Ref.TH.3188-CERN

THE $pd \rightarrow t\pi^+$ REACTION IN AN ISOBAR MODEL -
ANGULAR DISTRIBUTIONS

M.E. Sainio

CERN -- Geneva

and

Research Institute for Theoretical Physics,
University of Helsinki, Helsinki, Finland.

ABSTRACT

Various wave function effects are studied in the differential cross-section for the $pd \rightarrow t\pi^+$ reaction. These include both S and D waves for the deuteron and the triton as well as the production of p wave and d wave pions. For the triton a parametrized Faddeev wave function is employed and for the deuteron a wave function of Hulthen or McGee type. The two-body correlations considered are $NA(^5S_2, ^5D_2, ^5P_3)$ and $NN(^1D_2)$ which are found to be the most important partial waves in the $pp \rightarrow d\pi^+$ reaction.

1. - INTRODUCTION

The coupled channels isobar model of Green and Niskanen^{1),2)} has recently been extended to a microscopic description of the $pd \rightarrow t\pi^+$ reaction^{3),4)}. Previously, for the $pp \rightarrow d\pi^+$ reaction the model had shown itself to be rather successful in explaining not only the total cross-section but, to an even greater extent, also the various differential cross-section and polarization parameters⁵⁾. However, the attempts of Refs. 3) and 4) to reach a similar general agreement with experiment, as is the case for the $pp \rightarrow d\pi^+$ reaction, have turned out to be less successful quantitatively. It is hard to localize the reasons for this apparent failure of the calculations in Refs. 3) and 4). Are they mainly due to effects not included in the model through the use of poor nuclear wave functions for the initial state deuteron and final state triton, or is it not really a fair comparison to look at the differential cross-section directly? It should be remembered that in $pp \rightarrow d\pi^+$ the main experiment-theory confrontation has been with the parameters (γ) describing the cross-section as a series $\sum_{n=0} \gamma_{2n} \cos^{2n} \theta$. Therefore the differential cross-section fit could well be much worse than that for any of the parameters in a similar expansion for the $pd \rightarrow t\pi^+$ reaction. In the present paper the main aim is to study the wave function effects and to finally try to reach a better over-all agreement with the experimental information.

In contrast with the above microscopic model, another approach, which has often turned out to be rather successful in explaining the various pion production reactions more phenomenologically, is to relate a many-body cross-section to a two-body one. As an example, when the $A(p,\pi)A+1$ cross-section is expressed in terms of the $pp \rightarrow d\pi^+$ cross-section, this is the so-called quasideuteron model of Ruderman⁶⁾ who employed this technique for the $pd \rightarrow t\pi^+$ reaction. Within this model the basic assumption is obviously that the reaction mechanism for the $A(p,\pi)A+1$ reaction is the same as for the $pp \rightarrow d\pi^+$ reaction. Thus the cross-section can be expressed as a product of a kinematical factor, a form factor and the $pp \rightarrow d\pi^+$ cross-section

$$\frac{d\sigma}{d\Omega}(pA \rightarrow (A+1)\pi^+) = G |F|^2 \frac{d\sigma}{d\Omega}(pp \rightarrow d\pi^+).$$

This approach has been employed and developed by several authors such as Ingram et al.⁷⁾, Barry⁸⁾, Fearing⁹⁾⁻¹¹⁾, Locher and Weber¹²⁾ and Duck et al.¹³⁾. Wilkin¹⁴⁾ has chosen a somewhat different way of thinking by relating the $A(p,\pi^+)A+1$ reaction cross-section to that of the $A(d,p)A+1$ reaction. However, there the main aim has been to study deuteron stripping reactions. These and

other models of pion production have been discussed extensively in the recent reviews of Fearing¹⁵⁾ and Measday and Miller¹⁶⁾. Even though no attempt is made in these models to give a truly microscopic description of the basic cross-section ($pp \rightarrow d\pi^+$), the states chosen are dictated by the knowledge that the Δ is the dominant underlying production mechanism.

In Section 2 the basic model for the $pd \rightarrow t\pi^+$ reaction is presented. Section 3 contains a brief discussion of the coupled channels two-body model employed. In Section 4 realistic wave functions for both the deuteron and triton, including S and D waves, are taken into account. In Section 5 the numerical results are given and discussed.

2. - THE $pd \rightarrow t\pi^+$ MODEL

The dominant pion production mechanism in the intermediate energy region is the one through the $\Delta(1232)$ isobar. This is manifested by, for example, the typical resonance structure of the $pp \rightarrow d\pi^+$ total cross-section. Therefore, in the present model for the energies considered, the Δ , as well as the nucleonic pion production, must be included.

In Fig. 1 the basic reaction is shown and the three nucleons have been numbered. The centre-of-mass momenta for the initial state proton and final state pion are denoted by $\vec{k}_0$ and $\vec{q}$ respectively. Microscopically, the circle in Fig. 1 can be divided into the contributions in Fig. 2. The bubbles on the right-hand side of Fig. 2 denote the full solution of coupled Schrödinger equations with explicit Δ configurations generated by a $\pi + \rho$ exchange transition potential.

The differential cross-section for the $pd \rightarrow t\pi^+$ reaction can generally be written as

$$\frac{d\sigma}{d\Omega_\pi} = (2\pi)^4 \frac{E_p E_d E_t \omega}{s} \frac{q}{k_0} \frac{1}{6} \sum_{\text{spins}} |T_{if}|^2 \quad (2.1)$$

where E_p , E_d , E_t and ω are the proton, deuteron, triton and pion energies respectively, and s is the square of the total centre-of-mass energy in the initial pd system. Making use of the antisymmetry of the initial and final state nucleonic wave functions the transition matrix element can be written as³⁾

$$T_{if} = 2\sqrt{3} \langle \Psi_f(\vec{r}_0, \vec{r}_1, \vec{r}_2; \vec{r}_\pi) | H_I | \Psi_i(p(\vec{r}_0); d(\vec{r}_1, \vec{r}_2)) \rangle \quad (2.2)$$

where H_I is the two-body pion production operator acting on the pair 01 forming the circle in Fig. 1. In addition, here the initial state normalization factor has the approximate value $1/\sqrt{3}$.

