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LEPTON PAIR PRODUCTION AT LARGE TRANSVERSE MOMENTUM
IN SECOND ORDER QCD

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ABSTRACT

The production of Drell-Yan pairs at large transverse momentum is calculated up to $O(\alpha_s^2)$ for a non-singlet combination defined as the cross-section for an incident particle minus the analogous cross-section for an incident antiparticle. Analytic results are presented for all cross-sections and numerical estimates are given at various energies of experimental interest.

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1.- INTRODUCTION

In this paper we present the detailed predictions of the QCD parton model for the production of lepton pairs at large transverse momentum. This process has been previously considered by many authors¹⁾⁻⁴⁾ who have noted (inter alia) that the growth of the lepton pair transverse momentum with increasing energy is strong qualitative evidence for an underlying quantum field theory. The purpose of this paper is to consider this process through to second order in α_s . The calculation of higher order corrections is always necessary in an asymptotic series expansion to fix the precision of lower order estimates; in the case of the Drell-Yan process it is particularly pressing from a theoretical point of view since the corrections to the total cross-section have been found to be large^{5),6)}. Indeed, the existence of a large correction to the Drell-Yan total cross-section has led some authors⁷⁾ to speculate that the normalization of the transverse momentum differential cross-section should be changed by a multiplicative factor. Our results will demonstrate that these speculations are partially borne out.

For reasons of calculational expediency we consider a non-singlet cross-section formed by taking the difference between the cross-section with an incident particle and an incident antiparticle. Our results will therefore be valid for the cross-section differences $(\pi^-\pi^+)$ or $(\bar{p}p)$ scattering off a given target, but we shall argue that in certain kinematic regions of experimental interest our formulae give the numerically dominant contribution for any incident particle containing valence antiquarks. This is because at present energies the distribution of gluons and sea partons is considerably softer than the valence quark distributions, so that we may neglect the contributions of the former at larger values of $\tau = Q^2/S$.

Our paper contains five sections. In Section 2 we describe the implementation of the QCD improved parton model in such a way as to establish our notation. Section 3 briefly describes the calculation of the Feynman diagrams; the results of the calculations which are analytically somewhat complex are relegated to a series of Appendices. Section 4 presents the leading terms in the $q_T \rightarrow 0$ limit (often called the DDT formula). Section 5 contains a discussion of the ambiguities inherent in the implementation of our formula because of the nature of the QCD perturbation series, the steps which we take to resolve those ambiguities, and finally our numerical estimates.

2.- OPERATIONAL PROCEDURE

In this section we supply the details which are necessary for the reader to understand what we have done. Since the technique for the inclusion of higher order corrections into parton processes has been amply described in the literature^{6),8)}, a review of these topics would be superfluous.

The structure of the cross-section for large transverse momentum lepton pair production in the QCD improved parton model is illustrated in Fig. 1. The dominant terms in the hadronic cross-section differential in the rapidity y contain only one active parton per hadron and may be written as,

$$\frac{d\sigma}{dQ^2 dq_T^2 dy} = \sum_{i,j} \int dx_1 dx_2 \left(\sigma_{ij}(\alpha_s(M^2), p_1, p_2) \right) \Big|_{\substack{p_1 = x_1 P_1 \\ p_2 = x_2 P_2}} f_i^B(x_1, M^2) f_j^T(x_2, M^2) \quad (2.1)$$

where σ_{ij} is the QCD improved parton cross-section expressed in terms of momenta p_1, p_2 of the incoming partons and $f_i^B (f_j^T)$ are the distributions of the i^{th} (j^{th}) type of parton in the beam (target) particle. The quantity σ_{ij} appears in Eq. (2.1) evaluated at rescaled values of the hadron momenta and M^2 is a normalization scale taken to be of the order of the large invariants (Q^2, q_T^2) . Equation (2.1) which is exact up to corrections of order $(1/Q^2)$ is in fact more general than we shall need. In the non-singlet cross-section which we calculate, the sum over parton types is restricted considerably. Imposing the condition that the distribution of quarks in particle B is equal to the distribution of antiquarks in $\bar{B}$, the non-singlet cross-section becomes,

$$\frac{d\sigma^{B-\bar{B}}}{dQ^2 dq_T^2 dy} = \sum_{f,f'} \int dx_1 dx_2 \Delta_{ff'}(x_1, x_2) V_f^B(x_1, M^2) V_{f'}^T(x_2, M^2) \quad (2.2)$$

where Δ is the parton cross-section difference

$$\Delta_{ff'}(x_1, x_2) = \left(\sigma_{\bar{q}_f q_{f'}} - \sigma_{q_f q_{f'}} \right) \Big|_{\substack{p_1 = x_1 P_1 \\ p_2 = x_2 P_2}} \quad (2.3)$$

and the valence distributions are defined

$$V_f^H(x, M^2) = q_f^H(x, M^2) - \bar{q}_f^H(x, M^2) \quad (2.4)$$

The sums on f, f' in Eq. (2.2) run only over the quark flavours. Thus, by forming the non-singlet cross-section we have eliminated all dependence on the distributions of incoming gluons which is a major uncertainty in the estimation of the size of large transverse momentum processes. A side benefit is that after removing the diagrams including initial gluons, the number of Feynman diagrams which we have to calculate is greatly reduced.

The σ_{qq} and $\sigma_{q\bar{q}}$ are parton cross-sections calculated in perturbation theory from which the mass singularities have been exorcised using a factorization prescription which we now make more definite. Thus, the quantities σ_{ij} are completely finite and have a perturbation series in α_s ,

$$\sigma_{ij} = \frac{s d\tilde{\sigma}_{ij}}{dQ^2 dt du} = \frac{s d\sigma_{ij}^{(1)}}{dQ^2 dt} \delta(s+t+u-Q^2) + \frac{s d\sigma_{ij}^{(2)}}{dQ^2 dt du} \theta(s+t+u-Q^2) \quad (2.5)$$

where the superscripts indicate the order in α_s at which the terms appear. The relationship between the perturbatively calculated cross-section and the factorized cross-section Eq. (2.5) is fixed by the relationship between the bare and scale dependent parton densities

$$V_f^H(x, M^2) = \int_x^1 \frac{dz}{z} V_{f_0}^H\left(\frac{x}{z}\right) \left[\delta(z-1) + \frac{\alpha_s}{2\pi} R(z, M^2) \right] \quad (2.6)$$

This relationship between the scale dependent densities of Eq. (2.2) and the bare parton densities which appear in the perturbative calculation contains singular terms because of mass singularities; we choose to regulate these singularities by continuing the dimension of space-time^{*)} to n dimensions ($n=4-2\epsilon$). Equation (2.6) completely specifies the relation between the QCD improved parton cross-section $d\tilde{\sigma}$ and the perturbation theory cross-section $d\sigma$ at order α_s^2 .

$$\begin{aligned} \frac{s d\sigma}{dt du} = & \left\{ \frac{s d\sigma^{(1)}}{dt} \delta(s+t+u-Q^2) \right. \\ & \left. + \frac{\alpha_s}{2\pi} \int_0^1 dx_1 R(x_1, M^2) \frac{s d\sigma^{(1)}}{dt} \right|_{p_1 \rightarrow x_1 p_1} \delta(x_1(s+t-Q^2)+u) \end{aligned} \quad (2.7)$$

*) This method is explained in detail in Ref. 6).

$$\begin{aligned}
 & + \frac{\alpha_s}{2\pi} \int_0^1 dx_2 R(x_2, M^2) \left. \frac{s d\sigma^{(1)}}{dt} \right|_{k_2 \rightarrow x_2 k_2} \delta(x_2(s+u-Q^2)+t) \\
 & + \frac{s d\tilde{\sigma}^{(2)}}{dt du} \Bigg\} \quad (2.7) \\
 & \text{cont.}
 \end{aligned}$$

In the above formula we have suppressed a differential with respect to Q^2 . We are therefore considering directly the cross-section for production of a massive photon of fixed mass Q . The relationship of this cross-section to the true muon pair production cross-section is given in the next section. In Eq. (2.7) it is the left-hand side which we calculate in perturbation theory; however, it is the finite residue $d\tilde{\sigma}^{(2)}$ which we wish to extract from our calculation. The value of $d\tilde{\sigma}^{(2)}$ will depend on the particular choice made for R . Performing the trivial integrations in Eq. (2.7) we obtain,

$$\begin{aligned}
 \frac{s d\sigma}{dt du} &= \frac{s d\sigma^{(1)}}{dt} \delta(s+t+u-Q^2) \\
 &+ \frac{\alpha_s}{2\pi} \frac{1}{(s+t-Q^2)} R\left(\frac{-u}{s+t-Q^2}, M^2\right) \left. \frac{s d\sigma^{(1)}}{dt} \right|_{k_1 \rightarrow \frac{-u}{s+t-Q^2} k_1} \\
 &+ \frac{\alpha_s}{2\pi} \frac{1}{(s+u-Q^2)} R\left(\frac{-t}{s+u-Q^2}, M^2\right) \left. \frac{s d\sigma^{(1)}}{dt} \right|_{k_2 \rightarrow \frac{-t}{s+u-Q^2} k_2} \\
 &+ \frac{s d\tilde{\sigma}^{(2)}}{dt du}. \quad (2.8)
 \end{aligned}$$

In general, the function R has the form,

$$R(z, M^2) = \left\{ C_F P_{qq}(z) \left(-\frac{1}{\epsilon} - \ln 4\pi + \gamma_E + \ln\left(\frac{M^2}{\mu^2}\right) \right) + f_{qq}(z) \right\} \quad (2.9)$$

where the coefficient of the mass singularity pole is the Altarelli-Parisi function⁹⁾

$$P_{qq}(z) = \left\{ \frac{1+z^2}{(1-z)_+} + \frac{3}{2} \delta(z-1) \right\} \quad (2.10)$$

