A UNIFIED GAUGE AND GRAVITY THEORY
WITH ONLY MATTER FIELDS

D. Amati and G. Veneziano

CERN -- Geneva

ABSTRACT

We propose a purely fermionic action with gauge and general relativistic invariances. This implies a unified treatment of gravity and gauge theories without elementary metric tensor and gauge boson fields. At the quantum level this scale invariant theory generates, as vacuum properties, both a metric and a scale Λ which becomes related to the Planck mass. The analysis of the spectrum displays, besides the original fermions, massless composite vierbein and gauge bosons, as well as a set of states with masses of order Λ . In a low energy regime in which these heavy modes are not excited, the light sector is shown to be governed by a phenomenological action which coincides with the conventional gravity-gauge matter theory without cosmological term and with a Newton constant and gauge couplings determined by Λ . For increasing energies, the gauge interactions are predicted to grow towards their merging with gravity at Λ . Approaching Λ , the theory differs substantially from the conventional one, not even allowing a definition of a space-time metric and providing hints for a milder ultra-violet behaviour.

1. - INTRODUCTION AND OUTLINE OF THE PAPER

Within our present understanding, it is conceivable that all elementary interactions can be described by gauge theories. This common structure opened the way towards unification, first with the successful electroweak theory of Glashow-Weinberg-Salam and then by its more speculative merging with the $SU(3)$ theory of strong interactions in the so-called grand unified theories¹⁾ (GUT). The energy scale at which this GUT unification takes place, M_{GUT} , is very large, ranging between 10^{14} and 10^{17} GeV depending on the details of the actual theoretical scheme.

In spite of their doubtless appeal and partial success, GUTs are still facing hard, unresolved problems (origin of families, hierarchy problem, etc.). Even more frustrating has been the impossibility so far to include gravity in a unified scheme. Not only is gravity a gauge theory (of a space-time rather than of an internal symmetry), it is also expected to become strong and hence non-negligible at a scale (of order 10^{19} GeV) which is embarrassingly close to M_{GUT} itself.

On the other hand, whereas in usual GUTs electroweak and strong interactions are renormalizable interactions becoming equally weak at M_{GUT} , the gravitational interaction shows such a steady increase of strength with energy that its meaning as a quantum theory is itself doubtful. This difference is of course an obvious obstacle for unifying gravity with the other interactions. Another more formal difference is that, in the conventional approach, the gauge connection is considered as a fundamental field whereas in gravity it is given in terms of the fundamental metric or vierbein fields.

In this paper²⁾ we attempt to propose and discuss a theoretical framework in which all gauge symmetries are introduced in the same way, i.e., in terms of fundamental fermionic fields only. Gauge bosons, as well as gravitons, will be composite and their interactions will be automatically unified (and strong) at the same scale, the inverse radius of compositeness.

This unconventional unification scheme, which extends the one proposed in Ref. 3) in that it includes gravity, leads to the usual framework in a "low energy" regime that we shall carefully define later. This is because, as we move down from the unification mass, the effective low energy interactions will be the ones dictated by gauge invariance and the principle of equivalence, by the presence of the corresponding dynamically generated gauge bosons and by dimensional arguments. Conventional gauge theories and gravity appear then as phenomenological low energy approximations. Away from that regime, our theory will differ from

the conventional ones not even allowing, for instance, an unambiguous definition of a space-time metric. This last point clearly indicates the difference between our approach and those which try to generate quantum gravity from matter fluctuations in a background metric field⁴⁾. Let us remark that, as discussed in Ref. 3), the vector gauge theories appearing in the low energy regime have the property of going towards unification at higher energies in a crescendo. This sort of asymptotically non-free GUT has been proposed and shown to be satisfactory on pure phenomenological grounds in Ref. 5) and more recently by Cabibbo and Farrar⁶⁾. For us this sort of unification is an automatic consequence of gauge field compositeness.

In the preceding discussion we have taken the attitude that the gauge symmetries observed at low energies ($\lesssim 100$ GeV) are those relevant for our unification procedure.