If it is assumed that the pion is created through Δ decay, the operator H_I takes the form

$$H_I = \sum_{k=0,1} \frac{f^*}{\mu} \bar{S}_k \cdot \bar{q} \tau_k \cdot \phi(\vec{r}_k) \quad (2.3)$$

$$= \sum_{k=0,1} \theta(k)$$

where $\bar{S}$ and τ are the transition spin and isospin operators respectively for spin = isospin = 3/2 particles¹⁷⁾. Here $\phi(\vec{r}_k)$ is the pion field, f^* is the $N\Delta\pi$ coupling constant for which the value $f^{*2}/4\pi = 0.35$ from the Δ decay width is adopted, and μ is the pion mass in units of fm^{-1} . The one-body operator $\theta(k)$ defined by Eq. (2.3) for a transition from a singlet state for the pair 01 to a triplet state for the same pair in the triton is, for the reaction $pd \rightarrow t\pi^+ 2)$,

$$\theta(k) = \frac{q}{\sqrt{2\omega}} \frac{f^*}{\mu} \left\{ \bar{S}_k j_0\left(\frac{1}{2}qr\right) + \sqrt{10} j_2\left(\frac{1}{2}qr\right) [C_2(\hat{r}) \times \bar{S}_k]^{(1)} \right\} \cdot C_1(\hat{q}) T_{-1}(k) \quad (2.4)$$

where the functions C_ℓ are the unnormalized spherical harmonics¹⁸⁾. The replacements $\bar{S}_k \rightarrow \bar{\sigma}_k$, $T_{-1} \rightarrow \tau_{-1}$ and $f^* \rightarrow f$ yield the corresponding operator for pion production from nucleonic states. Equation (2.4) refers to p wave production, whereas d wave pion production is the one arising from the ${}^5P_3(N\Delta)$ state coupled to the ${}^3F_3(NN)$ partial wave for the pair 01. In this case the operator θ takes the form

$$\theta(k) = \frac{q}{\sqrt{2\omega}} \frac{f^*}{\mu} \sqrt{6} j_1\left(\frac{1}{2}qr\right) [\bar{S}_k \times C_1(\hat{r})]^{(2)} \cdot C_2(\hat{q}) T_{-1}(k) \quad (2.5)$$

for a transition to a triplet isosinglet state for the pair 01 in the triton.

For the production operators in Eqs. (2.4) and (2.5) the so-called Galilean corrections have been neglected. In Ref. 19) the argument is given that corrections of relativistic order do not have any a priori reason to be of Galilean-invariant form. On the other hand, the size of these corrections is not expected to be large as shown in Ref. 2) if a comparison with the $pp \rightarrow d\pi^+$ reaction serves as any guide.

It is now assumed that the dominant mechanism for pion production in the $pd \rightarrow t\pi^+$ reaction is

$$\begin{aligned}
 pd &\rightarrow [N_0(N_1 N_2)^{L=0, S=1, J=1; T=0}]_{J=\frac{3}{2}, T=\frac{1}{2}} \\
 &\rightarrow [(N_0 N_1)^{202; 1} N_2]^{\frac{3}{2}, \frac{1}{2}} \\
 &\rightarrow [(\Delta_0 N_1)^{022; 1} N_2]^{\frac{3}{2}, \frac{1}{2}} \\
 &\rightarrow [(N_0 N_1)^{011; 0} N_2]^{\frac{1}{2}, \frac{1}{2}} \pi \rightarrow t\pi^+
 \end{aligned} \tag{2.6}$$

This is the chain of processes coupling to the S wave triton. The ΔN configuration emerges from a transition potential acting within the pair (01).

The scheme coupling to the D wave triton is

$$\begin{aligned}
 &[(N_0 N_1)^{202; 1} N_2]^{\frac{3}{2}, \frac{1}{2}} + [(\Delta_0 N_1)^{222; 1} N_2]^{\frac{3}{2}, \frac{1}{2}} \\
 &\rightarrow [(N_0 N_1)^{211; 0} N_2]^{\frac{1}{2}, \frac{1}{2}} \pi \rightarrow t\pi^+
 \end{aligned} \tag{2.7}$$

Similarly, for the triplet odd case one obtains

$$\begin{aligned}
 pd &\rightarrow [N_0(N_1 N_2)^{011; 0}]^{\frac{5}{2}, \frac{1}{2}} \\
 &\rightarrow [(N_0 N_1)^{313; 1} N_2]^{\frac{5}{2}, \frac{1}{2}} \\
 &\rightarrow [(\Delta_0 N_1)^{123; 1} N_2]^{\frac{5}{2}, \frac{1}{2}} \\
 &\rightarrow [(N_0 N_1)^{011; 0} N_2]^{\frac{1}{2}, \frac{1}{2}} \pi \rightarrow t\pi^+
 \end{aligned} \tag{2.8}$$

which corresponds to d wave pion production. These assumptions are based on the experience of the relative importance of various channels in the $pp \rightarrow d\pi^+$ coupled channel calculations^{1), 2), 20)}, i.e., the dominant states are taken to be the same ones dominating the pp pion production. This is also the underlying assumption in the quasideuteron approach for single pion production.

With the chain of processes in Eq. (2.6) and assuming the full NN interaction only for the pair (01) and neglecting the pion distortions the transition matrix element has the form^{3), 4)}

$$\begin{aligned}
 & \langle \Psi_f | T | \Psi_i \rangle \\
 &= - \frac{6\sqrt{3}}{(2\pi)^4} \langle \frac{1}{2} m \ 1 M | \frac{1}{2} m + M \rangle \\
 & \times \sum_{m'} \langle 2 m' \ \frac{1}{2} m + M | \frac{3}{2} m' + m + M \rangle \\
 & \times \int d\vec{k} f(\vec{k}) Y_{2m'}^*(\hat{K}) \int d\vec{r} \frac{w(Kr)}{Kr} e^{i\delta_2(K)} \mathcal{M} \\
 & \times \int d\vec{s} e^{i\vec{s} \cdot (\vec{k} + \vec{Q})} \psi_t^*(\vec{r}, \vec{s})
 \end{aligned} \tag{2.9}$$

where $f(\vec{k})$ is the momentum space deuteron S state wave function, $w(kr)$ is the radial part of the ${}^5S_2(N\Delta)$ wave function and $\psi_t(\vec{r}, \vec{s})$ is the space part of the triton wave function, m and M are the initial proton and deuteron spin projections respectively, and $\delta_2(K)$ is the 1D_2 NN scattering phase shift at the laboratory energy $E_{lab} = 2 \hbar^2 K^2 / M_N$. The vectors $\vec{r} = \vec{r}_0 - \vec{r}_1$ and $\vec{s} = 1/2(\vec{r}_0 + \vec{r}_1) - \vec{r}_2$ are the Jacobi co-ordinates for the three-body system when nucleon 2 is taken to be the spectator. The momentum associated with $\vec{r}$ is $\vec{K} = 3/4 \vec{k}_0 - 1/2 \vec{k}$ and the effective momentum transfer is $\vec{Q} = 1/2 \vec{k}_0 - 1/3 \vec{q}$. The spin-isospin matrix element $\mathcal{M}$ of the one-body operator $\theta(0)$ has the form

$$\begin{aligned}
 \mathcal{M} = & \frac{q}{\sqrt{2\omega}} \frac{f^*}{\mu} \frac{1}{3} \sqrt{\frac{10}{3}} j_0(\frac{1}{2}qr) \\
 & \times C_{1 m' + m + M - m''}(\hat{q}) (-1)^{\frac{1}{2} - m' - m - M} \begin{pmatrix} \frac{3}{2} & 1 & \frac{1}{2} \\ m' + m + M & m' - m' - m - M & -m'' \end{pmatrix}
 \end{aligned} \tag{2.10}$$

where m'' is the spin projection of the final state triton. Here also the term proportional to $j_2(1/2 qr)$ has been neglected. In the previous work^{3),4)} the triton wave function $\psi(\vec{r}, \vec{s})$ had a simplified Gaussian or correlated Gaussian form. Here $\psi(\vec{r}, \vec{s})$ is the solution of the three-nucleon Faddeev equations²¹⁾ which was parametrized within a factorization approximation in Ref. 22). Further details will be given in Section 4.