The singular part of Eq. (2.9) is determined by the factorization theorem^{10),8)} and is hence unambiguous, but we have considerable liberty in the choice of the function $f_{qq}(z)$. We first define a scheme which for obvious reasons we will denote as "MS factorization" corresponding to the choice

$$f_{qq}^{\overline{MS}}(z) = 0. \quad (2.11)$$

The physical significance of this choice is small but it represents a useful intermediate step in terms of which to write the formulae. A better choice⁶⁾ when comparisons are made using input parton densities defined from deep inelastic lepton scattering is,

$$f_{qq}^{\overline{MS}}(z) = C_F \left[(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ - \frac{3}{2} \left(\frac{1}{1-z} \right)_+ - \frac{1+z^2}{1-z} \ln z + 3 + 2z - \left(\frac{2}{3} + \frac{\pi^2}{3} \right) \delta(z-1) \right] \quad (2.12)$$

Both of the above choices have the advantage that the valence value sum rules are preserved,

$$\int_0^1 dx V_f^H(x, H^2) = \int_0^1 dx V_{f_0}^H(x) \quad (2.13)$$

A third choice is more appropriate for internal comparisons between Drell-Yan experiments and corresponds to

$$f_{qq}^{\overline{MS}}(z) = C_F \left[2(1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ - \frac{1+z^2}{1-z} \ln z + \left(\frac{\pi^2}{3} - 4 \right) \delta(z-1) \right] \quad (2.14)$$

The above choice Eq. (2.12) defines parton distribution functions such that there are no corrections to the non-singlet total Drell-Yan cross-section at $O(\alpha_s)$. In practice the parton distribution functions are measurable from the rapidity differential cross-section rather than the total cross-section, but the difference between the corrections to these two quantities is so small^{6),11)} that Eq. (2.14) is theoretically a reasonable approximation to parton distribution functions determined from lepton pair production experiments alone.

Equation (2.8) is the basic formula for the extraction of $d\tilde{\sigma}$. It is clear from Eq. (2.10) that the functions R are defined as distributions. The technical details associated with the exact definition of the distributions are more properly discussed in the next section where we describe the actual calculation of the graphs.

3.- CALCULATION OF FEYNMAN GRAPHS

The simplest quark-antiquark diagrams which lead to the production of a Drell-Yan pair at large transverse momentum are shown in Fig. 2. To regulate the divergences which appear in subsequent orders we calculate in $n = 4 - 2\epsilon$ space-time dimensions¹²⁾. The result for the cross-section for the process

$$q + \bar{q} \rightarrow \gamma^* + G \quad (3.1)$$

is given by

$$\frac{sd\sigma^{(1)}}{dt du} = K e_f^2 \frac{\alpha_s}{s} T_0(Q^2, u, t) \delta(s+t+u-Q^2) \quad (3.2)$$

where T_0 is the lowest order matrix element squared,

$$T_0(Q^2, u, t) = \left[(1-\epsilon) \left(\frac{u}{t} + \frac{t}{u} \right) + \frac{2Q^2(Q^2-u-t)}{ut} - 2\epsilon \right] \quad (3.3)$$

where as before $Q^2 = q^2$, $s = (p_1 + p_2)^2$, $t = (p_1 - q)^2$, $u = (p_2 - q)^2$ (see Fig. 1). The constant K is given by

$$K = \frac{2\pi\alpha}{N_c} C_F \frac{(1-\epsilon)}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \left(\frac{sQ^2}{ut} \right)^\epsilon \quad (3.4)$$

In the above equations, α is the electromagnetic fine structure constant and α_s is the analogous quantity for the QCD coupling constant suitably rescaled so that it remains dimensionless in n dimensions.

$$\alpha_s \rightarrow \alpha_s (\mu^2)^\epsilon \quad (3.5)$$

Equation (3.2) is the differential cross-section for the production of a photon of mass Q . The corresponding cross-section for the production of a lepton pair is obtained by multiplying by an additional factor $(\alpha/3\pi Q^2)$. Thus we have that

$$\frac{sd\sigma^{(1)}}{dQ^2 dt du} = \frac{\alpha}{3\pi Q^2} K e_f^2 \frac{\alpha_s}{s} T_0(Q^2, u, t) \delta(s+t+u-Q^2) \quad (3.6)$$

The diagrams which constitute virtual gluon corrections to the above process are shown in Fig. 3. To the order to which we are working it is the interference of these diagrams with the diagrams of Fig. 2 which we must calculate. It is advantageous to evaluate the contribution of this interference to the matrix

element squared before performing the integration over the virtual gluon momentum. This allows a partial cancellation of the scalar numerator factors against the denominators in the Feynman integrals. This procedure leads more rapidly to a compact expression for the final answer which is given in Appendix A^{*)}. Here in the main text we display only the terms which are proportional to the lowest order expression.

$$\begin{aligned} \frac{s d\sigma^{\text{VIRTUAL}}}{dt du} &= K \frac{\alpha_s(M^2)}{s} \left\{ T_0(Q^2, u, t) \delta(s+t+u-Q^2) \cdot \right. \\ &\left[1 + \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi M^2}{Q^2} \right)^\epsilon \left[-\frac{(2C_F+N_C)}{\epsilon^2} - \frac{1}{\epsilon} \left(3C_F - 2C_F \ln\left(\frac{s}{Q^2}\right) + \frac{11}{6}N_C + N_C \ln\left(\frac{sQ^2}{ut}\right) - \frac{2}{3}T_R \right) \right. \right. \\ &\left. \left. + \left(\frac{2}{3}\pi^2 C_F - \frac{N_C}{6}\pi^2 - 8C_F - C_F \ln^2\left(\frac{s}{Q^2}\right) + \frac{N_C}{2} \ln^2\left(\frac{s}{Q^2}\right) - \frac{N_C}{2} \ln^2\left(\frac{ut}{Q^2}\right) + \right. \right. \right. \\ &\left. \left. \left. \left(\frac{11}{6}N_C - \frac{2}{3}T_R \right) \ln\left(\frac{M^2}{Q^2}\right) \right] \right] + \dots \right\} \end{aligned} \quad (3.7)$$

The above expression has been renormalized by including the appropriate counter-terms (see Appendix A). The remaining terms coming from the virtual diagrams, indicated by three dots in Eq. (3.7), are finite in the limit $\epsilon \rightarrow 0$. Equation (3.7) is valid for an arbitrary choice of the scale M^2 at which we evaluate the running coupling constant.

The diagrams giving the $O(\alpha_s^2)$ real corrections to the lowest order cross-section are shown in Figs. 4 - 6 for the processes,

$$q + \bar{q} \rightarrow \gamma^* + G + G \quad (3.8)$$

$$q + \bar{q} \rightarrow \gamma^* + q + \bar{q} \quad (3.9)$$

$$q + q \rightarrow \gamma^* + q + q \quad (3.10)$$

When we demand that the massive photon is at a large angle to the incoming beam direction, infra-red and collinear singularities, present in our calculation as

) This answer can also be obtained by crossing the formula for $\gamma^ \rightarrow q + \bar{q} + G$ in Ref. 13).

poles in ϵ , show up only in the diagrams of Fig. 4 and in the contribution to the matrix element squared of $(F_1 + F_2)$ in Fig. 5. It is therefore natural to group these diagrams together. These diagrams give rise to poles in ϵ after integration over the phase space of the gluons. Collinear divergences appear after integration over the angular variables of the two gluons (quark and anti-quark) system. Soft gluon singularities show up as poles in the variable s_2 defined as (cf. Fig. 1),

$$s_2 = (k_1 + p_2 - q)^2 \quad (3.11)$$

which is the invariant mass of the system recoiling against the massive photon. At lowest order the massive photon recoils against a single gluon and s_2 is hence equal to zero. In second order, in the limit when one of the gluons is soft, the singularity also occurs at $s_2 = 0$.

The poles in ϵ due to the soft singularities are made manifest by use of the identity,

$$\begin{aligned} \left(\frac{1}{s_2}\right)^{1+\epsilon} &= -\frac{1}{\epsilon} \delta(s_2) \left[1 - \epsilon \ln A + \frac{\epsilon^2}{2} \ln^2 A \right] \\ &+ \left(\frac{1}{s_2}\right)_{A+} - \epsilon \left(\frac{\ln s_2}{s_2}\right)_{A+} + O(\epsilon^2) \end{aligned} \quad (3.12)$$

where the distributions denoted by the subscript $A+$ are defined as follows,

$$\int_0^A ds_2 \frac{f(s_2)}{(s_2)_{A+}} = \int_0^A \frac{ds_2}{s_2} [f(s_2) - f(0)] \quad (3.13)$$

$$\int_0^A ds_2 \left(\frac{\ln s_2}{s_2}\right)_{A+} f(s_2) = \int_0^A \frac{ds_2}{s_2} [f(s_2) - f(0)] \ln(s_2) \quad (3.14)$$

In practice we shall find that the upper limit of integration A is fixed by the external variables of the problem.

For reasons of book-keeping, it is convenient to divide the calculation of the diagrams involving extra radiation of real partons into four pieces. We will write real contributions to the non-singlet cross-section as,

$$d\sigma^{\text{REAL}} = d\sigma_{\text{I}} + d\sigma_{\text{II}} + d\sigma_{\text{III}} - d\sigma_{\text{IV}} \quad (3.15)$$

The correspondence between the matrix elements squared and the above cross-section pieces is given in the Appendix. For the moment we direct our attention to $d\sigma_I$, which is the only piece containing mass singularities and hence which requires factorization^{*)}.

$$d\sigma_I \sim \left| \sum_{i=1}^8 G_i \right|^2 + \left| F_1 + F_2 \right|^2 \quad (3.16)$$

We write the result for this piece of cross-section in the same form as the $O(\alpha_s^2)$ pieces of Eq. (2.7).

$$\begin{aligned} \frac{s d\sigma_I}{dt du} = & \frac{\alpha_s}{2\pi} \int dx_1 R(x_1) \left. \frac{s d\sigma^{(1)}}{dt du} \right|_{p_1 \rightarrow x_1 p_1} + \frac{\alpha_s}{2\pi} \int dx_2 R(x_2) \left. \frac{s d\sigma^{(1)}}{dt du} \right|_{p_2 \rightarrow x_2 p_2} \\ & + \frac{s d\tilde{\sigma}_I}{dt du}. \end{aligned} \quad (3.17)$$

The first two terms of the right-hand side of Eq. (3.17) will be used to define the scale dependent parton densities as in Eq. (2.6). The singularities still present in $d\tilde{\sigma}_I$ cancel completely against the singularities in the virtual contributions. This feature is apparent in Eqs. (A.4) and (B.10) in the Appendices.

