Ref.TH.3206-CERN

GRAND UNIFICATION IN EXTENDED SUPERGRAVITY

John Ellis
CERN -- Geneva

Presented at the Second Europhysics Study Conference
on Unification of the Fundamental Interactions
Erice, Sicily, October 6 to 14, 1981

Ref.TH.3206-CERN
30 November 1981

INTRODUCTION

This paper is a review of attempts that have been made in the last couple of years to forge a connection between grand unified theories of elementary particle interactions and extended supergravities. My principal collaborators in this endeavour have been Mary K. Gaillard and Bruno Zumino¹⁻³, and as between us we have already written several reviews⁴⁻⁶ of our approach, this paper is intended to emphasize recent developments at the expense of older results.

The motivations for seeking grand unification in extended supergravity are well known. Present grand unified theories⁷ (GUTs) are clearly inadequate - one does not know which group to choose, GUTs do not explain the number of fermion generations, they have a hierarchy problem with elementary scalars and difficulties with strong CP violation, and even the minimal SU(5) GUT has at least 23 parameters, which we might like to reduce to 0, 1 or 3 depending on our religion. On the other hand, supersymmetry⁸ (susy) is the only way of combining internal symmetries with Lorentz invariance in a non-trivial way, and is so beautiful that it must be true. It may even be useful⁹, for example for "solving" the hierarchy problem by enabling one to "fix and forget" the Higgs potential. Recently proposed¹⁰ schemes of this type only use a simple $N = 1$ rigid susy, which is not true "superunification" as the fermionic charges do not carry internal indices. To do this one should go to extended ($N > 1$) susy. Furthermore, if one is to include gravity (and what else is there left to do if one believes in GUTs?) one must invoke local susy. We are therefore led to try¹⁻³ to extract a GUT from an extended

supergravity (ESG) theory¹¹. These are the only theories of gravitational interactions with matter fields which are not known to have unrenormalizable infinities. Maybe ESGs also have uncontrollable infinities, but $N = 4$ Yang-Mills theories are already known¹² to avoid many infinite pitfalls, so that one has motivation to hope that some ESG may be "it".

THE SPECTRA OF EXTENDED SUPERGRAVITY THEORIES

There are¹¹ few of them: $N = 2, 3, 4, 5, 6$ and $7 = 8$ with the particle spectra shown in Table 1. These all contain just one graviton, as many gravitinos as there are susy generators, and just the right number of spin one particles to gauge an internal $SO(N)$ symmetry [or $SO(6) \times O(2)$ in the case of $N = 6$]. It has long been

Table 1. Multiplets in extended supergravities

Helicity \ N =	2	3	4	5	6	7 = 8
2	1	1	1	1	1	1 = 1
3/2	2	3	4	5	6	7 + 1 = 8
1	1	3	6	10	15 + 1	21 + 7 = 28
1/2	-	1	4	10 + 1	20 + 6	35 + 21 = 56
0	-	-	1 + 1	5 + 5	15 + 15	35 + 35 = 70
-1/2	-	1	4	1 + 10	6 + 20	21 + 35 = 56
-1	1	3	6	10	1 + 15	7 + 21 = 28
-3/2	2	3	4	5	6	1 + 7 = 8
-2	1	1	1	1	1	1 = 1

known¹³ that the $N \leq 4$ theories could be gauged, but the gauging of $N = 5$ ¹⁴ to 8¹⁵ has only recently been demonstrated. Gauging introduces a dimensionless gauge parameter g , hitherto absent, and entails the introduction of a cosmological constant

$$\Lambda = O(g^2 m_P^4) \quad (1)$$

Is gauging $SO(N)$ necessary, desirable or useful? One may take the traditional point of view of the Swiss law that anything which is not forbidden is compulsory, or one may be so anticlerical as to refuse the offer of an arbitrary input parameter. Perhaps the $SO(N)$ gauge interactions could be useful for "preconfining" states in the original supermultiplets of Table 1 so that the observed particles are composites, a necessity to be emphasized subsequently. However, confinement is conventionally associated with a non-Abelian gauge coupling rising to $O(1)$ on some scale Λ_{NA} and giving bound states of radius $1/\Lambda_{NA}$. But the $SO(N)$ gauge coupling is known¹⁶ to have a vanishing β function at one-loop and it is suspected that this may continue to be true at the multiloop level¹². In this case there would be no Λ_{NA} for these interactions, and the bound state scale could only be provided by the dimensional coupling $O(m_P)$ of the supergravitational interactions, which would therefore have to play a crucial rôle in the "preconfinement" mechanism. What of the associated cosmological constant (1)? The present failure of the Universe to expand exponentially means that the cosmological constant averaged over large distance scales must be very small:

$$\langle \Lambda \rangle_{\text{large}} < O(10^{-29}) \text{ gm/cc} \approx 10^{-47} \text{ GeV}^4 \quad (2)$$

If susy exists at all, it must be broken at some energy scale $\geq 1 \text{ GeV}$, which guarantees that the cosmological constant must have been non-zero before symmetry breakdown, as only susy-invariant vacua have zero energy¹⁷. Indeed, it has been argued¹⁸ that the cosmological constant has to exhibit fluctuations of order m_P^4 on very small distances if it is to be negligibly small (1) on very large distance scales. While the gauging^{14,15} of higher N ESGs is very exciting, it is not obvious to us whether we should avail ourselves of the opportunity.