3. - THE TWO-BODY INPUT

To calculate the T matrix element in, for example, Eq. (2.9), two-body information is needed in the form of the ${}^5S_2(N\Delta)$ wave function $w(Kr)$.

One of the aims of the present work is to extend the $\Delta(1232)$ isobar model for pion production in pp collisions^{1),2),20)} to more complex reactions. The advantage gained this way would be a consistent microscopic prescription for a variety of reactions.

The basic ingredients of the nucleon-nucleon model applied here are, firstly, the explicit $N\Delta$ components of the wave function obtained by solving Schrödinger equations coupling NN and $N\Delta$ configurations through the transition potentials and, secondly, the inclusion of the Δ width in a phenomenological way by putting a complex mass for the energy of the Δ in these coupled equations. The insertion of a complex Δ mass can be understood in terms of the elimination of the explicit $NN\pi$ channel from the full calculation including all the channels NN , $N\Delta$ and $NN\pi$ ²⁰⁾.

For the particular case of the ${}^1D_2(NN)$ channel which is dominantly coupled to the ${}^5S_2(N\Delta)$ state the Schrödinger equations can be written as

$$\left[-\frac{d^2}{dr^2} + \frac{\epsilon}{r^2} - k^2 + \frac{M_N}{\hbar^2} V_1 \right] u(r) = -\frac{M_N}{\hbar^2} V_2 w(r) \quad (3.1)$$

$$\left[-\frac{d^2}{dr^2} - k^2 + \frac{M_N}{\hbar^2} \left\{ (M_\Delta - i\frac{\Gamma}{2} - M_N) + V_3 \right\} \right] w(r) = -\frac{M_N}{\hbar^2} V_2 u(r)$$

where $k^2 = M_N E_{\text{lab}}/2\hbar^2$. Here V_1 is the Reid soft core NN potential with the iterated $N\Delta$ diagrams excluded by the modification¹⁾

$$V_1 = V_{NN} + \frac{V_2^2}{\Delta E(k^2)} \quad (3.2)$$

where $\Delta E(k^2)$ is a parameter adjusted to fit the real part of the phase shift $\delta({}^1D_2)$. Furthermore, V_2 is the transition potential coupling the NN and $N\Delta$ channels. It consists of the π and ρ exchange parts including the $M_\Delta - M_N$ mass difference corrections introduced by Durso et al.²³⁾. In addition, the potential includes dipole form factors with the cut-off parameters $\Lambda_\pi = 5.6 \text{ fm}^{-1}$ and $\Lambda_\rho = 7.6 \text{ fm}^{-1}$ ²⁴⁾. The π and ρ coupling constants have the same values as in Ref. 2). For the diagonal $N\Delta$ interaction V_3 the approach of Ref. 1) is applied. For the Δ width the expression²⁵⁾

$$\Gamma = 142 \text{ MeV} \frac{\omega(q')}{\omega(q_R)} (q'R)^3 / (1 + (q'R)^2) \quad (3.3)$$

with $R = 0.81/\mu$ has been chosen. Here

$$q' = q (1 - \omega(q) / [2(M_N + \omega(q))]) \quad (3.4)$$

is the pion momentum including centre-of-mass corrections due to the bystander nucleon, $\omega^2(q) = \mu^2 + q^2$ and q is the pion momentum obtained from the $pp \rightarrow d\pi^+$ reaction kinematics. Also, q_R is the pion momentum at the resonance in the πN system. Fortunately, the results are not too sensitive to the details of Γ (20). Only the $pp \rightarrow d\pi^+$ cross-section at the resonance position seems to depend crucially on Γ .

Using the wave functions resulting from the coupled equations (3.1) the $pp \rightarrow d\pi^+$ cross-section shown in Fig. 3 is obtained. The solid line includes the contributions from the states $^1D_2(NN)$, $^5S_2(N\Delta)$ and $^5D_2(N\Delta)$. If the $^5P_3(N\Delta)$ state, which is coupled to the $^3F_3(NN)$ channel, is added the dashed line results. For this calculation the formulae of Ref. 2) are employed. From Fig. 3 it can be seen that the input wave functions account for about 80-85% of the total cross-section up to 560 MeV. However, above the resonance the discrepancy rises to a factor two. This is also the order of accuracy obtained in Ref. 2) in a more refined calculation for the total cross-section. The fact that asymmetries calculated in Ref. 2) are in much better agreement with experiment indicates that the discrepancy in the total cross-section is probably a normalization effect. Some possibilities of improving the model are discussed in Ref. 5).

4. - THE WAVE FUNCTION DEVELOPMENTS

When calculating T matrix elements for pion production [e.g., the one in Eq. (2.9)] three wave functions are needed for the various systems shown in Fig. 1. These are $\psi_t(\vec{r}, \vec{s})$ for the final state triton, $w(Kr)$ for the interacting $N\Delta$ system 01 and $f(\vec{k})$ for the initial state deuteron. The model for obtaining $w(Kr)$ was presented in the previous section. In this section the $\psi_t(\vec{r}, \vec{s})$ and $f(\vec{k})$ wave functions will be discussed.

The standard approach to a phenomenological three-nucleon wave function is to assume a simple functional form for it and then to adjust the unknown parameters to some measurable quantities, in particular the charge form factor.

An example of this type of fit is the one in Ref. 11). However, it is known that meson-exchange effects play an important role in the calculations for the charge form factor of the three-body system^{26),27)}. Thus the nucleonic three-nucleon wave function should not reproduce the experimental charge form factor. In the present calculation a solution of the Faddeev equations is employed. It is based on the work of Ref. 21) and was fitted to the Yukawa type of functions in Ref. 22). The solution of the three-body problem considered in Ref. 21) takes into account the interaction in the two-nucleon channels 1S_0 and $^3S_1 - ^3D_1$ giving five independent components for the wave function. Each of these channel wave functions was factorized as a product of terms representing the motion of the pair 01 and the motion of the centre-of-mass of the pair 01 with respect to the spectator particle 2. These approximate wave functions were then fitted to sums of Yukawa functions and the resulting parameters can be found in Ref. 22). The charge form factor of ^3He obtained using these fits agrees quite well with the result of the exact solution of the Faddeev equations upto the second minimum, i.e., $q \lesssim 8\text{fm}^{-1}$. From the five components the first three have been chosen for the present calculation. The other two are of minor importance as can already be seen from the normalization factors n_ν in Ref. 22). With this wave function the probabilities for the S, S' and D states in the triton are 89.4, 1.4 and 7.8% respectively, whereas the corresponding renormalized figures to ensure a total probability of unity are 90.7, 1.4 and 7.9%.

