy, though there is a difference. If θ is set to zero when the theory is renormalized at some large momentum scale μ , its renormalization when the scale μ is changed to some low value $O(1)$ GeV is finite and small: $\delta\theta = O(10^{-16})$ in the Kobayashi-Maskawa model¹²⁾, unlike the large $O(\alpha_m)$ renormalization of the light Higgs mass in GUTs without susy. However, the logarithmic divergence in $\delta\theta$ renders the Ansatz that $\theta = 0$ at some large momentum scale technically "unnatural".

In this paper we re-examine the question of θ and its renormalization in a susy theory, and also comment on other possible manifestations of CP violation such as the neutron electric dipole moment and the K^0 system. We argue that there is no renormalization of the θ parameter in a susy theory, and that a realistic model with susy broken need have a residual finite θ renormalization, neutron electric dipole moment and K^0 phenomenology^{7),12)} little different from the conventional Kobayashi-Maskawa model⁸⁾. The apparently miraculous absence of θ renormalization in a susy theory is due to the same no-renormalization theorem^{4),5)} that "solves" the gauge hierarchy problem. The renormalization of θ would be induced by chiral rotations on quark fields necessitated by the renormalization of the quark mass matrix: but this is absent in a susy theory because it is a $\int d^2\theta$ quantity.

To see how θ renormalization is generally related to mass renormalization, we recall some of the analysis of Ref. 12). At zeroth order in the non-strong interactions one writes the inverse quark propagator as

$$S_0^{-1}(p) = \not{p} - M \quad (1)$$

with M a real and diagonal matrix. We denote by $\Sigma(p)$ the sum of irreducible non-strong diagram contributions to the quark propagator. It has the Dirac decomposition

$$\Sigma(p) = A \not{p} L + B \not{p} R + M C L + D M R \quad (2)$$

where hermiticity imposes the constraints

$$A = A^\dagger, \quad B = B^\dagger, \quad C = D^\dagger \quad (3)$$

Combining (1) and (2) to get the full inverse quark propagator we see that the left and right-handed quark fields must undergo wave-function renormalizations

$$\psi_L = \frac{1}{\sqrt{1-A}} \psi_L^{\text{Ren}}, \quad \psi_R = \frac{1}{\sqrt{1-B}} \psi_R^{\text{Ren}} \quad (4)$$

and that the quark mass matrix becomes

$$M^{\text{Ren}} = \frac{1}{\sqrt{1-B}} M (1+C) \frac{1}{\sqrt{1-A}} \quad (5)$$

When one makes this mass matrix real and diagonal one makes a net $U(1)$ rotation through an angle

$$\delta\theta = \arg \det M^{\text{Ren}} \quad (6)$$

Because of the hermiticity of A and B (3) and the real and diagonal nature of M (1), it is apparent from Eq. (5) that

$$\delta\theta = \arg \det (1+C) = \text{Im Tr } C + \dots \quad (7)$$

In a supersymmetric theory a similar analysis applies when one calculates in superspace, with the quantities A and B being interpreted as wave function renormalization factors for the matter chiral superfields, and with the bonus that the squark mass matrix is diagonalized simultaneously with the quark mass matrix (5). The punch line is however that $C = 0$ in a susy theory because of the no-renormalization theorem^{4),5)}. This is immediately apparent from a superspace argument, where C is related to the supersymmetric mass counterterm. Equation (7) then tells us that

$$\delta\theta = 0 \quad (8)$$

A word or two about an alternative way of thinking about θ renormalization are perhaps relevant here. The normal gauge field kinetic term and the supersymmetric extension of the $F_{\mu\nu}\tilde{F}^{\mu\nu}$ coefficient of θ in conventional QCD can be regarded as the real and imaginary parts of

$$\int d^2\theta W_\alpha W^\alpha \quad (9)$$

where we define W_α in the conventional way. Looking at the expression (9) one might be tempted to conclude that it could not be renormalized because it does not have the $\int d^4\theta$ form yielded by superloops. However, it is equivalent to a $\int d^4\theta$ term apart from a total divergence in four space, and hence the no-renormalization theorem does not apply. In fact this line of argument about θ renormalization is not likely to be very fruitful because $F_{\mu\nu}\tilde{F}^{\mu\nu}$ and its supersymmetric extension do not renormalize through the conventional introduction of perturbative renormalization counterterms. Indeed $F_{\mu\nu}\tilde{F}^{\mu\nu}$ vanishes in perturbation theory because it is a total divergence. Instead, θ renormalization is induced by mass renormalization (7) and hence vanishes in a susy theory. For this reason it is technically "natural" to set $\theta = 0$ in a susy theory, and then compute the finite corrections which arise when susy is broken. We will do this shortly in the context of a supersymmetric extension of the Kobayashi-Maskawa model⁸⁾ of CP violation in the weak interactions, after first making a few remarks about alternative models.