One could take the alternative attitude that the gauge symmetry that unifies with gravity at M_{Planck} is the GUT interaction itself. In this case GUT would appear as an effective "low energy" theory valid at $E \approx M_{\text{GUT}} \ll M_{\text{Pl}}$ which could further break at lower energies down to $SU(3) \otimes SU(2) \otimes U(1)$ following one of the conventional GUT schemes. This scheme would necessitate several GUT families above M_{GUT} in order to reach a small unified gauge coupling at M_{GUT} . This idea is perhaps less appealing in our view, but shows the possibility of the co-existence of two very different unification mechanisms.

This paper is organized in the following way. In Section 2 we shall introduce our construction of theories invariant under a set of gauge transformations on the basis of only fundamental fermions. To avoid internal indices (colour, flavour, etc.) we shall limit our treatment to Lorentz and $U(1)$ gauge invariances (i.e., gravitation and QED), discussing only briefly in Section 4 the inclusion of other gauge or internal symmetries. After having written the invariant action, we shall discuss our strategy to analyze it through the introduction of a suitable set of auxiliary fields. In Section 3 we discuss the vacuum properties which are associated with homogeneous classical solutions. They are shown to generate a scale Λ through a spontaneous breaking of local Lorentz and general co-ordinate transformations. This scale, which appears as a vacuum property, represents the only dimensional quantity of the theory and it is in terms of it that all masses and energies will be measured.

In Section 4 we compute the quadratic fluctuations around the previous stationary field configuration up to second order derivatives. This will allow us to recognize induced mass and kinetic terms of the eigenmodes. A light sector

(massless bosons and arbitrarily light fermions) is identified, together with a heavy sector (masses of order Λ). If the heavy eigenmodes are not excited - as expected for all energies much smaller than Λ - the remaining light modes are shown to represent a spinor, a photon and a graviton described by an effective action which is the usual Einstein-Dirac-Maxwell action and where the Newton constant and the electric charge are given in terms of Λ . This relation can be described by saying that Λ is to be identified both with the Planck mass and the Landau pole position. We also extend this result to include further gauge symmetries.

In Section 5, after showing that our procedure coincides with a $1/N$ expansion, we discuss the high energy excitations. In the ultra-violet regime where their contribution is relevant, the theory differs eventually from the conventional one. Even a space-time metric cannot be defined, showing how, in our framework, the geometrical approach to gravity loses validity for distances approaching the Planck length. We also briefly discuss why this ultra-violet regime, appearing also in loop contributions to low energy processes, may have less pernicious consequences than in the usual Einstein theory which is perceived by us as a phenomenological low energy theory at the tree level.

Our paper may thus be considered as a proposal of an alternative, pregeometrical theory of general relativity which, besides being prepared to unify with gauge theories (as QED), has a chance to evade the ultra-violet pathologies of the conventional theory.

2. - THE INVARIANT ACTION

a) As discussed in the introduction, we shall limit our discussion in this section to general relativity and QED.

We begin by writing, in terms of a single spinor field^{*)} ψ , an action invariant under local $U(1)$ and Lorentz transformations. In their infinitesimal form these read^{**)}:

$$\psi(x) \longrightarrow (1 - i\alpha(x)) \psi(x) \quad ; \quad U(1) \quad (2.1a)$$

^{*)} We stress that the definition of a spinor is related to an $O(3,1)$ invariance at each point of space-time and does not make reference to a metric. Of course, $O(3,1)$ admits an invariant tensor $\eta_{\alpha\beta} = \text{diag}(1, -1, -1, -1)$, but this has no a priori implication on distances between space-time points.

^{**)} We adopt the normalizations and conventions of J.D. Bjorken and S.D. Drell, Relativistic Quantum Fields, McGraw-Hill (1965). The usual contraction of Lorentz indices through $\eta_{\alpha\beta}$ is tacitly understood.

$$\psi(x) \longrightarrow \left(1 - \frac{i}{4} \sigma_{\beta\gamma} \omega^{\beta\gamma}(x)\right) \psi(x); \quad O(3,1) \quad (2.1b)$$

Besides, we shall write the action as a space-time integral of a scalar density thus enforcing invariance under general co-ordinate transformations $x_\mu \rightarrow x'_\mu(x)$. Our first task will be to construct a covariant derivative.