The triton wave function for $J_z = 1/2$ can then be written as²⁸⁾

$$\begin{aligned} |\Psi(\vec{r}, \vec{s})\rangle = & |W_{\frac{1}{2}}^A\rangle \psi_{\frac{1}{2}}^{\frac{3}{2}}(\vec{r}, \vec{s}) + |W_{\frac{1}{2}}^+\rangle \psi_{\frac{1}{2}}^-(\vec{r}, \vec{s}) \\ & + |W_{\frac{1}{2}}^-\rangle \psi_{\frac{1}{2}}^+(\vec{r}, \vec{s}) \\ & + \sum_{m_\chi, m_\psi} \langle 2 m_\chi \frac{3}{2} m_\psi | \frac{1}{2} \frac{1}{2} \rangle [|W_{\frac{3}{2}}^+ m_\psi \rangle \psi_{\frac{3}{2}}^-(\vec{r}, \vec{s}) \\ & + |W_{\frac{3}{2}}^- m_\psi \rangle \psi_{\frac{3}{2}}^+(\vec{r}, \vec{s})] \end{aligned} \quad (4.1)$$

where the spin-isospin states are

$$\begin{aligned} |W_{\frac{1}{2}}^A\rangle &= \frac{1}{\sqrt{2}} \{ |[(N_0 N_1)^{0;1} N_2]^{\frac{1}{2}, \frac{1}{2}}\rangle - |[(N_0 N_1)^{1;0} N_2]^{\frac{1}{2}, \frac{1}{2}}\rangle \} \\ |W_{\frac{1}{2}}^+\rangle &= \frac{1}{\sqrt{2}} \{ |[(N_0 N_1)^{1;1} N_2]^{\frac{1}{2}, \frac{1}{2}}\rangle - |[(N_0 N_1)^{0;0} N_2]^{\frac{1}{2}, \frac{1}{2}}\rangle \} \\ |W_{\frac{1}{2}}^-\rangle &= \frac{1}{\sqrt{2}} \{ |[(N_0 N_1)^{0;1} N_2]^{\frac{1}{2}, \frac{1}{2}}\rangle + |[(N_0 N_1)^{1;0} N_2]^{\frac{1}{2}, \frac{1}{2}}\rangle \} \end{aligned} \quad (4.2)$$

$$\begin{aligned}
 |W_{\frac{3}{2}}^+\rangle &= - |[(N_0 N_1)^{1;1} N_2]^{\frac{3}{2}, \frac{1}{2}}\rangle \\
 |W_{\frac{3}{2}}^-\rangle &= |[(N_0 N_1)^{1;0} N_2]^{\frac{3}{2}, \frac{1}{2}}\rangle
 \end{aligned}
 \tag{4.2}$$

cont.

The superscripts indicate the symmetry property of the wave function. The completely antisymmetric state is denoted by A and the symmetric state by S. For the mixed symmetry states $\pm$ show the symmetry with respect to the exchange of the particles 0 and 1. The completely antisymmetric S wave part of the triton wave function can now be written as

$$\begin{aligned}
 |\bar{\Psi}^{\chi=0}(\bar{r}, \bar{s})\rangle &= |W_{\frac{1}{2}}^A\rangle \psi_{1/2}^S(\bar{r}, \bar{s}) \\
 &= |W_{\frac{1}{2}}^A\rangle \frac{1}{4\pi\sqrt{2}} \left\{ \tilde{f}(\bar{r}, \bar{s}) \right. \\
 &\quad \left. + \tilde{f}\left(-\frac{1}{2}\bar{r} + \bar{s}, -\frac{3}{4}\bar{r} - \frac{1}{2}\bar{s}\right) + \tilde{f}\left(-\frac{1}{2}\bar{r} - \bar{s}, \frac{3}{4}\bar{r} - \frac{1}{2}\bar{s}\right) \right\}
 \end{aligned}
 \tag{4.3}$$

where the short-hand notation

$$\tilde{f}(\bar{r}, \bar{s}) = \frac{1}{|\bar{r}| |\bar{s}|} \left\{ \eta_1 \varphi_1(|\bar{r}|) \chi_1(|\bar{s}|) - \eta_2 \varphi_2(|\bar{r}|) \chi_2(|\bar{s}|) \right\} \tag{4.4}$$

as well as the factorization approximation of Ref. 22) have been introduced.

With the wave function in Eq. (4.3) the T matrix element in Eq. (2.9) takes the form

$$\langle \bar{\Psi}_f | T | \bar{\Psi}_i \rangle = \frac{8\sqrt{30}}{(2\pi)^{7/2}} \langle \frac{1}{2} m \ 1 M | \frac{1}{2} m+M \rangle \frac{f^*}{\mu} \frac{1}{\sqrt{\omega}} q F(m+M, m'') \tag{4.5}$$

where

$$\begin{aligned}
 F(m+M, m'') &= \int d\bar{K} f\left(\frac{3}{2}\bar{k}_0 - 2\bar{K}\right) a(m+M, m'') e^{i\delta_2(K)} \\
 &\quad \times \int_0^\infty dr \ r^2 \frac{w(Kr)}{Kr} j_0\left(\frac{1}{2}qr\right)
 \end{aligned}
 \tag{4.6}$$

$$\begin{aligned}
 & \times \frac{\pi}{2\sqrt{2}} \left[n_1 \sum_{ij} a_i^{(1)} b_j^{(1)} \left\{ \frac{e^{-\alpha_i^{(1)} r}}{r} \frac{1}{|\vec{k} + \vec{Q}|^2 + \beta_j^{(1)2}} \right. \right. \\
 & \quad + \frac{2}{\pi |\vec{k} + \vec{Q}| r} \int_0^\infty dx \frac{\sin(2xr)}{x^2 + \frac{1}{16} [3|\vec{k} + \vec{Q}|^2 + 4\alpha_i^{(1)2} + 3\beta_j^{(1)2}]} \\
 & \quad \left. \left[Q_0 \left(\frac{x^2 + \frac{9}{16} |\vec{k} + \vec{Q}|^2 + \alpha_i^{(1)2}}{\frac{3}{2} x |\vec{k} + \vec{Q}|} \right) + Q_0 \left(\frac{x^2 + \frac{1}{16} |\vec{k} + \vec{Q}|^2 + \frac{1}{4} \beta_j^{(1)2}}{\frac{1}{2} x |\vec{k} + \vec{Q}|} \right) \right] \right\} \\
 & \quad - n_2 \sum_{ij} [(1) \rightarrow (2)] \Big] \quad (4.6)
 \end{aligned}$$