The other contributions in Eq. (3.15) either contain no collinear poles or cancel in the non-singlet combinations which we consider. Analytic expressions for them are given in Appendices C and D.

The output of our calculation is,

$$\Delta_{ff'}(x_1, x_2) = \frac{s d\sigma_{ff'qf}}{dt du} - \frac{s d\sigma_{qf'qf'}}{dt du} = \frac{s d\sigma^{\text{VIRTUAL}}}{dt du} + \frac{s d\tilde{\sigma}^{\text{REAL}}}{dt du} \quad (3.18)$$

which is completely finite as a consequence of the above arguments. It is this quantity which has to be inserted into the general formula Eq. (2.1) to obtain the prediction for massive photon production at large transverse momentum.

$$\frac{d\sigma}{dq_T^2 dy} = \int_0^1 dx_1 \int_0^A ds_2 \frac{1}{(x_1 S + U - Q^2)} \Delta_{ff'}(x_1, x_2) V_f^{\bar{B}}(x_1, u^2) V_{f'}^T(x_2, u^2) \quad (3.19)$$

where

*) The other terms in Eq. (3.15) require no factorization $d\sigma_{II} = d\tilde{\sigma}_{II}$, $d\sigma_{III} = d\tilde{\sigma}_{III}$, $d\sigma_{IV} = d\tilde{\sigma}_{IV}$.

$$x_2 = \frac{s_2 - Q^2 - x_1(\tau - Q^2)}{x_1 S + U - Q^2}, A = U + x_1(S + \tau - Q^2), B = \frac{-U}{(S + \tau - Q^2)} \quad (3.20)$$

where S , T and U are the hadronic variables,

$$S = (P_1 + P_2)^2, T = (P_1 - q)^2, U = (P_2 - q)^2 \quad (3.21)$$

After making a choice for the valence quark distributions inside the scattered hadrons we can use Eq. (3.19) to make numerical estimates.

4.- THE LEADING TERMS FOR $q_T^2 \ll Q^2$

The exact formulae for the parton cross-sections given in Appendices A-D can be used to extract the leading behaviour when $q_T^2 \ll Q^2$. In this limit the cross-section can be written as the product of the q_T integrated cross-section (with parton densities and running coupling constants evaluated at scale q_T^2) and a Sudakov-type form factor,

$$q_T^2 \frac{d\sigma}{dQ^2 dq_T^2 dy} = \frac{\partial}{\partial(\ln q_T^2)} \left[T(q_T^2, Q^2) \frac{d\sigma}{dQ^2 dy} \right] \quad (4.1)$$

This is the so-called DDT formula in which the form factor is given by¹⁴⁾⁻¹⁶⁾,

$$T(q_T^2, Q^2) = \exp \left[- \frac{\alpha_s(q_T^2)}{2\pi} C_F \ln^2 \left(\frac{Q^2}{q_T^2} \right) \right] \quad (4.2)$$

Terms which depend on the scale chosen for the running coupling constant or the parton densities make no contribution to the logarithmically leading terms in Eqs. (4.1) and (4.2) and hence will be neglected in the following treatment. The general expression for the rapidity differential cross-section is

$$\frac{d\sigma}{dQ^2 dq_T^2 dy} = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 \left. \frac{s \sigma_{ij}}{dQ^2 du dt} \right|_{\substack{p_1 = x_1 P_1 \\ p_2 = x_2 P_2}} \theta(s_2) f_i(x_1) f_j(x_2) \quad (4.3)$$

where s_2 , the invariant mass recoiling against the massive photon, is

$$s_2 = x_1 x_2 S + x_1(\tau - Q^2) + x_2(U - Q^2) + Q^2 \quad (4.4)$$

In analogy with the method of Ref. 17) we rewrite the region of integration in Eq. (4.3) as

$$\int_0^1 dx_1 \int_0^1 dx_2 \theta(s_2) \rightarrow \int_{\sqrt{\tau_+} e^y}^1 \frac{dx_1}{(x_1 S' + U - Q^2)} \int_0^{A_1} ds_2 + \int_{\sqrt{\tau_+} e^{-y}}^1 \frac{dx_2}{(x_2 S' + T - Q^2)} \int_0^{A_2} ds_2 \quad (4.5)$$

The limits of integration and their approximate forms in the limit $|q_T| \rightarrow 0$ are specified as follows

$$A_1 = U + x_1(S + T - Q^2) \rightarrow S(x_1 - \sqrt{\tau} e^y)(1 - \sqrt{\tau} e^{-y}) + O(q_T^2) \quad (4.6)$$

$$A_2 = \sqrt{\tau_+} e^y (x_2 S + T - Q^2) + Q^2 + x_2(U - Q^2) \rightarrow \sqrt{q_T^2 S} e^y (x_2 - \sqrt{\tau} e^{-y}) + O(q_T^2) \quad (4.7)$$

$$\tau_+ = \left(\tau + \frac{q_T^2}{S}\right)^{\frac{1}{2}} + \sqrt{\frac{q_T^2}{S}} \rightarrow \sqrt{\tau} + \frac{|q_T|}{\sqrt{S}} + O(q_T^2) \quad (4.8)$$

The denominators on the right-hand side of Eq. (4.5) when integrated give rise to leading terms in the limit $q_T \rightarrow 0$. Thus

$$(x_1 S + U - Q^2) \rightarrow S(x_1 - \sqrt{\tau} e^y) + O(q_T^2) \quad (4.9)$$

$$(x_2 S + T - Q^2) \rightarrow S(x_2 - \sqrt{\tau} e^{-y}) + O(q_T^2) \quad (4.10)$$

Using the above formula the extraction of the leading terms is now a simple exercise. In the lowest order in α_s the only term in the $q\bar{q}$ annihilation matrix element which has a pole in q_T^2 is,

$$T_0(Q^2, u, t) \delta(s_2) \rightarrow \frac{2Q^2(Q^2 - u - t)}{ut} \delta(s_2) \equiv \frac{2Q^2}{q_T^2} \delta(s_2) \quad (4.11)$$

Retaining only this piece we obtain after integration over x_1 and x_2 ,

$$\frac{q_T^2 d\sigma}{dQ^2 dq_T^2 dy} = \frac{4\pi\alpha^2}{3N_c Q^2 S} \left(\frac{\alpha_s C_F}{2\pi}\right) 2 \ln \frac{Q^2}{q_T^2} \left[\sum_f e_f^2 \left(q_f(x_1 = \sqrt{\tau} e^y) \bar{q}_f(x_2 = \sqrt{\tau} e^{-y}) + (1 \leftrightarrow 2) \right) \right] \quad (4.12)$$

in agreement with the first order expansion of Eq. (4.1). In the next order the logarithmically leading terms come from the following pieces of the parton cross-section,

$$\begin{aligned}
 \frac{s d\sigma}{dQ^2 dt ds_2} &= \frac{4\pi\alpha^2 C_F}{3N_c Q^2 s} \left(\frac{\alpha_s}{2\pi} \right)^2 \\
 &\left\{ T_0(Q^2, u, t) \delta(s_2) \left[-\frac{N_c}{2} \ln^2 \frac{ut}{Q^4} + N_c \ln^2 \frac{A}{Q^2} + \left(C_F - \frac{N_c}{2} \right) \ln^2 \frac{A}{ut} \right] \right. \\
 &+ T_0(Q^2, u, t) \left[2 C_F \ln \left(\frac{ut - Q^2 s_2}{(u-s_2)(t-s_2)} \right) \left(\frac{1}{s_2} \right)_{A+} + (8C_F - 2N_c) \left(\frac{\ln \left(\frac{s_2}{Q^2} \right)}{s_2} \right)_{A+} \right. \\
 &\left. \left. - (2C_F - N_c) \ln \left[\frac{(u-s_2)(t-s_2)(ut-Q^2 s_2)}{(Q^2)^4} \right] \left(\frac{1}{s_2} \right)_{A+} \right] + \frac{8C_F (Q^2)^3}{ut(ut-Q^2 s_2)} \ln \left(\frac{s_2(ut-Q^2 s_2)}{Q^2(u-s_2)(t-s_2)} \right) \right\}
 \end{aligned}
 \tag{4.13}$$

The first term in Eq. (4.13) has been taken from Eq. (A.4). All the other terms come from Eq. (B.10). None of the other cross-sections in Eq. (3.15) give rise to leading terms. Notice also that the choice of the scale M^2 is irrelevant when we consider only the leading logarithms. After performing the s_2 integration we may always replace,

$$\ln(A + q_T^2) \rightarrow \ln A
 \tag{4.14}$$

because the neglected terms are logarithmically subleading for both choices of A [Eqs. (4.6), (4.7)]. Inserting the parton cross-section Eq. (4.13) into Eq. (4.3) our final result is,

$$\frac{q_T^2 d\sigma}{dQ^2 dq_T^2 dy} = \frac{4\pi\alpha^2}{3N_c Q^2 s} \left(\frac{\alpha_s C_F}{2\pi} \right)^2 2 \ln^3 \left(\frac{Q^2}{Q^2} \right) \left[\sum_f e_f^2 \left(q_f(x_1 = \sqrt{t} e^y) \bar{q}_f(x_2 = \sqrt{t} e^{-y}) + (1 \leftrightarrow 2) \right) \right]
 \tag{4.15}$$

in agreement^{15),16)} with the second order expansion of Eq. (4.1).

The utility of Eq. (4.1) for numerical estimates has been questioned; a more correct treatment^{15),17)-20)} takes the conservation of transverse momentum into account exactly by performing a Fourier transform to impact parameter space. By incorporating transverse momentum conservation exactly we may resum some of the

next-to-leading terms in a region in which the leading contribution is suppressed by a form factor. Our computation can be used to investigate the pattern of next-to-leading contributions and to check that after resummation the residual terms are truly perturbative in size. The results of such an analysis will be presented in a future publication.