It is well known¹⁹ that the supermultiplets of Table 1 cannot be used directly for grand unification. They do not contain enough spin one particles, since

$$SO(8) \not\supset SU(3) \times SU(2) \times U(1) \quad (3)$$

let alone even a minimal GUT group. In fact the maximal promising subgroup is

$$SO(8) \supset SU(3) \times U(1) \times U(1) \quad (4)$$

which does not contain candidates for $W^\pm$ bosons, so that they could only be interpreted as composite states. Similar difficulties occur with the fermion spectra in Table 1: they do not contain candidates for known fermions such as the μ , τ , b , etc. Another trouble with $SO(N)$ ($N \neq 6, 10, \dots$) groups in general is that their representations available for fermions are necessarily real. GUTters' worry about this because they know that the observed fermions are complex: each generation is in a $(3, 2) + 2(\bar{3}, 1) + (1, 2) + (1, 1)$ of $SU(3) \times SU(2)$. Furthermore, such a chiral representation is necessary if one is to prevent spin 1/2 fermions from acquiring masses $\geq O(m_X) \gg O(m_W)$ in a natural way, i.e., by matching different helicity states with $|\text{helicity}| \leq J$ and identical internal symmetry properties to get a massive particle of spin J . Indeed, if one constructs the $N=8$ ESG by dimensional reduction from the $N=1$ theory in 11 dimensions in the most general way²⁰, one finds masses for particles of all spins $\leq 3/2$. This illustrates the dangers of real fermion representations, and may also suggest that one should seek an alternative formulation of the $N=8$ theory if one wants a sufficiently non-trivial spectrum of low-mass fermions.

So far we have seen that the $W^\pm$, μ , τ and b do not appear in the spectra of Table 1, and in the ESG framework can therefore only be interpreted as composite states²¹. One may also encounter problems if one seeks to identify Higgs fields with the elementary spin-zero states in Table 1. It has been argued¹⁸ in the context of quantum gravity that elementary scalars necessarily acquire masses $O(m_P)$ from their propagation through a foamy structure of space-time at short distances $O(1/m_P)$. Hence phenomenological scalar fields should probably also be composite. Furthermore, it has been suggested¹⁸ that the space-time foam in a supergravity theory may be even more singular, to such an extent that space-time becomes fractal at very short distances with Hausdorff dimension $\gamma = 6$. If there is no cut-off on these foamy fluctuations, they will give logarithmically divergent masses to elementary spin 1/2 particles. In this case all observed spin 1/2 particles would also have to be composite. It is amusing to speculate on an extension of this argument. If the effective dimension of space-time at short distances were $\gamma = 11$, the resulting super-duper-foam might give masses $O(m_P)$ to all elementary particles of spin $\leq 3/2$, in striking coincidence with the results²⁰ of dimensional reduction from the $N=1$ theory in 11 dimensions. Thus one might argue that all light particles of spin $\leq 3/2$ must necessarily be composite. This possibility will be pursued in the fourth section, but first we need to recall some more structure of ESGs.

When they obtained the unbroken $N = 8$ ESG in four dimensions by dimensional reduction from the $N = 1$ theory in 11 dimensions, Cremmer and Julia²² noticed that the Lagrangian could be written in a form invariant under both a global non-compact $E_{7(+7)}$ symmetry and also a local $SU(8)$ symmetry. Written in a manifestly gauge-invariant form, the theory contains 133 fundamental scalar fields in an adjoint representation of E_7 . One can remove 63 scalars by making an appropriate choice of $SU(8)$ gauge, leaving oneself with 70 physical scalar fields which parametrize the coset space $E_7/SU(8)$ [the maximal compact subgroup of non-compact $E_{7(+7)}$ is $SU(8)/Z_2$]. These 70 scalar fields are those shown in the last column of Table 1. The general structure of the theory is reminiscent of non-linear σ (or CP^{N-1}) models²³: it has long been known²⁴ that one can write consistent Lagrangians with a non-compact global symmetry if the non-compact parts are realized non-linearly in terms of the scalar fields. The $SU(8)$ gauge invariance of the Lagrangian is realized²² using an $SU(8)$ -covariant derivative

$$D_{\mu B}^A \equiv \delta_{\mu B}^A \partial_\mu + i Q_{\mu B}^A \quad (5)$$

with the $Q_{\mu B}^A$ vectors in an adjoint representation of $SU(8)$ which do not have a kinetic term in the Lagrangian. Hence they can be eliminated using the equations of motion, and seem to contain a quadratic dependence on the fundamental scalar fields. This is strikingly similar to what happens in the CP^{N-1} model²³:

$$\mathcal{L}_{CP^{N-1}} = - \sum_{i=1}^N (\partial_\mu - i v_\mu) z_i^* (\partial_\mu + i v_\mu) z_i \quad : \quad \sum_{i=1}^N |z_i|^2 = 1 \quad (6)$$

which manifestly possesses a local $U(1)$ gauge invariance as well as the global $SU(N)$ invariance, and whose equations of motion enable one to express

$$v_\mu(x) = \frac{1}{2} \sum_{i=1}^N z_i^* \overleftrightarrow{\partial}_\mu z_i \quad (7)$$

as a quadratic function of the fundamental scalar fields.

The $1/N$ expansion has been used to show²³ that in two dimensions quantum effects generate a kinetic term for the v_μ field, so that the gauge field becomes dynamical and generates a long range confining potential. The physical spectrum therefore contains bound states sitting in unitary representations of the global $SU(N)$ groups²⁵. A supersymmetric version of the two-dimensional CP^{N-1} model also contains a fermionic superpartner of the gauge field which also becomes dynamical. Furthermore, it has been shown recently²⁶ that the $1/N$ expansion in three dimensions yields similar results of a dynamical gauge boson and an $N = 1$ fermionic

superpartner. This model is perhaps more interesting than the two-dimensional example because in this case the gauge boson has a physical helicity state. The infra-red properties of two- and three-dimensional models are however different from those in four dimensions, and it may well be that a purely bosonic CP^{N-1} model in four dimensions would not perform the same trick of generating dynamical gauge bosons. However, our four-dimensional supergravity theory of course contains fermion fields as well, and it has recently been argued²⁷ that a non-linear fermion theory in four dimensions can yield dynamical, physical, composite gauge bosons.