and

$$\begin{aligned}
 a(m+M, m'') &= -\frac{4}{3} 4\pi \operatorname{Re} \left\{ \sum_{m'} \begin{pmatrix} 2 & \frac{3}{2} & \frac{1}{2} \\ m' & -m' - m - M & m + M \end{pmatrix} \right. \\
 & \times \begin{pmatrix} \frac{3}{2} & 1 & \frac{1}{2} \\ m' + m + M & m'' - m' - m - M & -m'' \end{pmatrix} Y_{1, m' + m + M - m''}(\hat{q}) \\
 & \quad \left. \times Y_{2, m'}^*(\hat{k}) \right\} \quad (4.7)
 \end{aligned}$$

which has the properties

$$\begin{aligned}
 a\left(\frac{1}{2}, \frac{1}{2}\right) &= -a\left(-\frac{1}{2}, -\frac{1}{2}\right) \\
 a\left(\frac{1}{2}, -\frac{1}{2}\right) &= a\left(-\frac{1}{2}, \frac{1}{2}\right) \quad (4.8)
 \end{aligned}$$

In Eq. (4.6) the parameters $a^{(v)}$, $b^{(v)}$, $\alpha^{(v)}$ and $\beta^{(v)}$ are those fitted in Ref. 22). In the D wave part of the triton the channel $v = 3$ in Ref. 22) contributes. Only the $\psi_{\frac{3}{2}}^{\frac{3}{2}}(\vec{r}, \vec{s})$ is taken into account because the $\psi_{\frac{3}{2}}^{\frac{3}{2}}(\vec{r}, \vec{s})$ does not include even waves for the pair (01) in the triton and thus the leading direct term does not appear in the matrix element for $\psi_{\frac{3}{2}}^{\frac{3}{2}}$. The $\ell = 2$ part now takes the form

$$\begin{aligned}
 |\Psi^{\ell=2}(\vec{r}, 3)\rangle &= \frac{m_2}{\sqrt{4\pi}} \sum_{m_2, m_3} \langle 2 m_2 \frac{3}{2} m_3 | \frac{1}{2} \frac{1}{2} \rangle \\
 & \times |W_{\frac{3}{2}}^{-}\rangle \\
 & \times \left[\frac{1}{|\vec{r}| |\vec{s}|} g_3(|\vec{r}|) \chi_3(|\vec{s}|) Y_{2, m_2}(\hat{r}) \right] \quad (4.9)
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{1}{2} \left\{ \frac{1}{1-\frac{1}{2}\bar{r}+\bar{s} \mid 1-\frac{3}{4}\bar{r}-\frac{1}{2}\bar{s}} \varphi_3(1-\frac{1}{2}\bar{r}+\bar{s}) \chi_3(1-\frac{3}{4}\bar{r}-\frac{1}{2}\bar{s}) \right. \\
 & \quad \times Y_{2m_2}(\widehat{-\frac{1}{2}\bar{r}+\bar{s}}) \\
 & \quad + \frac{1}{1-\frac{1}{2}\bar{r}-\bar{s} \mid 1-\frac{3}{4}\bar{r}-\frac{1}{2}\bar{s}} \varphi_3(1-\frac{1}{2}\bar{r}-\bar{s}) \chi_3(1-\frac{3}{4}\bar{r}-\frac{1}{2}\bar{s}) \\
 & \quad \left. \times Y_{2m_2}(\widehat{-\frac{1}{2}\bar{r}-\bar{s}}) \right\} \quad (4.9) \\
 & \text{cont.}
 \end{aligned}$$

The transition matrix element can then be calculated just as in the S wave case given in Eq. (4.5). The leading term has the same angular dependency given by $a(m+M, m'')$ except for a small correction term.

Taking into account the deuteron D state implies the replacement of the three-nucleon spin-isospin state

$$\begin{aligned}
 & \langle \frac{1}{2} m \ 1 M \mid \frac{1}{2} m+M \rangle [(\frac{1}{2} \frac{1}{2})^0; \frac{1}{2}]^{\frac{1}{2}, \frac{1}{2}} f(\bar{k}) \\
 \text{by} \quad & \langle \frac{1}{2} m \ 1 M \mid \frac{1}{2} m+M \rangle [(\frac{1}{2} \frac{1}{2})^0; \frac{1}{2}]^{\frac{1}{2}, \frac{1}{2}} f(\bar{k}) \quad (4.10) \\
 & - \langle \frac{1}{2} m \ 1 M \mid \frac{3}{2} m+M \rangle \frac{1}{\sqrt{2}} [(\frac{1}{2} \frac{1}{2})^0; \frac{1}{2}]^{\frac{1}{2}, \frac{1}{2}} Y_2(\hat{k})^{\frac{3}{2}, \frac{1}{2}} f_D(\bar{k})
 \end{aligned}$$

where $f_D(\bar{k})$ is the momentum space wave function for the deuteron D wave taken from the fit of McGee²⁹⁾. This particular component is chosen because it involves the singlet even 01 pair which then couples to the dominant ${}^5S_2(N\Delta)$ channel. The resulting deuteron D wave contributions do not interfere with the deuteron S wave part because of the different total spin.

In the d wave pion production the ${}^3F_3(NN)$ state for the pair 01 has to be considered. The spin-isospin part is now

$$\begin{aligned}
 & \langle \frac{1}{2} m \ 1 M \mid \frac{1}{2} m+M \rangle \frac{\sqrt{3}}{4} [(N_0 N_1)^{1,1} N_2]^{\frac{1}{2}, \frac{1}{2}} \\
 & + \langle \frac{1}{2} m \ 1 M \mid \frac{3}{2} m+M \rangle \frac{\sqrt{3}}{2} [(N_0 N_1)^{1,1} N_2]^{\frac{3}{2}, \frac{1}{2}} \quad (4.11)
 \end{aligned}$$

and this is coupled to the $Y_3(\hat{r})$ component for the pair 01. The ${}^3F_3(NN)$ channel is coupled to several $N\Delta$ states of which ${}^5P_3(N\Delta)$ is picked as the

dominant component since it has the lowest value of ℓ . The angular term corresponding to the function a in Eq. (4.7) is now much more complicated containing a multiple summation of terms proportional to $Y_2(\hat{Q})Y_3(\hat{k})$ and can be found in the Appendix.

5. - RESULTS

The main aim of the present article has been to investigate in the $pd \rightarrow t\pi^+$ reaction the importance of adopting realistic wave functions for the nuclei involved. The basic reaction mechanism employed is the same as in Ref. 3) and is shown in Figs. 1 and 2.