Defining

$$D_\mu = \partial_\mu + A_\mu + \sigma^{\beta\gamma} A_{\mu,\beta\gamma}, \quad (2.2)$$

$$A_\mu = \frac{1}{8} \text{Tr} (H^{-1} \vec{H}_\mu - \vec{H}_\mu H^{-1}) \quad (2.3)$$

$$A_{\mu,\beta\gamma} = \frac{1}{16} \text{Tr} (\sigma_{\beta\gamma} H^{-1} \vec{H}_\mu - \sigma_{\beta\gamma} \vec{H}_\mu H^{-1})$$

and the Dirac index matrices

$$H = \psi \bar{\psi}, \quad \vec{H}_\mu = \psi \overleftrightarrow{\partial}_\mu \bar{\psi} \quad (2.4)$$

it is easy to check that D_μ is a good covariant derivative under the transformations (2.1). This means that $D_\mu \psi(x)$ transforms as $\psi(x)$ in (2.1).

As a consequence, the composite operator

$$W_{\alpha\mu}(x) = \bar{\psi}(x) \frac{i}{2} [\gamma_\alpha, D_\mu] \psi(x) \equiv \frac{i}{2} \bar{\psi}(x) \gamma_\alpha \vec{D}_\mu \psi(x) - \frac{i}{2} \bar{\psi}(x) \overleftarrow{D}_\mu \gamma_\alpha \psi(x) \quad (2.5)$$

transforms as a usual vierbein, i.e., as a set of four four-vectors (one for each index α) under general co-ordinate transformations and, for each μ , as a vector under local Lorentz transformations. Therefore $\det W_{\alpha\mu}$, where the determinant refers to the 4×4 matrix of indices $\alpha \mu$, is a scalar density. This can still be multiplied by a scalar invariant which, due to the lack of a space-time metric tensor to saturate space-time indices, can only be ^{*)} an arbitrary scalar function $V(H)$. We are thus led to define

*) This is true if we require the scalar, and therefore our action density, to be a finite order polynomial in the derivatives.

$$A_f = \int d^4x (\det W_{\alpha\mu}) V(H) \quad (2.6)$$

as the most general relativistic and $U(1)$ invariant action of our single spinor system.

Let us notice that even if non-polynomial in ψ and $\bar{\psi}$ [due to H^{-1} in Eqs. (2.3)] A_f is polynomial of fourth order in the derivatives and only first order in the derivative with respect to any space-time co-ordinate. The four derivatives compensate the dimension of d^4x so that ψ is dimensionless and so is H . Thus $V(H)$ and therefore A_f are free of any dimensional parameters. We shall see that the arbitrariness of $V(H)$ will not jeopardize the possibility of analyzing the consequences of (2.6). $V(H)$ will indeed enter into the condition for the minimum of A_f in terms of H . As we shall see, the existence of a minimum will be important while its location (arbitrary for arbitrary V) will be irrelevant. Further details of V will influence the spectrum of the theory but in a way which will not affect its basic physical aspects.

We shall be interested in extending the single spinor case discussed up to now by introducing possible replicas, i.e., different fields ψ_i transforming analogously under the local transformation (2.1). In so doing we may assign a different weight K_i to the contribution of the i^{th} replica to the definition (2.4) of H that enters into the definition (2.2), (2.3) of the covariant derivative. This introduces therefore a set of dimensionless parameters K_i into the invariant action (2.6) where now

$$W_{\alpha\mu} = \frac{i}{2} \sum_i \bar{\psi}_i [\gamma_\alpha, D_\mu] \psi_i \quad (2.7)$$

D_μ being given by (2.3) in terms of

$$H = \sum_i K_i \psi_i \bar{\psi}_i, \quad \overleftrightarrow{H}_\mu = \sum_i K_i \psi_i \overleftrightarrow{\partial}_\mu \bar{\psi}_i \quad (2.8)$$