It therefore seems permissible to follow Cremmer and Julia in their conjecture²² that the physical spectra of four-dimensional ESGs contain dynamically generated composite gauge bosons corresponding to a local $SU(8)$ symmetry in the case of the $N = 8$ theory. Unless all susy is lost at the Planck mass, which does not happen in the two- and three-dimensional models, it is also plausible to conjecture¹ that at least some of the states in a supermultiplet containing the gauge fields $Q_{\mu B}^A$ will also become dynamical, physical, composite particles. Thus we suppose that all the observed and expected "elementary" particles: quarks, leptons, gauge bosons and Higgses, are all composites of the states seen in Table 1, which are now to be interpreted as preons bound on a distance scale of order $1/m_p$. It may be that the graviton is truly elementary: it is clearly a singlet of all the manifest and hidden internal symmetries of the ESG, and as such may escape whatever "preconfinement" mechanism removes the other preons from the low energy spectrum. We are therefore led to the geological stratigraphy of "elementary particles" laid out in Fig. 1, which is reminiscent of a wall of the Grand Canyon. However, the "elementarity" of the graviton is by no means essential: there are well known theorems²⁸ which guarantee that any massless spin two particle, even if composite, will have couplings identical with those of an elementary graviton at energies $\ll m_p$. Nevertheless, for definiteness we will pursue the scenario of Fig. 1.

GRAND UNIFICATION IN THE BOUND STATE SUPERMULTIPLY

Analogies with the known $N = 1$ and 2 cases, and other arguments¹⁻⁶, suggest that for general N the appropriate supermultiplet of bound states containing the vector fields $Q_{\mu B}^A$ (5) is

$$\left(\frac{3}{2}\right)^A, (1)^A_B, \left(\frac{1}{2}\right)^A_{[BC]}, (0)^A_{[BCD]}, \dots \left(\frac{3-N}{2}\right)^A \quad (8)$$

where the numbers in parentheses are helicities, and the lower indices are antisymmetrized up to N times, thereby fixing the lowest helicity. The supermultiplet obtained applying (8) to the case of $N = 8$ is shown in Table 2. A recent analysis²⁹ of embryonic Regge behaviour in the $2 \leftrightarrow 2$ scattering amplitudes of the $N = 8$

Fig. 1 The geological strata of "fundamental" particles, as seen by an observer from a large distance scale. At what level will the susy-bird land ?

ESG theory suggests the existence of Regge trajectories which could support physical massless states with the quantum numbers of Table 2, as well as of the original preon multiplet in Table 1. It is not possible to be sure that the residues of these Regge poles do not vanish on mass-shell, and the analysis of multiparticle scattering amplitudes would probably signal the existence of other trajectories, but this result may be a signal that we are on the right track.

The representations in parentheses in Table 2 are those obtained by tracing one upper and one lower index in (8). In contrast to theories with lower N , the spin one trace field $Q_{\mu A}^A$ does not have a special symmetry rôle in the $N = 8$ theory²²: the gauge symmetry is $SU(8)$, not $U(8)$. We therefore conjecture^{1,2} that this field does not manifest itself as a low mass bound state, and it is natural³⁰ to extend this conjecture to at least some of the other parenthesized trace representations in Table 2. Neglecting trace representations in general involves breaking at least some supersymmetries. Is there a profound connection between the fact that susy is not manifest at very low energies $O(1)$ GeV, and the absence of any reason for breaking susy if one uses bound states in supermultiplets with $N \leq 6$?

If one tries to identify the observed "elementary" particles with (at least a subset of) states in the supermultiplet (8), one immediately encounters a major problem of chirality. We recalled earlier on that GUTs want chiral fermion representations:

$$R(f_R) \neq R(f_L) \quad (9)$$

Table 2. The $N = 8$ supercurrent multiplet

Helicity	-3/2	-1	-1/2	0	+1/2	+1	+3/2	+2	+5/2
SU(8)	<u>8</u>	<u>63</u>	<u>216</u>	<u>420</u>	<u>504</u>	<u>378</u>	<u>168</u>	<u>36</u>	<u>8</u>
content		(+1)	(+8)	(+28)	(+56)	(+70)	(+56)	(+28)	

in order that $\bar{f}_R f_L$ not be a singlet of the GUT group, in which case every fermion could acquire a GUT-invariant mass $\geq m_X$. We are gratified that the multiplet of Table 2 is chiral, in contrast to the "preons" of Table 1. However, one can have too much of a good thing, and the representation (8) of Table 2 is not real or vector-like under the exact low energy gauge group $SU(3) \times U(1)$, which means that the normal rules of helicity matching would exclude any mass for many of these states¹.