Figure 4 shows the results of the $pd \rightarrow t\pi^+$ calculation at the three proton laboratory energies 400, 470 and 600 MeV^{*}). Here both the deuteron and the triton have only S wave components in the wave function. The Hulthen type form is chosen for the deuteron. For the triton three different parametrizations have been used. The dash-dotted and dashed lines represent the cross-sections for the Gaussian and correlated Gaussian triton wave functions respectively. These are the results given in Ref. 4). The solid line refers to the curve resulting from the transition matrix in Eq. (4.5) using the Faddeev wave function. The shapes of the cross-sections stay almost unaltered, only the normalizations differ from the earlier calculations with correlated Gaussian triton wave functions. In the previous calculation⁴⁾ a prominent dip appeared in the cross-section at the effective momentum transfer $Q \approx 1.8\text{fm}^{-1}$. The dip persists in the present work and, indeed, it seems to move slightly towards higher momentum transfer from $Q \approx 1.8\text{fm}^{-1}$ to $Q \approx 2.0\text{fm}^{-1}$. This is expected because the ^3He charge form factor reproduced by the wave function employed here also has its zero at a higher momentum transfer than the experimental dip²¹⁾. Some attempts were also made to incorporate the term proportional to $j_2(1/2\text{qr})$ in the one-body production operator given in Eq. (2.4). It seems to increase the cross-section slightly, a few per cent in the forward direction and slightly more in the backward direction. In addition, the importance of the S' state (the $\psi_{1/2}^+$ term) in Eq. (4.1) was investigated. It turned out that this term tends to reduce the cross-section by a few per cent, more in the backward direction than in the forward hemisphere. As a consequence of these considerations both effects, the $j_2(1/2\text{qr})$ term and the S' state were neglected from the rest of the calculations.

*₂) The experimental points³⁰⁾ have been scaled to the energies of interest using the multiplicative factor given in Ref. 3).

In Fig. 5 the effect of including the D wave part in the triton is shown. The solid lines represent the S wave calculations already given in Fig. 4. The dash-dotted lines show the cross-sections when both the S and D wave parts are included in the triton. Qualitatively these agree with the results reported in the previous article⁴⁾, where a simple Gaussian D wave correlation was employed. However, the size of this modification is smaller than before. The states for the pair (01) which couple to the D wave triton have been taken to be $^1D_2(NN)$ and $^5D_2(N\Delta)$. In addition, the effect arising from different forms for the deuteron wave function is demonstrated by the dashed line in Fig. 5. These lines are to be compared with the solid ones, because for them the D wave part of the triton was not included. The result of this comparison is the same as before⁴⁾, i.e., the details of the deuteron S wave function do not have much effect.

In Fig. 6 the influence of the D state of the deuteron is shown as well as the d wave pion production from the $^5P_3(N\Delta)$ state for the pair 01. The solid line represents the S and D wave triton calculation given in the previous figure. The dash-dotted line shows the effect of the D state of the deuteron. The results are in excellent agreement with the calculation of Fearing¹⁰⁾ who also obtained a slight reduction in the forward direction, mainly due to the S state renormalization, and a small enhancement in the backward direction. The dashed line gives the d wave pion contributions when added to the solid curve neglecting the D wave part of the deuteron. The most striking feature is the increase in the forward direction extending from 30% at 400 MeV initial proton laboratory energy to 90% at 600 MeV. In the backward hemisphere there appear small oscillations due to the $Y_2(\hat{q})$ factor in the production matrix element. In angles less than 90° this phenomenon is not seen as a result of the large background. The d wave pion production considered here includes two effects, one with the same total spin as the singlet production from the pair 01 and another with total spin = 3/2. The first of these amplitudes interferes with the production amplitudes corresponding to the solid line and the second adds coherently as does the effect from the deuteron D state. However, the latter interfere in the full calculation. In Fig. 7 the final results are shown including all the effects discussed above. Compared with the previous calculation⁴⁾, the over-all agreement with experiment is much better, but still at the backward angles the cross-sections are not properly reproduced, especially at higher energies. In the forward direction, the results are not that bad, keeping in mind that there are no adjustable parameters in the present calculation.

To explore the most important energy region of the two-body input, the integration over K , the relative momentum of particles 0 and 1 in the initial state, in Eq. (2.9) was performed in pieces for a set of three angles for the initial proton energies 400 and 470 MeV. The result of this study was that the peak position in the K integration is a function of both the incoming proton energy and the effective momentum transfer. The K value corresponding to the peak position in the $pp \rightarrow d\pi^+$ reaction appears in the $pd \rightarrow t\pi^+$ reaction as the maximum point for 400 MeV initial energy at about 150° and for 470 MeV at about 60° . From this it may be concluded that in fact the inadequate two-body input above the resonance, indicated by the $pp \rightarrow d\pi^+$ cross-section in Fig. 3, could be an important reason for too low cross-sections in the backward hemisphere.

Another way of plotting the results is shown in Fig. 8. Fig. 8a gives the experimental points³⁰⁾ as a function of the effective momentum transfer $Q = |1/2 \vec{k}_0 - 1/3 \vec{q}|$. Only a part of the data given in Ref. 30) is taken but no attempt has been made to judge which are the most reliable points. Instead, the disagreement between various experiments has been shown. The solid curve shows the result of the 400 MeV calculation given above in Fig. 7a. It seems to reproduce the trend of the experiments up to momentum transfer $Q \approx 320$ MeV/c. Above that, the experimental situation is unclear and it is not possible to say how good the results are. Figure 8b gives the curves for the other two energies considered here. The dashed line corresponds to the 470 MeV and the dash-dotted line to the 600 MeV calculations shown also in Figs. 7b and 7c respectively. A similar scaling phenomenon was observed by Källne et al.³¹⁾ in ${}^3\text{He}(\pi^-, n){}^2\text{H}$ measurements. If the behaviour of the curves in Fig. 8 is parametrized in terms of an exponential for the range $Q \lesssim 300$ MeV/c so that

$$\frac{d\sigma}{d\Omega} = C(E_{\text{lab}}) e^{-\alpha Q} \quad (5.1)$$

The parameter α adopts the values 0.046, 0.044 and 0.044/MeV/c for the energies 400, 470 and 600 MeV respectively. A fit by eye to the experimental points seems to give correspondingly numbers like 0.048, 0.044 and 0.055. The value $\alpha = 0.044$ corresponds to the figure $\lambda = 2/\alpha \approx 45$ MeV/c in the notation of Ref. 31) in good agreement with $\lambda = 48$ MeV/c quoted there. In addition, from Fig. 8b it can be seen that the highest curve is the one corresponding to the 470 MeV laboratory energy which is close to the resonance position $E_{\text{lab}} \approx 450$ MeV. The experimental points do not support this ordering of the lines. The angular integration yields the total cross-sections 42.4, 31.9 and 8.72 μb for initial laboratory energies 400, 470 and 600 MeV respectively.