We shall actually use this introduction of replicas in two ways. One will be to use N equal weights ($K_i=1, i=1, \dots, N$) in order to define a $1/N$ expansion. The other will be to enforce a Pauli-Villars type of regularization.

b) The quantum theory we are considering implies the functional integration over the fundamental fields $\psi_i, \bar{\psi}_j$ of the exponential of the action A_f of Eq. (2.6) to which source terms $\bar{J}_i \psi_i + j_i \bar{\psi}_i$ are tacitly added. This is obviously a complex task because of the high non-linearity of A_f . Our strategy is to introduce as many auxiliary bosonic fields as needed in order to render A bilinear in $\psi_i, \bar{\psi}_i$ and to perform next the fermion functional integral. This leads us to a new action which, though equivalent to the original one, is written only in terms of the auxiliary fields.

For this purpose we introduce four functional Lagrange multipliers $\xi^{\mu\alpha}, \lambda^{\mu,\beta\gamma}, \lambda^\mu$ and ϕ needed to enforce the definitions of $W_{\alpha\mu}, A_{\mu,\beta\gamma}, A_\mu$ and H as given in Eqs. (2.3), (2.7) and (2.8)*). The action written in terms of the original fermions and of the auxiliary bosonic fields, reads:

$$A_{f,b} = \int d^4x \det W \left\{ [V(H) - \xi^{\mu\alpha} W_{\alpha\mu} - \lambda^{\mu,\beta\gamma} A_{\mu,\beta\gamma} - \lambda^\mu A_\mu + \text{Tr}(H\phi)] + \right. \\ \left. + \sum_i \bar{\psi}_i \left(\xi^{\mu\alpha} \frac{i}{2} [\gamma_\alpha, (\partial_\mu + A_\mu + \sigma_{\beta\gamma} A_{\mu,\beta\gamma})] - K_i \phi + \right. \right. \\ \left. \left. + \frac{K_i}{4} H^{-1} \overleftrightarrow{\partial}_\mu \lambda^\mu + \frac{K_i}{16} (\sigma_{\beta\gamma} H^{-1} \overleftrightarrow{\partial}_\mu - H^{-1} \sigma_{\beta\gamma} \overleftrightarrow{\partial}_\mu) \lambda^{\mu,\beta\gamma} \right) \psi_i \right\} \quad (2.9)$$

The integration over $\psi_i, \bar{\psi}_i$ is now formally straightforward. However, in order to give meaning to the trlog obtained in that way, an ultra-violet regularization procedure must be introduced. It is crucial to do this while respecting all the invariances of the theory. We shall adopt a Pauli-Villars (PV) prescription by choosing replicas characterized by parameters K_i in Eq. (2.8) and by PV parameters C_i satisfying

$$\sum_i C_i K_i^{2n} = 0, \quad n = 0, 1, \dots \quad (2.10)$$

*) Notice that ϕ and H are dimensionless matrices in Dirac space, that $W_{\alpha\beta}, A_{\mu,\beta\gamma}, A_\mu$ have dimensions of a mass and that $\xi^{\mu\alpha}, \lambda^{\mu,\beta\gamma}$ and λ^μ have dimension m^{-1} .

extending the usual PV conditions^{*)}. In Appendix A we give an explicit construction of a PV set (with an infinite number of recurrences) satisfying (2.10) with $C_i = \pm 1$. We recall that, as usual, negative C_i represent PV ghosts. We shall discuss later the physical relevance of the regularization parameters.

We stress that in order to regularize the integration over the fermion fields we had to introduce into the theory a set of dimensionless parameters satisfying a set of conditions. The important matter, nevertheless, is that all invariances of the theory have been preserved. In particular, our regularization has not broken conformal invariance unlike what inevitably happens when mass and kinetic terms in the action have the same degree of homogeneity in $\psi, \bar{\psi}$.