There is also a problem with axial anomalies. Since $SO(N)$ is a safe group, gauging it can never be upset by anomalies, but gauging the effective low-energy $SU(N)$ is not necessarily safe, and the bound state fermions (8) in Table 2 do indeed seem to yield anomalies. Römer³¹ has given a general formula for computing the axial anomaly due to a fermion in an (n,m) representation of the Lorentz group with internal group Casimir $C(n,m)$:

$$C(n,m) \times \frac{(m+1)(n+1)}{6} [m(m+2) - n(n+2)] \frac{g^2}{32\pi^2} F_{\mu\nu}^a F_{\mu\nu}^a \epsilon^{\mu\nu\rho\sigma} \quad (10)$$

which gives the anomalies of Table 3 when it is applied to the traceless fermions in Table 2. Unfortunately, it is not clear what sense to make of this calculation. Although one can set up consistent gauge interactions for spin 3/2 particles, this is only possible if they are massive, and one has no consistent field theoretical formalism at all for the interactions of spin 5/2 particles. Furthermore, Coleman and Grossman³² have given S-matrix arguments that only triangle diagrams containing massless spin 1/2 fermions can contribute to the anomaly. However, it is clear from Table 3 that restricting ourselves to these states does not help us.

Table 3. Anomalies from the bound state supermultiplet

Helicity	3/2	1/2	-1/2	-3/2	-5/2
Group factor	-1	+3	+75	+55	-1
Spin factor	+3?	+1	-1	-3?	-5??
Product	-3?	+3	-75	-165?	5??

We conclude on the basis of these arguments that the full supermultiplet (8) of Table 2 is too chiral. We must find some way either of discarding some helicity states, or else of adding in more supermultiplets. It is easy to convince oneself that the normal rules for giving $SU(3) \times U(1)$ invariant masses to fermions probably require an infinite number of complete supermultiplets^{5,33} and this has been proven under certain restrictive assumptions³⁴. We will turn in the fifth section to a possible scenario for an infinite set of supermultiplets. For now we focus in the alternative of discarding unwanted helicity states from Table 2.

It has been argued^{1,2,35} that the only particles which can consistently have masses $\ll m_p$ are a set forming a renormalizable field theory, which means that they can include no state with helicity > 1 , that the only states with helicity one must be gauge vector bosons, and that the spin 1/2 fermions must be free of anomalies with respect to the low-energy effective gauge group. In addition to the conventional renormalizable interactions there may be non-renormalizable interactions whose couplings are inverse powers of the Planck mass. Such interactions would probably cause baryons to decay³⁶ and neutrinos to have masses³⁷ even independently of GUT interactions, and might help to explain the magnitudes and ratios of the u, d and electron masses³⁸. We have looked^{1,2} for subsets of the states in Table 2 which could form a phenomenologically acceptable low-energy gauge theory. Some popular GUT groups such as $SO(10)$ and E_6 are not subgroups of $SU(8)$ and so cannot be obtained. Also it turns out that any subsets of Table 2 whose fermions are vector-like with respect to $SU(3) \times U(1)$ and free of $SU(6)$ or $SU(7)$ anomalies in fact contain unacceptable completely real fermion representations. This philosophy therefore solves the problem of choosing the GUT group: it must be $SU(5)$. Furthermore, the $N = 8$ bound state supermultiplet is the only representative of the set (8) which has a sufficiently rich fermion spectrum to accommodate three generations^{1,2}. We are therefore led to

$$(N = 8 \text{ ESG}) \rightarrow SU(8) \rightarrow SU(5) \quad (11)$$

as the unique way of embedding grand unification in extended supergravity.

Our first example of a candidate superGUT was obtained² by taking the maximal set of chiral $SU(5)$ anomaly-free left-handed fermions from among the traceless fermions in Table 2:

$$(\underline{45} + \overline{\underline{45}}) + 4(\underline{24}) + 9(\underline{10} + \overline{\underline{10}}) + 3(\underline{\overline{5}} + \underline{5}) + 9(\underline{1}) + 3(\underline{\overline{5}} + \underline{10}) \quad (12)$$

We were encouraged to see that this spectrum contained three generations of chiral $SU(5)$ fermions, in accord with phenomenological prejudices based on the successful prediction³⁹ of the mass of the

bottom quark and the constraint of three (or at the most four) light neutrinos allowed by cosmological nucleosynthesis calculations⁴⁰. It is also amusing that the spectrum (12) contains so many fermions which are real with respect to SU(5) that the reduced SU(5) β function is

$$\beta_{\text{SU}(5)} = +147\frac{1}{2} \text{ instead of } -55 + 4N_G \quad (13)$$

once one gets above all the particle thresholds. The β function (13) is so large that $g_{\text{SU}(5)}$ may become $O(1)$ at energies $E \sim m_P^{1,41}$, as illustrated in Fig. 2. Getting such an increase in $g_{\text{SU}(5)}$ only requires $\langle\beta\rangle \approx +70$ in the energy range between m_X and m_P , so that the trick of Fig. 2 can be played even if some of the fermions in (12) have masses $O(m_P)$. It is heartening that there is no longer a need for non-perturbative supergravity effects to generate a small effective coupling constant $\alpha_{\text{GUT}} \equiv g_{\text{SU}(5)}^2/4\pi = O(10^{-2})$: this is taken care of by the renormalization group evolution between m_X and m_P . We already knew⁴² that the consistency of GUTs required

$$\frac{1}{120} < \alpha_{\text{em}} < \frac{1}{170} \quad (14)$$

and now we have a qualitative explanation how it got there: largely thanks to the existence of many high-mass particles.

Fig. 2 Possible evolution of the GUT coupling constant at energy scales between m_X and m_P .

One can construct many alternative³⁰ superGUTs within the general philosophy outlined above. For example, one might postulate a set of bound state supermultiplets which enables one to use the normal helicity-matching rules to dispose of all unwanted states of low helicity, while tolerating the apparent existence of light states of high helicity. Alternatively, one might take non-maximal subsets of the traceless spin 1/2 fermions in Table 2, or perhaps include some particles from the trace representations. Here I would like to mention³ some alternative superGUTs which accommodate some desirable variations on the GUT theme which have been proposed in the recent literature.