6. - CONCLUSIONS

The aim of the present study has been to investigate carefully various wave function effects in the $pd \rightarrow t\pi^+$ reaction within the coupled channels Δ isobar model. The outcome of these attempts has been a better agreement with experiment than before for a certain range of the effective momentum transfer Q . However, for higher values of Q the disagreement still persists. This may be a reflection of the fact that the present two-body model does not yield a good understanding of the $pp \rightarrow d\pi^+$ reaction above the resonance region actually probed by the large angle $pd \rightarrow t\pi^+$ production. The two-body input was not adjusted to reproduce the $pp \rightarrow d\pi^+$ cross-section because this could put too much emphasis on certain partial waves and thus lead to erroneous strength of some features in the cross-section. A striking feature, in addition, is the close similarity of the final curves given in Fig. 7 and those presented by dashed lines in Fig. 4 showing the result of the full S wave calculation with the correlated Gaussian triton wave function. This could be a reflection of the fact that direct fit to experiment to determine the parameters of a wave function could simulate a lot of effects when the resulting wave function is applied to a microscopic calculation. Of course, the reliability of this kind of simulation is not at all clear.

The crucial ingredient in the two-body model employed is the width Γ of the Δ resonance. In this width the energy dependence comes via the pion momentum in Eq. (3.4) which is dictated by the πd kinematics in the final state. This prescription may not be the best possible one. Also, some non-local or energy-dependent effects in the $NN \rightarrow N\Delta$ transition potential could lead to different results at these high energies. The pion distortions are not expected to have a large effect in the present approach. Through the coupled equations part of the rescattering within the system (01) has been taken into account. The possibility of the pion becoming real was considered in Ref. 20) using a complex $N\Delta - \Delta N$ interaction. This did not alter the results significantly. The graph which has not been discussed so far is the one with pion rescattering from the spectator particle 2. If it is assumed that this should go through the Δ again, the influence is expected, due to spin-isospin reasons as discussed in Ref. 32), to be small. Another distortion effect, that of the proton in the initial state, has not been taken into account. In principle, one could solve the two-body equations in the presence of an additional averaged interaction, which would simulate the presence of the spectator. However, this kind of approach would introduce arbitrary parameters into the model.

ACKNOWLEDGMENTS

My sincerest thanks go to Dr. A.M. Green for his patient guidance and numerous comments concerning the manuscript. I also wish to acknowledge useful discussions and correspondence with Dr. Ch. Hajduk. In addition I thank the Emil Aaltonen Foundation for a grant and the National Research Council for Sciences in Finland (Particle Physics Committee) for supporting my stay at CERN.

APPENDIX

The inclusion of the D wave part of the triton [Eq. (4.9)] leads to the emergence of the following modification to the amplitude in Eq. (4.6)

$$F_D(m+M, m'') = n_3 \int d\bar{K} f\left(\frac{3}{2}\bar{k}_0 - 2\bar{K}\right) e^{i\delta_2(K)}$$

$$\begin{aligned} & \int_0^\infty dr r^2 j_0\left(\frac{1}{2}qr\right) \left[a(m+M, m'') \right. \\ & \left\{ \frac{6}{\sqrt{10}} \frac{f}{f^*} \frac{u(Kr)}{Kr} \frac{\pi}{2} \sum_{ij} a_i^{(3)} b_j^{(3)} \left(\frac{e^{-d_i^{(3)}}}{r} \left(1 + \frac{3}{d_i^{(3)}r} + \frac{3}{(d_i^{(3)}r)^2} \right) \right. \right. \\ & \times \frac{1}{|\bar{k}+\bar{Q}|^2 + \beta_j^{(3)2}} + \frac{4}{\pi} \int_0^\infty dx x^2 j_2(2xr) \\ & \left. \left[x^2(q_0 - q_2) - \frac{9}{20} \times |\bar{k}+\bar{Q}|(q_1 - q_3) \right] \right) \\ & - \sqrt{\frac{7}{10}} \frac{v(Kr)}{Kr} \frac{\pi}{2} \sum_{ij} a_i^{(3)} b_j^{(3)} \left(\frac{e^{-d_i^{(3)}}}{r} \left(1 + \frac{3}{d_i^{(3)}r} + \frac{3}{(d_i^{(3)}r)^2} \right) \right. \\ & \times \frac{1}{|\bar{k}+\bar{Q}|^2 + \beta_j^{(3)2}} + \frac{4}{\pi} \int_0^\infty dx x^2 j_2(2xr) \\ & \left. \left[x^2\left(q_0 - \frac{2}{7}q_2\right) - \frac{3}{5} \times |\bar{k}+\bar{Q}|(2q_1 - \frac{3}{14}q_3) \right. \right. \\ & \left. \left. + \left(\frac{3}{4}\right)^2 \frac{5}{7} |\bar{k}+\bar{Q}|^2 q_2 \right] \right\} \\ & - \frac{12}{\sqrt{10}} \left(-\frac{f}{f^*} ad(m+M, m'') \frac{u(Kr)}{Kr} \right. \\ & \left. + \frac{1}{2\sqrt{7}} add(m+M, m'') \frac{v(Kr)}{Kr} \right) \\ & \left. \int_0^\infty dx x^2 j_2(2xr) \left[\left(x^2 + \frac{9}{16} |\bar{k}+\bar{Q}|^2 \right) q_2 \right. \right. \\ & \left. \left. - \frac{3}{20} \times |\bar{k}+\bar{Q}|(7q_1 + 3q_3) \right] \right] \end{aligned}$$

where $u(Kr)$ and $v(Kr)$ are the $^1D_2(NN)$ and $^5D_2(N\Delta)$ wave functions respectively and

$$g_n = -G [Q_n(A) + (-1)^n Q_n(B)]$$

and

$$A = \frac{x^2 + \frac{9}{16} |\bar{k} + \bar{q}|^2 + (\alpha_i^{(3)})^2}{\frac{3}{2} x |\bar{k} + \bar{q}|}$$

$$B = \frac{x^2 + \frac{1}{16} |\bar{k} + \bar{q}|^2 + \frac{1}{4} (\beta_j^{(3)})^2}{\frac{1}{2} x |\bar{k} + \bar{q}|}$$

$$G = \frac{1}{2x |\bar{k} + \bar{q}| (\alpha_i^{(3)})^2} \cdot \frac{1}{x^2 + \frac{1}{16} [3 |\bar{k} + \bar{q}|^2 + 4 (\alpha_i^{(3)})^2 + 3 (\beta_j^{(3)})^2]}$$