The condition (2.10) allows us to give a meaning to the formal expression arising from integration over the fermion fields and hence to the effective bosonic action A_b defined by

$$\exp A_b \equiv \int \prod_i \mathcal{D}\psi_i \mathcal{D}\bar{\psi}_i \exp A_{f,b} \quad (2.11)$$

Using (2.9) for $A_{f,b}$ we immediately obtain

$$\begin{aligned} A_b = & \int d^4x \det W \left[V(H) - \xi^{\mu\alpha} \lambda_{\alpha\mu} - \lambda^{\mu,\beta\gamma} A_{\mu,\beta\gamma} - \lambda^\mu A_\mu + \text{Tr}(H\phi) \right] + \\ & + \sum_i c_i \text{Tr} \log \left\{ \xi^{\mu\alpha} \frac{i}{2} [\gamma_\alpha (\partial_\mu + A_\mu + \sigma_{\beta\gamma} A_{\mu,\beta\gamma})] - k_i \phi + \right. \\ & \left. + \frac{k_i}{4} H^{-1} \overleftrightarrow{\partial}_\mu \lambda^\mu + \frac{k_i}{16} (\sigma_{\beta\gamma} H^{-1} \overleftrightarrow{\partial}_\mu - H^{-1} \sigma_{\beta\gamma} \overleftrightarrow{\partial}_\mu) \lambda^{\mu,\beta\gamma} \right\} \end{aligned} \quad (2.12)$$

^{*)} The reason why our regularization contains an infinite series of poles is due to the different weight each PV has in H and H_μ , on one side, and in W and A on the other [cf. Eqs. (2.3), (2.7) and (2.8)]. This leads to i dependent couplings of λ^μ , $\lambda^{\mu,\alpha\beta}$ and ϕ . These are the couplings that break chirality and generate loops whose regularization necessitates the infinite number of PV regulators. An alternative approach would have been to define all composite boson operators with equal weight and to introduce PV "mass terms" by hand. Although a finite number of conditions (2.10), and therefore a finite number of regulators, would then suffice we would not have been able to protect the original fermion from getting a large mass from the v.e.v. of H .

We have therefore translated our initial action, written in terms of only fermions, into the action (2.12) written in terms of only bosons. We shall see that this language will be better suited for analyzing the spectrum of the theory. Before doing that, let us stress that among the variety of boson fields we are left with, none has the properties of a metric field. We have two fields $W_{\alpha\mu}$ and $\xi^{\mu\alpha}$ which could be associated with lower and upper index vierbeins and could be used to define space-time metric tensors such as $W_{\alpha\mu} W_{\alpha\nu}$ and $\xi^{\mu\alpha} \xi^{\nu\alpha}$. However, these two tensors cannot be identified with $g_{\mu\nu}$ and $g^{\mu\nu}$ unless one is the inverse of the other. We thus see that the possibility of defining a metric needs the identification of W with ξ^{-1} up to a proportionality constant. This is not of course the case in our theory since W and ξ are independent fields. We shall be able nevertheless to determine an asymptotic kinematical regime in which that identification will be correct, thus allowing the definition of a space-time metric as an approximate concept.

3. - VACUUM PROPERTIES AS CLASSICAL SOLUTIONS

We shall attempt the analysis of our action (2.12) written in terms of auxiliary bosonic fields by a semi-classical treatment, which consists of finding stationary points of the action and expanding around them. We note that this procedure becomes justified as a $1/N$ expansion if N identical replicas of each fermion plus regulator families are considered. A factor N then multiplies the trlog term of (2.12) playing the role of the $1/N$ factor of the other terms in the action. We shall verify in Section 5 that our semiclassical (loop) expansion does indeed generate a series in $1/N$. We notice at this point that the measure in the bosonic functional integral, which we have systematically ignored, would contribute to the effective action with terms with no $1/N$ or N factor, therefore not contributing to our semiclassical expansion.