Global $N = 1$ supersymmetric GUTs^{9,10} may "solve" the hierarchy problem thanks to the celebrated "no-renormalization" theorem⁴³. In our original superGUT we broke all supersymmetries in the acts of discarding trace representations and taking the maximal renormalizable subset of the traceless states in Table 2. Alternatively one could keep an $N = 1$ susy in the effective theory, but it turns out³ that the maximal susy GUT does not contain three generations. The maximal three-generation susy GUTs contain the following chiral supermultiplets

$$\begin{aligned} \text{with traces} \quad : \quad & (\underline{45} + \overline{\underline{45}}) + 4(\underline{24}) + 3(\underline{10} + \overline{\underline{10}}) + 3(\underline{5} + \overline{\underline{5}}) + \\ & + 6(\underline{1}) + 3(\underline{5} + \overline{\underline{10}}) \end{aligned} \quad (15a)$$

$$\begin{aligned} \text{without traces:} \quad & (\underline{45} + \overline{\underline{45}}) + 4(\underline{24}) + 2(\underline{10} + \overline{\underline{10}}) + 1(\underline{5} + \overline{\underline{5}}) + \\ & + 3(\underline{1}) + 3(\underline{5} + \overline{\underline{10}}) \end{aligned} \quad (15b)$$

which are sufficient to construct a viable susy GUT. Another recent GUT variation is to solve the strong CP problem by embedding a global, anomalous $U(1)$ symmetry into the theory à la Peccei-Quinn⁴⁴ and breaking it spontaneously at an energy scale $O(m_X)$. Such a theory contains a very light ($m \sim 10^{-8}$ eV), very weakly interacting ($g_{\text{eff}} \sim 10^{-15}$) axion which is essentially unobservable⁴⁵. The minimal $SU(5)$ example of such a GUT contains a complex $\underline{24}$ and two $\underline{5}$'s of Higgses in addition to the conventional $\underline{5} + \overline{\underline{10}}$ generations of fermions. The $SU(8)$ $\underline{420}$ of scalar fields contained in Table 2 are clearly sufficient to accommodate these Higgs fields, and hence a global Peccei-Quinn $U(1)$ symmetry can be imposed on the superGUT if so desired. However, we do not yet see a natural way in which such a $U(1)$ symmetry could emerge without being imposed by hand.

These examples of alternative superGUTs indicate that it is possible to extract a variety of phenomenologically interesting models using the general philosophy outlined previously. However, all these models beg the question of how to dispose of unwanted helicity states from the bound state supermultiplet (8) of Table 2, and we turn to this question in the next section.

It was emphasized in the previous section that the bound state supermultiplet (8) is highly chiral, and one cannot give masses to all the unwanted helicity states using the normal rules of matching states of $|\text{helicity}| \leq J$ with identical internal quantum numbers so as to get a massive state of spin J . So what happens to the unwanted helicity states of $|\text{helicities}| \geq 1, = 1/2$? And a related question: how do the dynamics select which (if any) renormalizable subset of particles from among the states available with $|\text{helicity}| \leq 1/2$? We can identify three distinct possibilities for the unwanted helicity states:

- A - They were never bound in the first place.
- B - Some of them do exist as massless states.
- C - They all exist, and have found "partner" helicity states to "eat" and become massive.

After a few remarks about the first two possibilities we come to the final solution which we prefer³.

A - If they had bound with low mass, the unwanted states would have made the theory unrenormalizable. Perhaps the unknown bound state equations which should bind the preons in these channels develop some related singularities that prevent the binding from occurring? There is no theorem that every composite channel must necessarily exhibit a bound state pole, and one would not discard the quark model if no obvious resonance were found in some obscure $\bar{q}q$ channel. There is indeed a theorem⁴⁶ that one cannot have massless bound states with $|\text{helicity}| > 1$ (or $> 1/2$) in theories with a Lorentz covariant energy momentum tensor (or conserved current). Unfortunately, gravity (and gauge) theories do not satisfy these conditions of Lorentz covariance, nor a fortiori do the ESGs of interest⁴⁷. It therefore seems that the theorem may be too stringent to be useful, which is just as well as it would have forbidden our desired massless composite gauge bosons if it had been applicable !

B - Some unwanted helicity states may exist with zero (or low) mass if they have very small couplings. General theorems⁴⁸ on kinematical singularities in $2 \leftrightarrow 2$ scattering amplitudes imply³ that the coupling constants of massless high-spin particles must contain inverse powers of masses. In our case these can only be m_p^{-n} , and hence such particles must have negligible interactions at low energies $E \ll m_p$. In general a $2 \leftrightarrow 2$ scattering amplitude

$$F(1+2 \leftrightarrow 3+4) = (\sqrt{s})^{\lambda_s} (\sqrt{-t})^{\lambda_t} (\sqrt{-u})^{\lambda_u} \hat{F}(s, t) \quad (16)$$

where F is dimensionless, s , t and u are the conventional Mandelstam variables, $\hat{F}$ is a kinematical singularity-free reduced amplitude, and

$$\begin{aligned}\lambda_s &\equiv |\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4|, & \lambda_t &\equiv |\lambda_1 - \lambda_2 - \lambda_3 + \lambda_4| \\ \lambda_u &\equiv |\lambda_1 - \lambda_2 + \lambda_3 - \lambda_4|\end{aligned}\quad (17)$$