and

$$ad(\frac{1}{2}, \frac{1}{2}) = 4\pi \frac{2}{3\sqrt{5}} \operatorname{Re} (Y_{2-1}^*(\hat{R}) Y_{1-1}(\hat{q}) + \frac{2}{\sqrt{3}} Y_{20}^*(\hat{R}) Y_{10}(\hat{q}))$$

$$ad(\frac{1}{2}, -\frac{1}{2}) = 4\pi \frac{2}{3\sqrt{5}} \sqrt{2} \operatorname{Re} (Y_{2-1}^*(\hat{R}) Y_{10}(\hat{q}) + \frac{1}{\sqrt{3}} Y_{20}^*(\hat{R}) Y_{11}(\hat{q}))$$

$$add(\frac{1}{2}, \frac{1}{2}) = -4\pi \frac{1}{\sqrt{5}} \operatorname{Re} (Y_{2-1}^*(\hat{R}) Y_{1-1}(\hat{q}) - \frac{5}{3} Y_{21}^*(\hat{R}) Y_{11}(\hat{q}))$$

$$add(\frac{1}{2}, -\frac{1}{2}) = -4\pi \frac{2\sqrt{2}}{3\sqrt{5}} \operatorname{Re} (-\frac{5}{2} \sqrt{2} Y_{2-2}^*(\hat{R}) Y_{1-1}(\hat{q}) + \frac{1}{\sqrt{3}} Y_{20}^*(\hat{R}) Y_{11}(\hat{q}))$$

with the properties in Eq. (4.8).

With the D state of the deuteron included

$$\langle \frac{1}{2} m 1 M | \frac{1}{2} m+M \rangle a(m+M, m'') f_s(\bar{k})$$

has to be replaced by

$$\langle \frac{1}{2} m 1 M | \frac{3}{2} m+M \rangle d(m+M, m'') f_D(\bar{k})$$

where

$$d(m+M, m'') = 4\sqrt{\frac{4\pi}{6}} (-1)^{1+m+M} \text{Re} \left\{ \sum_{m', m_2} \begin{pmatrix} 2 & \frac{1}{2} & \frac{3}{2} \\ m' & m+M-m_2 & m_2-m'-m-M \end{pmatrix} \begin{pmatrix} \frac{3}{2} & 1 & \frac{1}{2} \\ m'+m+M-m_2 & m''-m'-m-M+m_2 & -m'' \end{pmatrix} \begin{pmatrix} \frac{1}{2} & 2 & \frac{3}{2} \\ m+M-m_2 & m_2 & -m-M \end{pmatrix} Y_{1, m'+m+M-m''-m_2}(\hat{q}) Y_{2, m'}^*(\hat{R}) Y_{2, m_2}(\hat{k}) \right\}$$

For the d wave pion production the amplitude F needs the modification

$$\langle \frac{1}{2} m \ 1 M \mid \frac{1}{2} m+M \rangle w(Kr) \text{ to } a$$

has to be replaced by

$$e^{i(\delta_3 - \delta_2)} \left[\langle \frac{1}{2} m \ 1 M \mid \frac{1}{2} m+M \rangle \left(-\frac{1}{3} \sqrt{\frac{7}{3}} w(Kr) \times j_1\left(\frac{1}{2} q r\right) f(m+M, m'') \right) + \langle \frac{1}{2} m \ 1 M \mid \frac{3}{2} m+M \rangle \times \left(\frac{1}{3} \sqrt{\frac{7}{6}} w(Kr) j_1\left(\frac{1}{2} q r\right) f f(m+M, m'') \right) \right]$$

where $w(Kr)$ is now the ${}^5P_3(N\Delta)$ wave function and

$$f(m+M, m'') = -\frac{32\pi}{5} \operatorname{Re} \left\{ \sum_{m'} \begin{pmatrix} 3 & \frac{5}{2} & \frac{1}{2} \\ m' & -m'-M-m & m+M \end{pmatrix} \begin{pmatrix} \frac{5}{2} & 2 & \frac{1}{2} \\ m'+m+M & m''-m'-m-M & -m'' \end{pmatrix} Y_{2m'+m+M-m''}(\hat{q}) \right. \\ \left. \times Y_{3m'}^*(\hat{k}) \right\}$$

and

$$ff(m+M, m'') = \frac{32\pi}{5} \sqrt{10} \operatorname{Re} \left\{ \sum_{m'} \begin{pmatrix} 3 & \frac{5}{2} & \frac{3}{2} \\ m' & -m'-m-M & m+M \end{pmatrix} \begin{pmatrix} \frac{5}{2} & 2 & \frac{1}{2} \\ m'+m+M & m''-m'-m-M & -m'' \end{pmatrix} Y_{2m'+m+M-m''}(\hat{q}) Y_{3m'}^*(\hat{k}) \right\}.$$

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FIGURE CAPTIONS

- Figure 1 : The basic pion production mechanism.
- Figure 2 : The various types of graphs taken into account in the present model.
- Figure 3 : The total cross-section for the $pp \rightarrow d\pi^+$ reaction given by the present model. The solid line refers to the p wave pion production and the dashed line includes a component of d wave production.
- Figure 4 : The differential cross-section for the $pd \rightarrow t\pi^+$ reaction at the three incoming proton laboratory energies a) 400 MeV, b) 470 MeV and c) 600 MeV. Both the triton and the deuteron wave function include only the S wave part; for the deuteron that of Hulthen and for the triton that of uncorrelated Gaussian (dash-dotted line), correlated Gaussian (dashed line) or the parametrized Faddeev (solid line) form.
- Figure 5 : The effect of various wave function modifications. The solid line as in Fig. 4. The dashed line shows the effect when the deuteron wave function has the form of McGee²⁹⁾. The dash-dotted line includes both the S and the D wave Faddeev components of the triton.
- Figure 6 : The influence of the deuteron D wave part as well as the d wave pions emerging from the $^5P_3(N\Delta)$ state for the pair 01 is shown. The solid line as the dash-dotted line in Fig. 5. The dashed line shows the modification due to d wave pions. The dash-dotted line includes both S and D waves of the initial state deuteron wave function.
- Figure 7 : The differential cross-section for the $pd \rightarrow t\pi^+$ reaction for the three proton energies a) 400 MeV, b) 470 MeV and c) 600 MeV when the various effects shown in separate figures before are combined.
- Figure 8 : The differential cross-sections shown as a function of the effective momentum transfer Q. In Fig. a) the experimental points for the three energies of interest are plotted as well as the theoretical prediction for $E_{lab} = 400$ MeV. In Fig. b) the curves are shown, in addition to the 400 MeV case (solid line), for $E_{lab} = 470$ MeV (dashed line) and $E_{lab} = 600$ MeV (dash-dotted line).

Fig. 1

Fig. 2

Fig. 3

Fig. 4c

Fig. 5a

Fig. 6a

Fig. 8a

Fig. 8b