Susy must be broken in the real world, and this may be achieved either softly or spontaneously. In the case of soft susy breaking¹⁾, the squarks can acquire an arbitrary mass matrix and mixing relative to their quark erstwhile partners¹³⁾. In the most general case Fig. 1 makes a contribution to ImTrC (2) and hence (7) to θ renormalization of order

$$\delta\theta_{\text{soft}} = \sum_q O\left(\frac{\alpha_s}{\pi} \text{ or } \frac{\alpha}{\pi}\right) \text{Im}(UV^\dagger)_{qq} \frac{\Delta m_{sq}^2}{m_{sq}^2 \text{ or } m_{\tilde{q}}^2} \frac{m_{\tilde{q}}}{m_q} \quad (10)$$

where U and V are the gaugino couplings to left- and right-handed quark-squark combinations and the alternative denominators apply when $m_{sq} \gg m_{\tilde{q}}$ or vice versa. It is very difficult to see how the quantity (10) could be less than the present experimental upper limit^{7),11)} on θ of order 2×10^{-9} coming from the neutron electric dipole moment. For instance, with $m_{sq} \approx m_{\tilde{q}} \approx O(100)$ GeV, $m_q = O(10)$ MeV $O(\alpha/\pi) = O(10^{-3})$, $\text{Im}(UV^\dagger)_{qq} = O(10^{-1})$ we get the bound

$$\frac{\Delta m_{sq}^2}{m_{sq}^2 \text{ or } m_{\tilde{q}}^2} \leq O(10^{-9}) \quad (11)$$

There is also a direct contribution to the neutron electric dipole moment coming from the diagram shown in Fig. 2. It gives¹⁴⁾

$$\delta d_n = \left(-\frac{2}{3} \text{ or } +\frac{1}{3}\right) \frac{\alpha}{12\pi} \left(\frac{m_q}{m_{sq}^2}\right) \text{Im}(UV^\dagger)_{qq} \quad (12)$$

which would be $O(10^{-24}$ to $10^{-25})$ e-cm and hence compatible with experiment if we take the previous set of parameters. The result (11) suggests very strongly that there is a symmetry forbidding a one-loop contribution to θ renormalization. This might either be a global $U(1)$ symmetry¹⁵⁾ à la Peccei and Quinn¹⁶⁾, which ensures $\theta = 0$ to every order in perturbation theory, or perhaps a weaker symmetry which only guarantees $\text{Im}(UV^\dagger)_{qq} = 0$ and hence removes one-loop contributions to θ and to d_n .

This is exactly what occurs in models where susy is violated spontaneously via the D term of a new $\tilde{U}(1)$ gauge group with identical hypercharges $\tilde{Y}$ for all quarks of the same charge^{13),17)}. This reality cancellation also occurs in many models with spontaneous susy breaking of the F type³⁾. In these theories θ renormalization and the neutron electric dipole moment arise in two-loop order as in the conventional Kobayashi-Maskawa model of CP violation. We expect the magnitude of θ renormalization to exhibit a similar pattern to that found in the KM model:

$$\delta\theta_{\text{spont.}} = O\left(\frac{\alpha}{\pi}\right)^2 S_1^2 S_2 S_3 \sin\delta \left(\frac{\Delta m_{sq}^2}{m_{sq}^2 \text{ or } m_{\tilde{W}}^2} \text{ or } \frac{\Delta m_q^2}{m_W^2} \right)^2 \quad (13)$$