The external kinematical factors in (16) clearly mean that $\hat{F}$ contains inverse powers of masses in general. Consider for example $(1/2, -1/2) \leftrightarrow (3/2, 1/2)$ scattering: using (17) one finds that

$$F = (us)\hat{F} \quad (18)$$

and if one goes to a direct channel (photon or gluon?) pole $1/s$ in $\hat{F}$ one sees that its coefficient, and hence the $(3/2, 1/2, \gamma$ or $g)$ coupling, must be $O(m_P^{-2})$. This line of argument works directly for all massless states of $|\text{helicity}| > 1$, and also for massless states of $|\text{helicity}| = 1$ unless they have the specific gauge form of couplings³. By this reasoning one can argue that all unwanted states of $|\text{helicity}| \geq 1$ would have negligible couplings at low energies, but one cannot wish away unwanted $|\text{helicity}| = 1/2$ states in the same way, and so this is only a partial solution to our problem.

Even if the direct interactions of unwanted helicity states are undetectable at low energies, one might worry³ that they could have an indirect cosmological effect. If they were once in thermal equilibrium and then decoupled because of their weakening interactions as the temperature fell, they would still be present in the Universe during cosmological nucleosynthesis. Their energy-momentum would necessarily accelerate the primordial expansion rate² and might thereby enhance the cosmological ${}^4\text{He}$ production rate above astrophysically acceptable limits. The amount of speed-up is⁴⁹

$$\xi^2 - 1 = \frac{4}{43} (N_B + \frac{7}{8} N_F) \left(\frac{43}{4N_D} \right)^{4/3} \quad (19)$$

where N_B and N_F are the numbers of species of decoupled bosons and fermions, and N_D is the total number of residual interacting degrees of freedom at the epoch of decoupling. From primordial ${}^4\text{He}$ nucleosynthesis one deduces

$$\xi^2 - 1 < 0.15 \quad (20)$$

which is satisfied when (here we correct Ref. 3)

$$(N_B + \frac{7}{8} N_F) < 0.07 N_D^{4/3} \quad (21)$$

This is because a large value of N_D degrades (19) the energy of every unseen boson and fermion sufficiently for their total contribution to the Universe's energy density to be negligible. The condition (21) is satisfied for all the unwanted N_B and N_F if one puts into N_D all the desired low-mass states included for example in the candidate superGUT (12) with its associated Higgs scalars. Therefore the presence of all the unwanted helicity states in Table 2 as weakly-interacting states of low mass is compatible with the nucleosynthesis constraints, though it should again be emphasized that we do not know how to guarantee that all the unwanted $|\text{helicity}| = 1/2$ states have negligibly weak interactions at low energies.

C - Maybe all the unwanted helicity states have large masses ? To arrange this within the normal helicity-matching rules we need extra helicity states to "eat" the unwanted ones, and probably an infinite set of bound state supermultiplets. We propose³ that this infinite set may be obtained from unitary representations of the global non-compact $E_{7(+7)}$ symmetry in the physical spectrum. As a word of caution it should be mentioned that the gauged version¹⁵ of the $N = 8$ ESG has a local $SO(8) \times SU(8)$ symmetry, but not a compact $E_{7(+7)}$ symmetry. Therefore this proposal breaks down if it turns out that $SO(8)$ is gauged. Why should one think that the physical spectrum might contain unitary representations of $E_{7(+7)}$? It has already been mentioned that in two-dimensional CP^{N-1} models the physical spectrum contains²⁵ unitary representations of the global $SU(N)$ symmetry, and the same is also true in three-dimensional models²⁶. There are also some indications⁵⁰ of an infinite spectrum in two-dimensional models based on a global non-compact $SO(N,1)$ group, redolent of a unitary representation. So if $E_{7(+7)}$ is present in the Lagrangian it may well show up in the physical spectrum.

It seems³ that the relevant $E_{7(+7)}$ representations may have the non-linear structure

$$\{\lambda_{\max}, R\} \times \sum_{n=0}^{\infty} [\phi_{ABCD}]^{(n)} \quad (22)$$

where $\{\lambda_{\max}, R\}$ denotes an $SU(8)$ supermultiplet (e.g., $\{3/2, \bar{8}\}$ in the case of Table 2), the ϕ_{ABCD} are the 70-dimensional scalars of Table 1, and the n -fold product is completely symmetric. If $SU(8)$ is broken down to $SU(6)$ or a subgroup thereof, as we proposed in the fourth section, it turns out³ that at every value of helicity present in the original supermultiplet the representation (22) contains an infinite number of examples of every representation of the unbroken subgroup. This enables us to find partner helicity states for every unwanted helicity state $\hat{R}(\lambda)$, for if we look at any λ' : $|\lambda'| \leq \lambda$, we can always find in (22) another example of the same representation $\hat{R}$ with helicity λ' . The representations of $E_{7(+7)}$ contained in (22) with any given helicity are not irreducible: this need not surprise us since the $SU(8)$ representations at any given helicity in the supermultiplet (8) of Table 2 are

themselves not reducible. To proceed further we must understand better the appropriate infinite-dimensional unitary representations⁵¹ of $E_{7(+7)}$. We expect that the reducibility is forced upon us by susy, in the same way as in Table 2. To see how this comes about, we need to know how to combine susy with the global non-compact symmetry into a more complete algebraic structure, and then look for its representations. We have started^{3,52} by looking at the $N = 4$ theory which has $SU(4) \times SU(1,1)$ global and $U(4)$ local symmetries. It contains complex scalar fields z in the coset space $SU(1,1)/U(1)$, on which the $SU(1,1)$ actions are

$$L_+ z = i, \quad L_- z = iz^2, \quad L_0 z = -iz \quad (23)$$

The $SU(1,1)$ transformations do not commute with susy transformations:

$$[L_+, Q_L] = -i\bar{z}Q_L, \quad [L_-, Q_L] = -izQ_L, \quad [L_0, Q_L] = iQ_L \quad (24)$$

which suggests that the full algebra contains generators of the form

$$(f_L(\bar{z}) + g_L(\bar{z})z) Q_L, \quad (f_R(z) + g_R(z)\bar{z}) \bar{Q}_R \quad (25)$$