5.- CHOICE OF PARAMETERS AND NUMERICAL RESULTS

This section contains our numerical analysis of the large q_T muon pair production cross-section. Before presenting these results we discuss which quantities are really determined by perturbative QCD and their sensitivity to the values of the parameters chosen.

It is now known¹⁾⁻⁴⁾ that no agreement can be reached between experimental data and the QCD results at $O(\alpha_s)$ without the introduction of an intrinsic parton transverse momentum of the order of the nucleon mass ($q_{T \text{ INT}} \sim 1 \text{ GeV}$). In the jargon of deep inelastic scattering this corresponds to a higher twist effect which vanishes as an inverse power of the energy of the process. If we believe that QCD is the correct theory of hadron forces then higher twist effects are a non-perturbative consequence of the interaction of quarks and gluons. Unfortunately, the extraction of numerical results for the size of these terms directly from QCD is at present beyond our analytic power. We are therefore left with an intrinsic transverse momentum parameter which has a significant effect on the magnitude of the cross-section at present energies. The influence of this parameter persists out to large values of the muon pair transverse momentum because the perturbative cross-section is a rapidly falling function of q_T . Only at very large energies and transverse momenta such as at the collider will the intrinsic transverse momentum effects become negligible. At these energies the genuine QCD high transverse momentum distribution will be measurable. Naturally at low energies the average transverse momentum also depends on the momentum smearing parameter.

The most satisfactory response to this sensitivity to the low transverse momentum region would be to extend the validity of our formula down to lower values of q_T . Our formulae are valid in the region $Q \sim q_T \gg \Lambda$. In the previous section we have isolated the leading term in the limit $Q \gg q_T \gg \Lambda$ for which there is a resummation formula. It will require further work to isolate all the logarithmic terms in this limit and resum them. If we carried this programme through to the end we would find that a smaller value of the transverse momentum smearing was required¹⁵⁾.

In the meantime the best that we can do is to assume a reasonable parametrization for the intrinsic transverse momentum and investigate the sensitivity to this choice in such a way as to isolate the features which are independent of the transverse momentum smearing. Following Ref. 2) we introduce the intrinsic transverse momentum according to

$$\frac{d\sigma^{REG}}{dQ^2 dq_T^2} = \frac{d\sigma}{dQ^2} f(q_T^2) + \frac{1}{\pi} \int d^2 k_T \frac{d\sigma}{dQ^2 dk_T^2} \left[f((q_T - k_T)^2) - f(q_T^2) \right] \quad (5.1)$$

where the transverse momentum smearing function is normalized such that,

$$\int_0^\infty dq_T^2 f(q_T^2) = 1 \quad (5.2)$$

The regularized cross-section Eq. (4.1) has the following properties,

$$\frac{d\sigma}{dQ^2} = \int_0^\infty dq_T^2 \frac{d\sigma^{REG}}{dQ^2 dq_T^2} \quad (5.3)$$

and

$$\langle q_T^2 \rangle = \langle q_T^2 \rangle_{INT} + \langle q_T^2 \rangle_{QCD} \quad (5.4)$$

where the average transverse momenta are defined respectively as

$$\langle q_T^2 \rangle_{INT} = \int_0^\infty dq_T^2 q_T^2 f(q_T^2) \quad (5.5)$$

$$\langle q_T^2 \rangle_{QCD} = \int_0^\infty dq_T^2 q_T^2 \left(\frac{d\sigma}{dQ^2 dq_T^2} / \frac{d\sigma}{dQ^2} \right) \quad (5.6)$$

At asymptotic values of S , Q^2 and q_T^2 , Eq. (5.1) reduces to the perturbative QCD cross-section. Taking the regularized cross-section given by Eq. (5.1) we find that the slope of the regularized and pure QCD cross-sections as a function of q_T^2 (at fixed τ , S) are equal to a very good approximation for $q_T^2 \gg \langle q_T^2 \rangle_{INT}$. We expect this to be true for any reasonable parametrization of the intrinsic transverse momentum. The dominant terms in the growth of the average transverse with S at fixed τ are entirely given by the QCD part

$$\frac{\partial \langle q_T^2 \rangle}{\partial S} = g(\alpha_s(Q^2), \tau) + O(1/S) \quad (5.7)$$

We choose the following analytic form for the smearing function

$$f(q_T^2) = \frac{1}{\langle q_T^2 \rangle_{INT}} \exp \left[- \frac{q_T^2}{\langle q_T^2 \rangle_{INT}} \right] \quad (5.8)$$

The first conclusion which we draw is that it is not possible, even after the inclusion of intrinsic transverse momentum, to fit all experimental data for $d\sigma/dQ^2 dq_T^2$ and the average transverse momentum using the $O(\alpha_s)$ results alone²¹⁾. This is illustrated in Fig. 7 for the case of π^-N scattering. However, there is a slight ambiguity in Eq. (5.1) which we must first discuss. There are two possible choices for the total Drell-Yan cross-section which we insert. The first choice illustrated in Fig. 7a is to choose $d\sigma/dQ^2$ to be the total Drell-Yan cross-section calculated in order $O(1)$ and $O(\alpha_s)$ for the lower and upper bands respectively. However, the normalization of the total Drell-Yan cross-section is known to be in disagreement with the lowest order parton model result. A possible objection to Fig. 7a could therefore be that it is merely displaying this well-known disagreement rather than showing that the $O(\alpha_s^2)$ corrections are needed also at large transverse momentum. That this is not the case can be seen from Fig. 7b where we have chosen $d\sigma/dQ^2$ in Eq. (5.1) to be a representation of the experimentally observed cross-section. The $O(\alpha_s)$ result calculated in this manner is shown to lie significantly below the data at large q_T . We therefore conclude that second order corrections are required to remove the discrepancy.

We give precise meaning to the above statement by making several further comments which fix our procedure and numerical parameters. Our parametrization of the form of the parton densities at a certain scale is given in Table 1. These densities satisfy valence value sum rules and are a representation of the densities measured in deep inelastic lepton hadron scattering²²⁾. Published data²³⁾ in the perturbative region ($Q^2 \sim q_T^2 \gg \Lambda^2$) probe the parton densities in the large x region. A small error in the value of the distribution function in this region modifies the cross-section significantly. For example, for $S = 282 \text{ GeV}^2$, $Q^2 = 110 \text{ GeV}^2$ and $q_T^2 = 9 \text{ GeV}^2$, a change of 10% in the distribution function exponent ρ produces a change of 60% in the cross-section (cf. Fig. 7a). For lower values of Q^2 and q_T^2 at fixed S the change is less dramatic but still remains important.

For this reason the inclusion of scaling violations in our parton densities as we scan through different values of q_T is superfluous. If we included scaling

violations and chose the scale of the parton densities to depend on q_T the effective exponent ρ would change as we change q_T . However, this slow variation of the exponent ρ is small (in the range of q_T which the data span) compared to the initial uncertainty in its determination. We therefore ignore scaling violation of the parton densities (except for the parton densities used to make predictions for ISR energies).

The scale M^2 which determines the strength of the running coupling constant (and in principle the scale of the parton densities) is another free parameter. In order to suppress large logarithms in succeeding orders we must choose M^2 to be a combination of the large invariants in the process (S, Q^2, q_T^2). Different choices significantly modify the size of the second order corrections with respect to the first order. This can be seen by comparing Figs. 7a and 8 which correspond to the two extreme choices $M^2 = Q^2$ and $M^2 = (q_T^2 + 2 \langle q_T^2 \rangle_{\text{INT}})$ respectively; for all other figures we have used the compromise choice,

$$M^2 = \frac{1}{2}(Q^2 + q_T^2) \quad (5.9)$$

The choice of scale $M^2 = (q_T^2 + \text{constant})$ expresses most clearly the fact that the scale size of the interaction changes with the transverse momentum of the produced lepton pair. This choice implies that at lower energies the coupling constant $\alpha_s(q_T^2)$ is almost never in the perturbative regime. Of course $M^2 = q_T^2$ cannot be ruled out by this reasoning, but we have preferred to take a more positive approach and adopt the compromise value given by Eq. (5.9).

By choosing the values of M^2 above we do not preclude the possibility of more rational choices of scale²⁴⁾. However, as we shall see below the correction which we obtain is large so that tuning M^2 , for example, so that the α_s^2 correction vanishes, would surely carry $\alpha_s(M^2)$ out of the perturbative range. We therefore require a physically motivated resummation before such a procedure can be applied.

The running coupling constant at scale M^2 is given by the expression²⁵⁾

$$\frac{\alpha_s(M^2)}{2\pi} = \frac{1}{b_0 \ln M^2/\Lambda^2} - \frac{b_1 (\ln \ln(M^2/\Lambda^2) - \ln \ln(M_0^2/\Lambda^2))}{b_0^3 \ln^2(M^2/\Lambda^2)} \quad (5.10)$$

where

$$b_0 = \frac{11N_c - 4T_R}{6}, \quad b_1 = \left(\frac{17}{6} N_c^2 - \frac{5}{3} N_c T_R - C_F T_R \right) \quad (5.11)$$

Equation (5.10) corresponds to a particular definition of Λ^*). For $M^2 = M_0^2$ where M_0^2 is some value in the perturbative range leading and non-leading values of α_s coincide at fixed Λ . Taking $M_0 = 7.5$ GeV we find that the difference between leading values of α_s and Eq. (5.10) is small for all values of M^2 which we have considered. The choice Eq. (5.10) has been made because whatever intuition we have about the best behaviour of the perturbation series is in terms of $\alpha_s(M^2)$ and not in terms of powers of $\ln M^2/\Lambda^2$ ²⁵⁾.

The only other remaining ambiguity is the physical process in terms of which we choose to normalize the parton distribution functions beyond the leading logarithms. The densities in Table 1 correspond to deep inelastic scattering densities and hence in the definition of the theoretical parton cross-sections we must normalize to deep inelastic scattering ⁵⁾. The alternative procedure requiring different experimental input is to normalize the parton densities using the q_T integrated Drell-Yan process with the scale of the parton densities taken to be Q^2 .