Since in a spontaneously broken model $\Delta m_{sq}^2 = \Delta m_q^2$, we replace $(\Delta m_{sq}^2) \rightarrow m_S^2$ or m_C^2 in the numerator of Eq. (13) [cf. Ref. 12] and therefore find that

$$\delta\theta_{\text{spont.}} = O\left(\frac{\alpha}{\pi}\right)^2 S_1^2 S_2 S_3 \sin\delta m_S^2 m_C^2 \left(\frac{1}{m_{sq}^2 \text{ or } m_{\tilde{W}}^2 \text{ or } m_W^2} \right)^2 \quad (14)$$

which is $O(10^{-16})$ if $m_{sq}, m_{\tilde{W}} \geq O(100)$ GeV, and hence phenomenologically acceptable. Such a value of θ would contribute $O(10^{-31} \text{ to } 10^{-32})$ e-cm to the neutron electric dipole moment. We find also that in a spontaneously broken susy theory the direct contribution to the neutron electric dipole moment will be of the same order as that in the usual KM model if $m_{sq}, m_{\tilde{W}} \geq O(100)$ GeV. (Recall that $d_n = 0$ in an exactly supersymmetric theory⁶⁾). We expect contributions of order 10^{-30} e-cm from both the two-loop diagrams of Fig. 3a¹²⁾ and the penguin diagrams of Fig. 3b¹⁸⁾. We therefore expect if susy is spontaneously broken on the conventional weak interaction scale that the phenomenology of θ and d_n will resemble that in the standard KM model, with the conceptual advantage that it is technically natural to set $\theta = 0$ above the scale of susy breakdown.

However, one can imagine a scenario in which θ is appreciably larger than in the KM model. This could happen if the susy breaking scale is significantly higher than the scale of $SU(2) \times U(1)$ breaking, and there are other sources of θ renormalization in between and at the thresholds for other interactions which are independent of the KM model for CP violation in the weak interactions, as discussed for example in Ref. 19). However, it remains to be seen whether a model of this type which performs the essential duty of protecting the gauge hierarchy can also contribute to θ substantially more than Eq. (14).

A final remark concerns CP violation in the $K_1 - K_2$ mass matrix. Evidently in a spontaneously broken theory one has the super-box diagram of Fig. 4 which is subject to the same (super)-GIM cancellations as in the conventional KM model. If $m_{\tilde{W}}^2 \geq m_{sq}^2 \gg \Delta m_{sq}^2 = m_t^2 - m_c^2$ we expect a contribution to the imaginary part of the $|\Delta S| = 2$ operator which is

$$O\left(\frac{1}{m_{\tilde{W}}^2}\right)\left(\frac{\alpha}{\pi}\right)^2(S_1^2 S_2 S_3 \sin\delta)\left(\frac{m_t^2 - m_c^2}{m_{sq}^2}\right)^2 \quad (15)$$

and likely to be dominated by the conventional box diagram. We therefore expect no change in the conventional phenomenology⁷⁾ of CP violation in the neutral kaon system.

We conclude by stressing once more the important information about susy theories offered to us by the neutron electric dipole moment. In the absence of some extra symmetry, models¹⁾ of soft susy breaking tend to give too large a value of d_n , because of excessive θ renormalization. Even if this is removed¹⁵⁾, one-loop diagrams in softly broken theories risk predicting $d_n = O(10^{-25})$ e-cm, close to the present experimental upper limit. If d_n is as small as the old Kobayashi-Maskawa prediction $O(10^{-30})$ e-cm^{12),18)}, this would favour models with spontaneous susy breaking on a scale $O(m_W)$. If d_n is found to lie somewhere between these extremes, either one should appeal to a complicated Higgs structure at a scale $O(m_W)$, or else one should appeal to models^{2),3)} with susy spontaneously broken at a scale considerably larger than $O(m_W)$. The neutron electric dipole moment is clearly a sensitive susy seismometer.

ACKNOWLEDGMENTS

We would like to thank Jacques Prentki for useful discussions.

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FIGURE CAPTIONS

- Figure 1 : One-loop diagram which may contribute to θ renormalization in a softly broken susy model.
- Figure 2 : One-loop diagram which may contribute to d_n in a softly broken susy model.
- Figure 3 : a) Generic two-loop superdiagram which can contribute to d_n and b) contribution to d_n from low-lying intermediate states (indicated by a dotted line) and a penguin (indicated by a shaded circle).
- Figure 4 : Super-box diagram contribution to the imaginary part of the $K_1 - K_2$ mass matrix.

Fig. 1

Fig. 2

Fig. 3

Fig. 4