This structure begins to look like a graded version of a Kac-Moody⁵³ algebra

$$[Q_a^u, Q_b^m] = i f_{abc} Q_c^{n+m}; \quad n, m \in \mathbb{Z} \quad (26a)$$

obtained from a conventional Lie algebra of local charges

$$[Q_a(\theta), Q_b(\theta')] = f_{abc} Q_c(\theta) (\theta - \theta') \quad 0 \leq \theta \leq 2\pi \quad (26b)$$

by making the Fourier transform

$$Q_a^n \equiv \int_0^{2\pi} d\theta e^{in\theta} Q_a(\theta) \quad (27)$$

It will be interesting⁵² to see whether "graded Kac-Moody algebras" exist in ESGs, and then see whether they play any important physical rôle.

Even if the helicity-matching works as we have proposed above, we have no indication why which preferred finite subset of helicity states should be left massless by the dynamics. However, at least the mechanism discussed here indicates that it is not impossible to imagine zapping all the unwanted helicity states up to large masses.

The central problem in these attempts to embed grand unification in extended supergravity using composite fields remains the disposal of the unwanted helicity states. There is clearly a need for more detailed dynamical studies like those being done to look at confinement in gauge theories. We need to know how (if at all) the dynamics selects which renormalizable effective theory at low energies $E \ll m_P$.

We must figure out whether it is necessary or useful to gauge the internal $SO(N)$ symmetry, and also the mirror question whether the possible non-compact global symmetries play an important rôle. The connection with conventional GUTs is still very obscure, and in particular the question of which supersymmetries to break at what energy scales. All at 10^{19} GeV? Retain an $N = 1$ susy down to $O(10^2)$ GeV to "solve" the hierarchy problem in a susy GUT? At what scale⁵⁴ is susy no longer local ?

Clearly there is food for thought, but the stakes are high. To paraphrase Hawking⁵⁵, perhaps we have the ultimate theory, and all we have to do is learn how to solve it.

ACKNOWLEDGEMENTS

In addition to a big thank you for my principal collaborators on this subject, Mary K. Gaillard and Bruno Zumino, I also wish to acknowledge valuable insights gleaned from Murat Günaydin, A. Kabelschacht and D.V. Nanopoulos.

REFERENCES

1. J. Ellis, M.K. Gaillard, L. Maiani and B. Zumino, "Unification of the Fundamental Particle Interactions", ed. by S. Ferrara, J. Ellis and P. Van Nieuwenhuizen, (Plenum Press, N.Y., 1980), p. 69.
2. J. Ellis, M.K. Gaillard and B. Zumino, Phys. Lett. 94B:343 (1980).
3. J. Ellis, M.K. Gaillard and B. Zumino, LAPP preprint TH-44/CERN TH.3152 (1981), to appear in Acta Physics Polonica.
4. J. Ellis, "First Workshop on Grand Unification", ed. by P. Frampton, S.L. Glashow and A. Yildiz, (Math. Sci. Press, Brookline, 1980), p. 287.
5. B. Zumino, Proc. 1980 Madison Int. Conf. on High Energy Physics, ed. by L. Durand and L.G. Pondrom, (A.I.P., N.Y., 1981), p. 964; and "Superspace and Supergravity", ed. by S.W. Hawking and M. Roček, (Cambridge University Press, Cambridge, 1981), p. 423.

6. M.K. Gaillard, talk presented at the Heisenberg Symposium, München 1981, LBL preprint LBL-13371 (1981).
7. J. Ellis, "Gauge Theories and Experiments at High Energies", ed. by K.C. Bowler and D.G. Sutherland, (Scottish Universities Summer School in Physics, Edinburgh, 1981), p. 201 and Lectures presented at the Les Houches Summer School, August 1981, LAPP preprint TH-48/CERN TH.3174 (1981), to appear in the proceedings;
P. Langacker, Phys. Rep. 72C:185 (1981); and
Talk presented at the Int. Symp. on Lepton and Photon Interactions at High Energies, Bonn 1981, Univ. of Pennsylvania preprint UPR-0180 T (1981).
8. Y.A. Gol'fand and E.O. Likhtman, J.E.T.P. Lett. 13:323 (1971);
D. Volkov and V.P. Akulov, Phys. Lett. 46B:109 (1973);
J. Wess and B. Zumino, Nucl. Phys. B70:39 (1974).
9. D.V. Nanopoulos, these proceedings.
10. S. Dimopoulos and H. Georgi, Harvard University preprint HUTP-81/A022 (1981);
N. Sakai, Tohoku University preprint TU/81/225 (1981).
11. For a review, see
P. van Nieuwenhuizen, Phys. Rep. 68C:189 (1981).
12. L.V. Avdeev, O.V. Tarasov and A.A. Vladimirov, Phys. Lett. 96B:94 (1980);
M.T. Grisaru, M. Roček and W. Siegel, Phys. Rev. Lett. 45:1063 (1980);
L. Caswell and D. Zanon, Phys. Lett. 100B:152 (1980).
13. D.Z. Freedman and A. Das, Nucl. Phys. B120:221 (1977);
E. Cremmer and J. Scherk (unpublished).
14. B. de Wit and H. Nicolai, Nucl. Phys. B188:98 (1981); see also these proceedings.
15. B. de Wit and H. Nicolai, CERN preprint TH.3183 (1981).
16. T.L. Curtright, Phys. Lett. 102B:17 (1981)
17. B. Zumino, Nucl. Phys. B89:535 (1975).
18. S.W. Hawking, Talk presented at the Heisenberg Symposium, München 1981; and
S.W. Hawking, D.N. Page and C.N. Pope, Phys. Lett. 86B:175 (1979) and Nucl. Phys. B170 (FS1):283 (1980).
19. M. Gell-Mann, Talk presented at the 1977 Washington Meeting of the American Physical Society (unpublished).
20. S. Ferrara and B. Zumino, Phys. Lett. 86B:279 (1979) and references therein.
21. See also
T.L. Curtright and P.G.O. Freund, "Supergravity", ed. by P. Van Nieuwenhuizen and D.Z. Freedman, (North Holland, Amsterdam, 1979), p. 167.
22. E. Cremmer and B. Julia, Nucl. Phys. B159:141 (1979).
23. A. D'Adda, P. Di Vecchia and M. Lüscher, Nucl. Phys. B146:63 (1978) and B152:129 (1979);
E. Witten, Nucl. Phys. B149:285 (1979).
24. J. Ellis, Nucl. Phys. B21:217 (1970).