We are now ready to discuss the agreement between our formulae and experimental results. Strictly speaking our formulae are valid only for the non-singlet difference $(\pi^- - \pi^+)N$, whereas most of the existing data are for π^-N scattering. However, at large values of τ , gluon and sea contributions can be neglected and the valence-valence approximation which we have calculated is sufficient. To test the validity of this hypothesis we have calculated the gluon and sea contributions using the known $O(\alpha_s)$ formulae only at $S = 282$ GeV² and $Q^2 = 52.5$ GeV². We find that these contributions are $3 \sim 5\%$ of the total $O(\alpha_s)$ result, so that by neglecting them we expect to introduce an uncertainty of only this order of magnitude.

Assuming the valence-valence approximation we can apply our results to π^-N scattering ^{**)}. The first step is to calibrate the intrinsic transverse momentum by imposing that the average transverse momentum is in agreement with the experimental data ²⁵⁾. Fixing $M^2 = Q^2$, we find that to reproduce $\langle |q_T| \rangle \sim 1$ GeV we must take $\langle q_T^2 \rangle_{INT} \sim 1$ GeV². This is the value taken in Fig. 7a, b which demonstrates that we cannot simultaneously fit the transverse momentum distribution and the average transverse momentum with the $O(\alpha_s)$ terms alone. Any attempt to bring the distribution into agreement by increasing the intrinsic transverse will also increase the average transverse momentum. The $O(\alpha_s^2)$ results seem to lead to a better agreement. Figure 8 displays the $O(\alpha_s^2)$ results only with $M^2 = (q_T^2 + 2\langle q_T^2 \rangle_{INT})$ and with $\langle q_T^2 \rangle_{INT} = 0.64$ and 1 GeV².

^{*)} This is not the definition proposed by Bardeen et al. Ref. 26).

^{**)} Unless stated otherwise the nucleon is taken to be 60% neutron, 40% proton.

Note that the 'perturbative' correction is large - more than 100%. In the case of the total Drell-Yan cross-section it has been argued²⁷⁾ that the largest part of the higher order corrections can be resummed to all orders in α_s . The largest part of the $O(\alpha_s)$ Drell-Yan cross-section is a factor which multiplies the lowest order result.

$$\left. \frac{d\sigma}{dQ^2} \right|_{O(\alpha_s)} \sim \left(1 + \frac{\alpha_s}{2\pi} C_F \pi^2 \right) \frac{d\sigma^{(0)}}{dQ^2} + O(\alpha_s) \quad (5.12)$$

where $O(\alpha_s)$ indicates corrections which are small at presently investigated values of Q^2 and S . The residual corrections can become large at the boundaries of phase space but in these regions the formula are probably unreliable anyway because of the influence of higher twist effects. Assuming that Eq. (5.12) isolates the dominant terms we may postulate also for the transverse momentum cross-section

$$\frac{d\sigma}{dQ^2 dq_T^2} = \exp \left(\frac{\alpha_s}{2\pi} C_F \pi^2 \right) \frac{d\sigma^{(1)}}{dQ^2 dq_T^2} + O(\alpha_s^2) \quad (5.13)$$

By examination of our formulae (Appendix A) we see that for $q\bar{q}$ annihilation there is a large correction proportional to the lowest order cross-section $\alpha_s/2\pi C_F \pi^2$. Of course the cross-section contains many other terms. In Fig. 9 we display separately the contributions to the cross-section to display the numerical importance of the various terms. The contributions $d\sigma_{II}$, $d\sigma_{III}$, $d\sigma_{IV}$ (diagrams of Figs. 5 and 6) are less than 5% of the total cross-section in all the cases we have investigated.

We now discuss the predictions for the cross-section differences for which our formulae are really valid. Figures 10 and 11 display the results for $(\pi^-\pi^+)N$ scattering for $S = 376 \text{ GeV}^2$. Assuming that the slope of the average transverse momentum squared with S is entirely given by the QCD perturbative part we find [cf. Eq. (5.7)]

$$g^{\pi^-\pi^+}(\alpha_s(Q^2), \tau) \approx (1.7 \sim 2.0) \times 10^{-3} \quad (5.14)$$

for $\sqrt{\tau} = 0.275$ and $S = (282-530) \text{ GeV}^2$. The value of this parameter is substantially the same if we use the lowest order formula alone. The average transverse momentum $\langle |q_T| \rangle$ is expected to grow approximately linearly with $\sqrt{S}$, but the value of the slope depends on the particular method of inclusion of the intrinsic transverse momentum smearing. Only when $\langle q_T^2 \rangle \gg \langle q_T^2 \rangle_{INT}$ is the slope predicted by QCD. We therefore do not present results for this quantity.

In the preceding discussion we have used the definition of the parton densities taken from deep inelastic scattering. It is also interesting to compare the results obtained by using parton distribution functions extracted from the measurement of the Drell-Yan cross-section $d\sigma/dQ^2 dy$. In practical terms this means that we must use the same densities as given in Table 1, but multiply by a factor of 2.4²⁸⁾.

The comparison between the lowest order and the second order cross-section is shown in Fig. 12. It is important to note that the correction is small enough to induce confidence in the validity of the perturbative method but that it still gives a significant effect.

In Fig. 13 we plot the ratio

$$K(q_T) = \frac{\frac{1}{q_T} \frac{d\sigma}{dQ dq_T} \Big|_{O(\alpha_s^2)}}{\frac{1}{q_T} \frac{d\sigma}{dQ dq_T} \Big|_{O(\alpha_s)}} \quad (5.15)$$

for $(\pi^-\pi^+)N$ as a function of q_T . The ratio is by no means constant; there is a fall-off with decreasing q_T . This fact is theoretically important since it tends to argue against a simple resummation of the type suggested in Eq. (5.13). However, given the limited transverse momentum resolution of present experiments it will be hard to observe since it is the ratio of two distributions which fall rapidly with q_T .

Our formulae can be used to make numerical predictions for the non-singlet cross-section $\bar{p} - p$. As before we expect that the contribution of sea quarks and gluons will be negligible at higher values of τ . Our formulae will therefore be a reasonable approximation for the case of incident antiprotons alone. In Fig. 14 the antiproton non-singlet cross-section is shown at $\sqrt{S} = 16.4$ GeV. Similar curves at $\sqrt{S} = 63$ GeV are shown in Fig. 15.

CONCLUSIONS

We have presented formulae for the production of lepton pairs at large transverse momentum. On the phenomenological side our results can be summarized by saying that the production of lepton pairs is enhanced above the $O(\alpha_s)$ prediction and that the enhancement factor falls with increasing q_T . In terms of parton densities defined in the Drell-Yan process itself the correction is within the perturbative range and shows the same behaviour as a function of q_T . On the theoretical side our formulae provide a stringent check which any resummation of leading terms has to reproduce.

APPENDIX A - $q + \bar{q} \rightarrow \gamma^* + G$

In this series of appendices we display the explicit analytic results for the graphs which we have calculated. The first process which we consider is the lowest order process (Fig. 2). The cross-section for this process ($|\Sigma_1 L_1|^2$) is given by

$$\frac{s d\sigma^{(1)}}{dt} = e_f^2 K \frac{\alpha_s}{s} T_0(Q^2, u, t) \quad (A.1)$$

where the matrix element squared calculated in $n = 4 - 2\epsilon$ dimensions is

$$T_0(Q^2, u, t) = \left[(1-\epsilon) \left(\frac{u}{t} + \frac{t}{u} \right) + \frac{2Q^2(Q^2 - u - t)}{ut} - 2\epsilon \right] \quad (A.2)$$

and constant factors are defined as

$$K = \frac{2\pi\alpha C_F}{N_c} \frac{(1-\epsilon)}{\Gamma(1-\epsilon)} \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \left(\frac{sQ^2}{ut} \right)^\epsilon, \quad C_F = \frac{4}{3}, \quad N_c = 3. \quad (A.3)$$

The radiative corrections to this process are given by the diagrams of Fig. 3.

The result of the calculation of the diagrams

$$\left| \sum_{i=1}^2 L_i + \sum_{i=1}^n V_i \right|^2$$

is, (dropping terms of higher orders in α_s and ϵ),

$$\begin{aligned} \frac{s d\sigma^{\text{VIRTUAL}}}{dt} = e_f^2 K \frac{\alpha_s(\mu^2)}{s} \left\{ T_0(Q^2, u, t) \left[1 + \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu^2}{Q^2} \right)^\epsilon \right. \right. \\ \left. \left. \left(-\frac{(2C_F + N_c)}{\epsilon^2} - \frac{1}{\epsilon} \left(3C_F - 2C_F \ln \frac{s}{Q^2} + \frac{11}{6} N_c + N_c \ln \frac{sQ^2}{ut} - \frac{2}{3} T_R \right) \right) \right. \right. \\ \left. \left. + \frac{\alpha_s}{2\pi} T_0(Q^2, u, t) \left[\frac{2\pi^2}{3} C_F - \frac{N_c \pi^2}{6} - 8C_F - C_F \ln^2 \frac{s}{Q^2} + \frac{N_c}{2} \left(\ln^2 \frac{s}{Q^2} - \ln^2 \frac{ut}{Q^4} \right) \right. \right. \right. \\ \left. \left. \left. + \left(\frac{11}{6} N_c - \frac{2}{3} T_R \right) \ln \frac{\mu^2}{Q^2} \right] \right. \right. \\ \left. \left. + \frac{\alpha_s}{2\pi} F(s, t, u) \right\} \quad (A.4) \end{aligned}$$