25. H. Haber, I. Hinchliffe and E. Rabinovici, Nucl. Phys. B172:458 (1980).
26. E.R. Nissimov and S.J. Pacheva, Comptes Rendus de l'Académie Bulgare des Sciences 32:1475 (1979); Lett. Math. Phys. 5:67, 333 (1980).
27. D. Amati, R. Barbieri, A.C. Davis and G. Veneziano, Phys. Lett. 102B:408 (1981); see
A.C. Davis, these proceedings.
28. S. Weinberg, "Lectures on Elementary Particles and Quantum Field Theory", ed. by S. Deser, M. Grisaru and H. Pendleton (M.I.T. Press, Cambridge, 1970).
29. M.T. Grisaru and H.J. Schnitzer, Brandeis University preprint (1981); see
M.T. Grisaru, these proceedings.
30. See however
J.-P. Derendinger, S. Ferrara and C.A. Savoy, Nucl. Phys. B188:77 (1981) and
C.A. Savoy, these proceedings;
J.E. Kim and H.S. Song, Seoul National University preprint (1981).
31. H. Römer, Phys. Lett. 83B:172 (1979).
32. S. Coleman and B. Grossman, private communication (1981).
33. M. Gell-Mann, unpublished (1980).
34. P.H. Frampton, Phys. Rev. Lett. 46:881 (1981)
35. M. Veltman, Acta Physica Polonica B12:437 (1981).
36. Ya.B. Zeldovich, Phys. Lett. 59A:254 (1976).
37. R. Barbieri, J. Ellis and M.K. Gaillard, Phys. Lett. 90B:249 (1980).
38. J. Ellis and M.K. Gaillard, Phys. Lett. 88B:315 (1979).
39. D.V. Nanopoulos and D.A. Ross, CERN preprint TH-3173 (1981) and references therein.
40. G. Steigman, Bartol Research Foundation preprint BA-81-20 (1981) and references therein; also these proceedings.
41. M. Glück and E. Reya, Phys. Lett. 105B:30 (1981).
42. J. Ellis and D.V. Nanopoulos, Nature 292:436 (1981).
43. J. Wess and B. Zumino, Phys. Lett. 49B:52 (1974);
J. Iliopoulos and B. Zumino, Nucl. Phys. B76:310 (1974);
S. Ferrara, J. Iliopoulos and B. Zumino, Nucl. Phys. B77:413 (1974).
44. R.D. Peccei and H.R. Quinn, Phys. Rev. Lett. 38:1440 (1977) and
Phys. Rev. D16:1791 (1977).
45. M. Dine, W. Fischler and M. Srednicki, Phys. Lett. 104B:199 (1981);
M.B. Wise, H. Georgi and S.L. Glashow, Phys. Rev. Lett. 47:402 (1981).
46. S. Weinberg and E. Witten, Phys. Lett. 96B:59 (1980).
47. M.K. Gaillard and B. Zumino, LAPP preprint TH-37 (1981).
48. J.P. Ader, M. Capdeville and H. Navelet, Nuov. Cim. 56A:315 (1968).

49. G. Steigman, K.A. Olive and D.N. Schramm, Phys. Rev. Lett. 43:239 (1979);
D.A. Olive, D.N. Schramm and G. Steigman, Nucl. Phys. B180: (FS2),497 (1981).
50. E. Rabinovici, unpublished (1980).
51. M. Günaydin and C. Saçlioğlu, Bonn University preprint Bonn HE-81-11 (1981).
52. J. Ellis, M.K. Gaillard, M. Günaydin and B. Zumino, dormant interest (1981).
53. V. Kac, Math. U.S.S.R. Izvestiya Ser. Math. 32:1271 (1968);
R. Moody, Bull. Am. Math. Soc. 73:217 (1967) and J. of Algebra 10:211 (1968).
54. R. Barbieri, S. Ferrara and D.V. Nanopoulos, CERN preprint TH.3159 (1981).
55. S.W. Hawking, Lucasian Inaugural Lecture "Is the end in sight for theoretical physics?", (Cambridge University Press, Cambridge, 1980).