where the function F is given by,

$$\begin{aligned}
 F(s, t, u) = & \left\{ C_F \left(\frac{s}{s+u} + \frac{s}{s+t} + \frac{s+t}{u} + \frac{s+u}{t} \right) \right. \\
 & + \ln \left| \frac{t}{Q^2} \right| \left(C_F \frac{(4s^2 + 2st + 4su + tu)}{(s+u)^2} + N_C \frac{t}{(s+u)} \right) \\
 & + \ln \left| \frac{u}{Q^2} \right| \left(C_F \frac{(4s^2 + 2su + 4st + tu)}{(s+t)^2} + N_C \frac{u}{(s+t)} \right) \\
 & + (2C_F - N_C) \left(2 \ln \left(\frac{s}{Q^2} \right) \left(\frac{s^2}{(t+u)^2} + \frac{2s}{(t+u)} \right) - \frac{Q^2(t^2 + u^2)}{tu(t+u)} \right) \\
 & - (2C_F - N_C) \left(\frac{s^2 + (s+u)^2}{ut} R_1(s, t) + \frac{s^2 + (s+t)^2}{ut} R_1(s, u) \right) \\
 & \left. + N_C T_0(Q^2, u, t) R_2(t, u) \right\}
 \end{aligned} \tag{A.5}$$

and the functions $R_{1,2}$ are defined as,

$$\begin{aligned}
 R_1(s, t) &= \left[\ln \left(\frac{s}{Q^2} \right) \ln \left(\frac{t}{Q^2 - s} \right) + \frac{1}{2} \ln^2 \frac{Q^2}{s} - \frac{1}{2} \ln^2 \left(\frac{Q^2 - t}{Q^2} \right) + \text{Li}_2 \left(\frac{Q^2}{s} \right) - \text{Li}_2 \left(\frac{Q^2}{Q^2 - t} \right) \right] \\
 R_2(t, u) &= \left[\frac{1}{2} \ln^2 \left(\frac{Q^2 - t}{Q^2} \right) + \frac{1}{2} \ln^2 \left(\frac{Q^2 - u}{Q^2} \right) + \text{Li}_2 \left(\frac{Q^2}{Q^2 - t} \right) + \text{Li}_2 \left(\frac{Q^2}{Q^2 - u} \right) \right]
 \end{aligned} \tag{A.7}$$

and

$$\text{Li}_2(x) = - \int_0^x \frac{dz}{z} \ln(1-z) \tag{A.8}$$

The above result can be obtained by crossing from the results of Ref. 13) for the process $\gamma^* \rightarrow q + \bar{q} + G$. Ultra-violet renormalization has been performed in Eq. (A.4) by addition of a counterterm appropriate for the $\overline{\text{MS}}$ subtraction scheme²⁵⁾,

$$\frac{d\sigma^{\text{CT}}}{dt} = e_f^2 K \frac{\alpha_s}{s} T_0(Q^2, u, t) \frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} (4\pi)^\epsilon \left(\frac{2}{3} T_R - \frac{11}{6} N_C \right) \frac{1}{\epsilon} \tag{A.9}$$

and the coupling constant has been replaced by the running coupling constant

$$\alpha_s \rightarrow \alpha_s(\mu^2) = \alpha_s \left(1 - \frac{\alpha_s}{2\pi} \left(\frac{11}{6} N_C - \frac{2}{3} T_R \right) \ln \frac{\mu^2}{\mu^2} + \dots \right) \tag{A.10}$$

APPENDIX B - $q + \bar{q} \rightarrow \gamma^* + G + G$

This appendix mainly gives the results of real gluon corrections to the process $q + \bar{q} \rightarrow \gamma^* + G + G$. However, as a question of book-keeping convenience it is helpful to include certain graphs for the process $q + \bar{q} \rightarrow \gamma^* + q + \bar{q}$. The results in this appendix refer to the series of graphs $|\sum_{i=1}^8 G_i|^2 + |F_1 + F_2|^2$. The kinematic variables for this inclusive process containing three particles in the final state are as follows

$$p_1 + p_2 \rightarrow q + k_1 + k_2 \quad (B.1)$$

where

$$s = (p_1 + p_2)^2, \quad t = (p_1 - q)^2, \quad u = (p_2 - q)^2 \quad (B.2)$$

and the variable,

$$s_2 = (p_1 + p_2 - q)^2 \quad (B.3)$$

is not independent being related by

$$s + t + u = Q^2 + s_2 \quad (B.4)$$

The invariant phase space for three particles in the final state is most easily written down in the centre-of-mass frame of the recoiling gluon pair

$$k_1 = \frac{\sqrt{s_2}}{2} (1, \dots, \cos\theta_2 \sin\theta_1, \cos\theta_1) \quad (B.5)$$

$$k_2 = \frac{\sqrt{s_2}}{2} (1, \dots, -\cos\theta_2 \sin\theta_1, -\cos\theta_1) \quad (B.6)$$

where the dots stand for (n-3) equal and opposite angular components. The Lorentz invariant phase space is given by

$$(PS)^{(3)} = S_f \int \frac{d^n k_1}{(2\pi)^{n-1}} \frac{d^n k_2}{(2\pi)^{n-1}} \frac{d^n q}{(2\pi)^{n-1}} \delta^+(k_1^2) \delta^+(k_2^2) \delta^+(q^2 - Q^2) (2\pi)^n \delta^n(p_1 + p_2 - q - k_1 - k_2)$$

where S_f is the statistical factor. After integration over irrelevant angles in Eqs. (B.5), (B.6) the phase space becomes

$$(PS)^{(3)} = \frac{S_f}{2^8 \pi^4 s} \frac{(4\pi)^{2\epsilon}}{\Gamma(1-2\epsilon)} \int dt du \left(\frac{s}{s_2(ut - s_2 Q^2)} \right)^\epsilon \int d\theta_2 \sin^{-2\epsilon} \theta_2 \int d\theta_1 \sin^{1-2\epsilon} \theta_1 \quad (B.8)$$

In the region of emission of soft or collinear gluons the cross-section contains divergences which are factored as described in the text. The result of the calculation of the diagrams $|\sum_{i=1}^8 G_i|^2 + |F_1 + F_2|^2$ may be written

$$\begin{aligned} \frac{s d\sigma_I}{dt du} &= \frac{\alpha_s}{2\pi} \int dx_1 R(x_1, M^2) \frac{s d\sigma^{(4)}}{dt} \Big|_{p_1 \rightarrow x_1 p_1} \delta(x_1(s+t-Q^2)+u) \\ &+ \frac{\alpha_s}{2\pi} \int dx_2 R(x_2, M^2) \frac{s d\sigma^{(4)}}{dt} \Big|_{p_2 \rightarrow x_2 p_2} \delta(x_2(s+u-Q^2)+t) \\ &+ \frac{s d\tilde{\sigma}_I}{dt du} \end{aligned} \quad (B.9)$$

where

$$R(x, M^2) = -\frac{1}{\epsilon} C_F P_{q\bar{q}}(x) \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu^2}{M^2}\right)^\epsilon$$

We find that the result for the $\overline{MS}$ mass singularity factored cross-section is

$$\begin{aligned} \frac{s d\tilde{\sigma}_I}{dt du} &= e_f^2 \frac{K}{s} \alpha_s \left\{ T_0(Q^2, u, t) \delta(s_2) \left[\frac{\alpha_s}{2\pi} \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu^2}{Q^2}\right)^\epsilon \right. \right. \\ &\left. \left[\frac{(2C_F + N_c)}{\epsilon^2} + \frac{1}{\epsilon} \left(3C_F + 2C_F \ln \frac{Q^2}{s} + N_c \ln \frac{Q^2 s}{ut} + \frac{11N_c}{6} - \frac{2}{3} T_R \right) \right] \right. \\ &+ \frac{\alpha_s}{2\pi} \left[\left(\frac{11}{6} N_c - \frac{2}{3} T_R \right) \ln \frac{Q^2}{A} + \left(\frac{67}{18} N_c - \frac{10}{9} T_R \right) + N_c \ln^2 \frac{Q^2}{A} + 2C_F \ln \left(\frac{ut}{A^2} \right) \ln \left(\frac{M^2}{Q^2} \right) \right. \\ &\left. \left. + \left(C_F - \frac{N_c}{2} \right) \left(\frac{\pi^2}{3} + \ln^2 \left(\frac{A^2 s}{Q^2 ut} \right) \right) - 3C_F \ln \left(\frac{M^2}{Q^2} \right) \right] \right] \\ &+ \frac{\alpha_s}{2\pi} T_0(Q^2, u, t) \left[\frac{\frac{2}{3} T_R - \frac{11}{6} N_c}{(s_2)_{A+}} + \frac{2C_F}{(s_2)_{A+}} \ln \left(\frac{(ut - Q^2 s_2)}{(u-s_2)(t-s_2)} \right) + 4C_F \left(\frac{\ln \frac{s_2}{A^2}}{s_2} \right)_{A+} \right. \\ &\left. \left. + \left(C_F - \frac{N_c}{2} \right) \left(4 \left(\frac{\ln \left(\frac{s_2}{A} \right)}{s_2} \right)_{A+} + \frac{2}{(s_2)_{A+}} \ln \left(\frac{s^2 A^2}{(u-s_2)(t-s_2)(ut - Q^2 s_2)} \right) \right) \right] + \right. \end{aligned} \quad (B.10)$$

$$\begin{aligned}
& + e_f^2 \frac{k}{s} \frac{\alpha_s^2}{2\pi} \left\{ C_F \left[\frac{s}{(t-s_2)^2} - \frac{2}{(t-s_2)} - \frac{s}{ut} + \frac{2Q^2 u + t s_2 - 2Q^2 s_2}{t(ut - Q^2 s_2)} + \frac{2Q^2(t-u)}{tu(t-s_2)} \right] \right. \\
& \quad + N_c \left[-\frac{11}{6} \frac{(s+Q^2)}{ut} + \frac{s^2(3s_2-4t)}{2tu(t-s_2)^2} - \frac{2s}{u(t-s_2)} + \frac{Q^2}{3t^2} \right] \\
& \quad + T_R \left[\frac{2}{3} \frac{(s+Q^2)}{ut} - \frac{2}{3} \frac{Q^2}{t^2} \right] \\
& \quad + 2 \left(C_F - \frac{N_c}{2} \right) \ln \left(\frac{s s_2}{ut - Q^2 s_2} \right) \left[\frac{(Q^2 - u)^2}{u(u-s_2)(t-s_2)} + \frac{(Q^2 - u)^2}{ut(t-s_2)} \right] \\
& \quad + 2 \left(C_F - \frac{N_c}{2} \right) \ln \left(\frac{s s_2}{(u-s_2)(t-s_2)} \right) \left[\frac{Q^2 + s}{ut} \right] \\
& \quad + C_F \ln \left(\frac{(ut - Q^2 s_2)}{(u-s_2)(t-s_2)} \right) \left[\frac{4Q^2(Q^2 - t)^2}{ut(ut - Q^2 s_2)} + \frac{2(Q^2 + s) - s_2}{(ut - Q^2 s_2)} + \frac{(s_2 - 2s)}{ut} \right] \\
& \quad + C_F \ln \left(\frac{u^2}{s_2} \right) \left[\frac{1}{ut(ut - Q^2 s_2)} \left[4ut(u - Q^2) - 4Q^2(u - Q^2)^2 - ut s_2 \right] \right. \\
& \quad \quad \left. + \frac{(Q^2 - u)}{(t-s_2)^2} + \frac{(2Q^2 - u)}{t(t-s_2)} - \frac{2}{t} + \frac{Q^2}{t^2} \right] \Bigg\} \\
& + e_f^2 \frac{k}{s} \frac{\alpha_s^2}{2\pi} \left\{ (u \leftrightarrow t) \right\}
\end{aligned}$$

(B.10)
cont.

The only tricky features of the above formulae are distributions in the variable s_2 which we have defined in order to make the poles manifest. The variable s_2 is the invariant mass of the two gluons (quark-antiquark pair) against which the massive photon recoils. As s_2 tends to zero we approach the soft emission limit. The singularities which cancel against the corresponding singularities of the virtual emission graphs occur in this region. The distributions, defined in the text, are in a suitable form to be included in the subsequent convolution integrals which we perform to calculate the hadronic cross-section.

APPENDIX C - $q + \bar{q} \rightarrow \gamma^* + q + \bar{q}$

The diagrams of Fig. 5 will be treated in this section. The diagrams of the top row contribute only if the incoming quark and antiquark are identical. The contributions of the various interferences are given in Table 2. It is not necessary to calculate to contributions of the diagrams in the last row of the table since when we form the non-singlet cross-section they cancel with contributions coming from the H diagrams when the statistical factor is taken into account.

The result for the diagrams $|F_3 + F_4|^2$ is as follows

$$\begin{aligned} \frac{s d\sigma_{II}}{dt du} = & \frac{K}{s} \sum_f e_f^2 \frac{\alpha_s^2}{2\pi} C_F \left\{ \frac{1}{2(s+Q^2-s_2)} \left(\frac{u}{s} - 1 \right) - \frac{5}{4s} \right. \\ & + \frac{1}{\lambda^2} \left[\frac{u}{(s+Q^2-s_2)} \left[-2s + \frac{3}{2} \frac{u}{s} (t-u) + 4u - 2t \right] + \frac{u}{2s} [2s + 2s_2 + t - u] \right] \\ & + \frac{3u^2(u-t)}{\lambda^4} \left[\frac{(2s-u-t)}{(s+Q^2-s_2)} - \frac{(s+s_2)}{s} \right] \\ & + \frac{1}{\lambda} \ln \frac{(s+Q^2-s_2+\lambda)}{(s+Q^2-s_2-\lambda)} \left[\frac{1}{(s+Q^2-s_2)} \left(\frac{2u^2}{s} + \frac{5}{2}u + \frac{3}{2}s \right) + \frac{1}{s} \left(\frac{3}{4}s + u - \frac{1}{2}s_2 \right) \right]^{(C.1)} \\ & + \frac{1}{\lambda^3} \ln \frac{(s+Q^2-s_2+\lambda)}{(s+Q^2-s_2-\lambda)} \left[\frac{u^2}{(s+Q^2-s_2)} \left(3u - t - \frac{u^2}{s} + \frac{t^2}{s} - 2s \right) \right. \\ & \quad \left. + \frac{u}{s} \left(2s_2 t - ut - 2s_2^2 + 4us_2 - 3u^2 + 2ss_2 - us \right) \right] \\ & + \frac{1}{\lambda^5} \ln \frac{(s+Q^2-s_2+\lambda)}{(s+Q^2-s_2-\lambda)} \left[\frac{3(u^2-t^2)u^2}{(s+Q^2-s_2)} (s-Q^2) + 3 \frac{u^2 Q^2}{s} (u-t)(u+t-2s_2) \right] \Big\} \\ & + \left\{ (u \leftrightarrow t) \right\} \end{aligned}$$

and λ is defined as

$$\lambda = [(u+t)^2 - 4Q^2 s_2]^{\frac{1}{2}} \quad (C.2)$$

The result for the set of diagrams indicated by

$$2 \left(\sum_{i=1}^4 F_i \right)^* \left(\sum_{i=5}^8 F_i \right)$$

is given by

$$\begin{aligned} \frac{sd\sigma_{III}}{dt du} = & e_f^2 \frac{K}{s} \frac{\alpha_s^2}{2\pi} \left(C_F - \frac{N_c}{2} \right) \\ & \left\{ \left[\frac{3us - 6Q^2s - 8s_2Q^2}{2su^2} + \frac{(t-u-s)}{2s_2u} + \frac{s^2}{t(u-s_2)^2} + \frac{1}{2su} (5t-3u) - \frac{(3s^2+2tu)}{tu(u-s_2)} \right] \right. \\ & + \frac{1}{\lambda^2} \left[\frac{2ts_2}{su} (t-u-s_2) + \frac{2st^2}{us_2} + \frac{2s_2^2}{s} + t \frac{(2t-3s-s_2)}{u} + s-s_2 \right. \\ & \left. \left. + \frac{t(s+s_2)(u^2-t^2)}{2uss_2} \left(1 + \frac{3s}{(s+Q^2-s_2)} \right) \right] \right. \\ & + \frac{1}{\lambda^4} \left[\frac{3}{2} \frac{st^2(s+s_2)}{us_2} (t^2+ut-2u^2) \right] \left[\frac{1}{\lambda} \ln \frac{(s+Q^2-s_2+\lambda)}{(s+Q^2-s_2-\lambda)} - \frac{2}{(s+Q^2-s_2)} \right] \\ & + \frac{1}{\lambda} \ln \frac{(s+Q^2-s_2+\lambda)}{(s+Q^2-s_2-\lambda)} \left[\frac{t(t+s) + \frac{1}{2}s^2}{us_2} + \frac{4s_2^2 - 2s_2t + t^2}{us} + \frac{5u - 6s_2}{s} + \frac{1}{s_2}(u+3s) \right. \\ & + \frac{s-2Q^2}{2u} + \frac{4s+2s_2}{(s+Q^2-s_2)} + \frac{t}{\lambda^2} \left(t-Q^2 - \frac{su}{s_2} + \frac{t}{u} (t-s_2-s) + \frac{ts}{us_2} (t-2s) \right) \left. \right] \\ & + 2 \ln \left[\frac{su}{(s+u-s_2)(u-s_2) + (ut-Q^2s_2)} \right] \frac{s_2^2 + (u-s_2)^2}{su(s+Q^2-s_2)} \\ & + \ln \left[\frac{su^2}{Q^2(u-s_2)^2} \right] \left(\frac{u+t+2s}{us} + \frac{(t^2+2st+2s^2)}{tus_2} - \frac{2Q^2}{u^2} \right) \left. \right\} \\ & + e_f^2 \frac{K}{s} \frac{\alpha_s^2}{2\pi} \left(C_F - \frac{N_c}{2} \right) \left\{ (u \leftrightarrow t) \right\} \end{aligned} \quad (C.3)$$

and λ has the same meaning as before [Eq. (C.2)].

APPENDIX D - $q + q \rightarrow \gamma^* + q + q$

The final set of diagrams which we have to calculate are the interferences

$$2 \sum_{i=1}^4 H_i^* \sum_{j=5}^8 H_j$$

The result is

$$\begin{aligned} \frac{s \, d\sigma_{\text{IV}}}{dt \, du} = & e_f^2 \frac{k}{s} \frac{\alpha_s^2}{2\pi} \left(C_F - \frac{N_c}{2} \right) \left\{ \frac{4 s_2 Q^2 - 2u(t+u)}{s u^2} \right. \\ & + \ln \left[\frac{(u-s_2)^2}{s s_2} \right] \left(\frac{(u+t)^2 - 2s_2(Q^2-s)}{u t s} \right) \\ & + \ln \left[\frac{(u-s_2)(s+Q^2-s_2)-su}{us} \right] \left(\frac{2s_2(Q^2-s)-(u+t)^2}{u t s} - \frac{2Q^2}{u^2} + \frac{2(Q^2-s-s_2)}{u(u-s_2)} \right. \\ & \left. \left. + \frac{4s^2}{u(u-s_2)(s+Q^2-s_2)} \right) \right. \\ & - \ln \left[\frac{Q^4(u-s_2)^2(t-s_2)^2}{u t ((s+Q^2-s_2)(u-s_2)-us)((s+Q^2-s_2)(t-s_2)-ts)} \right] \left(\frac{s}{2ut} \left[1 + \left(1 + \left(\frac{u+t}{s} \right)^2 \right) \right] \right. \\ & + Q^2 \left(\frac{Q^2-2s-2t}{ut s} + \frac{1}{u^2} \right) \left. \right) + \ln \left[\frac{(u-s_2)^2 t ((s+Q^2-s_2)(t-s_2)-ts)}{(t-s_2)^2 u ((s+Q^2-s_2)(u-s_2)-us)} \right] \frac{Q^2}{t^2} \\ & \left. + e_f^2 \frac{k}{s} \frac{\alpha_s^2}{2\pi} \left(C_F - \frac{N_c}{2} \right) \left\{ (u \leftrightarrow t) \right\} \right\} \end{aligned} \quad (D.1)$$

The contribution of these diagrams to lepton pair production has been considered also in Ref. 29).

$V_u^P(x, M_0^2) = u(x, M_0^2) - \bar{u}(x, M_0^2) = \frac{2\Gamma(\rho+3)}{\Gamma(\frac{1}{2}) \Gamma(\rho+1)} (1-x)^\rho x^{-1/2}$
$V_d^P(x, M_0^2) = d(x, M_0^2) - \bar{d}(x, M_0^2) = \frac{\Gamma(\rho+5)}{\Gamma(\frac{1}{2}) \Gamma(\rho+2)} (1-x)^{\rho+1} x^{-1/2}$
$V^{\pi^+}(x, M_0^2) = u(x, M_0^2) - \bar{u}(x, M_0^2) = \bar{d}(x, M_0^2) - d(x, M_0^2) = \frac{0.75}{\sqrt{x}}(1-x)$
$\int_0^1 V_u^P(x) dx = 2, \int_0^1 V_d^P(x) dx = 1, \int_0^1 V^{\pi^+}(x) dx = 1$
$M^2 = Q^2, q_T^2 + 2\langle q_T^2 \rangle_{INT}, \frac{1}{2}(Q^2 + q_T^2), M_0 = 7.5 \text{ GeV}$
$N_C = 3, C_F = \frac{4}{3}, T_R = \frac{1}{2}n_f = 2$
$\langle q_T^2 \rangle_{INT} = 0.64 \sim 1.0 \text{ GeV}^2$

Table 1 : Definitions used for numerical estimates

Diagrams	Value for identical incoming quarks	Value for non-identical incoming quarks
$ F_1 + F_2 ^2$	Contribution to $d\tilde{\sigma}_I$ Eq. (B.10)	0
$2\text{Re}(F_1 + F_2)^*(F_3 + F_4)$	0 (Charge conjugation)	0
$ F_3 + F_4 ^2$	$d\sigma_{II}$ Eq. (C.1)	0
$2\text{Re}(\Sigma_{i=1}^4 F_i^*)(\Sigma_{j=5}^8 F_j)$	$d\sigma_{III}$ Eq. (C.3)	0
$ \Sigma_{i=5}^8 F_i ^2$	$ \Sigma_{i=1}^4 H_i ^2 + \Sigma_{i=5}^8 H_i ^2$	$ \Sigma_{i=1}^4 H_i ^2$

Table 2

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FIGURE CAPTIONS

- Figure 1 : The process hadron 1 + hadron 2 $\rightarrow \mu^+ \mu^- + X$ in the context of the QCD parton model.
- Figure 2 : Lowest order diagrams which contribute to the process $q + \bar{q} \rightarrow \gamma^* + G$. The gluon is denoted by a curly line and the photon by a wavy line.
- Figure 3 : Diagrams which contribute to the process $q + \bar{q} \rightarrow \gamma^* + G$ in next-to-leading order. The order α_s^2 contribution of these diagrams is given by their interference with the diagrams of Fig. 2.
- Figure 4 : Diagrams which contribute to the process $q + \bar{q} \rightarrow \gamma^* + G + G$.
- Figure 5 : Diagrams which contribute to the process $q + \bar{q} \rightarrow \gamma^* + q + \bar{q}$. For the case of non-identical quarks only the bottom row of diagrams contributes whereas for identical quarks both rows of diagrams are necessary.
- Figure 6 : Diagrams which contribute to the process $q + q \rightarrow \gamma^* + q + q$. For the case of distinguishable quarks the bottom row of diagrams is absent.
- Figure 7 : a) $1/q_T d\sigma^{\pi^+N}/dQdq_T$ vs. q_T for $S = 282 \text{ GeV}^2$, $Q^2 = 52.5 \text{ GeV}^2$ and $M^2 = Q^2$; the quark densities are defined in Table 1. The gluon-sea contribution which we estimate to be $\leq 5\%$ has been neglected. The intrinsic transverse momentum is set equal to $\langle q_T^2 \rangle_{\text{INT}} = 1 \text{ GeV}^2$. The lower dashed region corresponds to the $O(\alpha_s)$ formula and the upper dashed region to the formula including $O(\alpha_s^2)$. The bands of values correspond to variations of the exponent ρ in the parametrization of the distribution functions (cf. Table 1) $\rho = 2.95 \pm 0.15$. The upper line corresponds to the exponentiated formula Eq. (5.13) with a median value for ρ . The solid triangles are experimental points²³⁾.
b) The solid curves reproduce the curves given in Fig. 7a for the smallest value of the parameter ρ , with all other parameters as above. The dashed curve is the lowest order result regularized using Eq. (5.1) with the total cross-section taken to be the experimentally measured result.

Figure 8 : $1/q_T d\sigma^{\pi^-N}/dQdq_T$ vs q_T . All parameters are chosen as in Fig. 7 except that $M^2 = (q_T^2 + 2\langle q_T^2 \rangle_{INT})$. Only the full results with $O(\alpha_S^2)$ are shown. The full curve corresponds to $\langle q_T^2 \rangle_{INT} = 0.64 \text{ GeV}^2$ and the dashed curve to $\langle q_T^2 \rangle_{INT} = 1.0 \text{ GeV}^2$. Experimental points as in Fig. 7.

Figure 9 : Contributions of the various pieces appearing in the formulae of Appendices A and B to the second order corrections. Shown plotted is the quantity

$$K(q_T^2, y=0) = d\sigma^{\pi^-N}/dQ^2 dq_T^2 dy \left| \begin{array}{c} O(\alpha_S^2) \\ \hline \end{array} \right| d\sigma^{\pi^-N}/dQ^2 dq_T^2 dy \left| \begin{array}{c} O(\alpha_S) \end{array} \right|$$

$S = 282 \text{ GeV}^2$, $Q^2 = 52.5 \text{ GeV}^2$. M^2 is chosen to be $1/2(Q^2 + q_T^2)$. The various contributions are

- δ_1) Lowest order contribution $\propto (\alpha_S/2\pi C_F \pi^2)$
- δ_2) All other terms proportional to the lowest order matrix element.
- σ) Terms containing the distributions $(s_2)_{A+}$ and $[\ln(s_2/A)/(s_2)]_{A+}$.
- γ) Terms coming from the particular definition of parton densities Eq. (2.12).
- λ) All other terms.

The dashed line is the sum of all terms except δ_1 . Using a different choice for M^2 (e.g., $M^2 = q_T^2 + 2\langle q_T^2 \rangle_{INT}$) all the curves become steeper in q_T .

Figure 10 : $1/q_T d\sigma^{(\pi^-\pi^+)N}/dQ dq_T$ vs q_T for $S = 376 \text{ GeV}^2$ at various values of Q^2 . $\langle q_T^2 \rangle_{INT} = 1 \text{ GeV}^2$ and $M^2 = 1/2(Q^2 + q_T^2)$. Only the full $O(\alpha_S^2)$ curves are shown.

Figure 11 : $\langle q_T \rangle$ and $\langle q_T^2 \rangle$ vs $\sqrt{\tau}$ for $(\pi^-\pi^+)N$ where $\sqrt{S} = 19.4$

$$\langle q_T^2 \rangle = \int dq_T^2 q_T^2 \frac{d\sigma^{(\pi^-\pi^+)N}}{dQ^2 dq_T^2} / \frac{d\sigma^{(\pi^-\pi^+)N}}{dQ^2}$$

and similarly for $\langle |q_T| \rangle$. $\langle q_T^2 \rangle_{\text{INT}} = 1 \text{ GeV}^2$ and $\alpha_s = \alpha_s \left[(Q^2 + q_T^2)/2 \right]$. The upper lines are the $O(\alpha_s^2)$ results; the lower lines are the $O(\alpha_s)$ results.

- Figure 12 : $1/q_T d\sigma^{\pi^-N}/dQ dq_T$ vs q_T ; $\langle q_T^2 \rangle = 1 \text{ GeV}^2$, $M^2 = Q^2$, $Q^2 = 52.6 \text{ GeV}^2$, $S = 282 \text{ GeV}^2$. The curves are plotted using the Drell-Yan parton densities defined in the text. The full line shows the $O(\alpha_s)$ result; the dashed-dotted line is the full result containing $O(\alpha_s^2)$ terms.
- Figure 13 : The function $K(q_T)$ (defined in the text) vs. q_T . The upper curves (solid and dashed) are obtained by including an intrinsic transverse momentum $\langle q_T^2 \rangle_{\text{INT}} = 1 \text{ GeV}^2$. The lower curves (dashed-dotted and dashed-crossed) are the pure QCD results. The scale of the running coupling constant M^2 is set equal to $(Q^2 + q_T^2)/2$ for all curves. Notice that using the Drell-Yan parton densities the correction is within the perturbative range and becomes negative at large values of q_T . $\sqrt{S} = 19.4 \text{ GeV}$, $Q^2 = 30.25 \text{ GeV}^2$.
- Figure 14 : $1/q_T d\sigma^{(\bar{p}-p)N}/dQ dq_T$ vs. q_T for $\sqrt{S} = 16.8 \text{ GeV}$ and $Q = 5.3 \text{ GeV}$ and 7.1 GeV . The scale of the running coupling constant was taken to be $M^2 = (Q^2 + q_T^2)/2$ and $\langle q_T^2 \rangle_{\text{INT}} = 1 \text{ GeV}^2$. Only the full $O(\alpha_s^2)$ curves are shown.
- Figure 15 : $1/q_T d\sigma^{(\bar{p}-p)P}/dQ dq_T$ vs. q_T for $\sqrt{S} = 63 \text{ GeV}$ and $Q = 5 \text{ GeV}$ and 12 GeV . The scale of the running coupling constant and of the parton densities was taken as $M^2 = Q^2$. The parton densities were evolved to the values $Q = 5 \text{ GeV}$ and $Q = 12 \text{ GeV}$ by performing anti-Mellin transform numerically⁵⁾. The initial conditions were,

$$V_u^P(x, M_0 = 7.5 \text{ GeV}) = \frac{2.27}{\sqrt{x}} (1-x)^3$$

$$V_d^P(x, M_0 = 7.5 \text{ GeV}) = \frac{1.29}{\sqrt{x}} (1-x)^4$$

FIG. 1

FIG. 2

FIG. 4

FIG. 5

FIG. 6

FIG. 7a

FIG. 7 b

FIG. 8

FIG. 9

FIG. 10

FIG. 11 a

FIG. 11 b

FIG. 12

FIG. 13

FIG. 14

FIG. 15