Let us now discuss the stationarity condition which implies the vanishing of the first derivatives of A_b with respect to all auxiliary fields. The only first order functional derivative not involving the trlog in (2.11) is

$$\frac{\delta A_b}{\delta W_{\alpha\mu}} = \det W \cdot \left\{ g^{\mu\alpha} (W^{-1})^{\mu\alpha} (V(H)) - \xi^{\mu\alpha} W_{\alpha\mu} - \lambda^\mu A_\mu - \lambda^{\mu\beta\gamma} A_{\mu,\beta\gamma} + \text{Tr}(\phi H) \right\} \quad (3.1)$$

Its vanishing gives a proportionality between $\langle \xi \rangle$ and $\langle W \rangle^{-1}$ so that neither $\langle \xi \rangle$ nor $\langle W \rangle$ can be zero, implying a spontaneous breaking of local Lorentz invariance. In order to discuss further the stationarity conditions we encounter the difficulty of having to evaluate derivatives of the tr log operator. This operator is a functional of the classical field configurations. Therefore the classical equations will themselves depend on the class of functions among which one looks for solutions. Let us remark that these equations also contain the sources, should we have explicitly included them in our treatment.

We will be interested in a vacuum that preserves global Poincaré invariance. We therefore will look for classical homogeneous solutions (i.e., space-time independent) in the absence of external sources. In this case the evaluation of the tr log expression and its derivatives is simple.

Let us first discuss the stationarity condition on H . The presence of H^{-1} in A_b implies that $H \sim 0$ configurations should be strongly damped. Indeed, the vanishing of $\delta A_b / \delta H$ equates $V'(H)$ with an expression that, depending on the fields, becomes singular for $H \rightarrow 0$. We assume that this equation, which has a well-defined meaning for $H \neq 0$, may be satisfied by some value of H . This may eventually imply some condition on $V(H)$ which we assume to be met^{*}). By taking into account Lorentz invariance we thus write

$$\langle H \rangle = v \mathbb{1} \quad (3.2)$$

and as we shall see, the actual value of v as long as it is non-zero will be immaterial.

Continuing our analysis, it is easy to see that the four equations

$$\frac{\delta A_b}{\delta \chi^\mu} = \frac{\delta A_b}{\delta A_\mu} = \frac{\delta A_b}{\delta \chi^{\mu, \beta\gamma}} = \frac{\delta A_b}{\delta A_{\mu, \beta\gamma}} = 0 \quad (3.3)$$

are solved by

^{*}) This will appear even more reasonable after realizing that $V(H)$ plays the role of a driving term in an effective potential for H singular at $H = 0$.

$$\langle X^\mu \rangle = \langle A_\mu \rangle = \langle X^{\mu, \alpha\beta} \rangle = \langle A_{\mu, \alpha\beta} \rangle = 0 \quad (3.4)$$

in accordance with global Lorentz invariance. The remaining two stationary conditions give

$$\begin{aligned} \frac{\delta A_b}{\delta \phi} = 0 &\Rightarrow v \det \langle W \rangle = -\frac{1}{4} \sum_i C_i \int \frac{d^4 p}{(2\pi)^4} T_r \frac{k_i}{\langle \xi \rangle^{\mu\alpha} \gamma_\alpha p_\mu - k_i \langle \phi \rangle} = \\ &= -\frac{1}{\det \langle \xi \rangle} \frac{1}{u} \sum_i C_i \int \frac{d^4 r}{(2\pi)^4} \frac{k_i^2 u^2}{r^2 - k_i^2 u^2} \end{aligned} \quad (3.5)$$

$$\begin{aligned} \frac{\delta A_b}{\delta \xi^{\mu\alpha}} = 0 &\Rightarrow \langle W_{\alpha\mu} \rangle \det \langle W \rangle = -\sum_i C_i \int \frac{d^4 p}{(2\pi)^4} T_r \frac{\gamma_\alpha p_\mu}{\langle \xi \rangle^{\nu\beta} \gamma_\beta p_\nu - k_i \langle \phi \rangle} = \\ &= -\frac{1}{\det \langle \xi \rangle} \langle \xi^{-1} \rangle_{\beta\mu} \cdot \sum_i C_i \int \frac{d^4 r}{(2\pi)^4} T_r \frac{\gamma_\alpha r^\beta (r + k_i u)}{r^2 - k_i^2 u^2} = \\ &= -\frac{1}{\det \langle \xi \rangle} \langle \xi^{-1} \rangle_{\alpha\mu} \sum_i C_i \int \frac{d^4 r}{(2\pi)^4} \frac{r^2}{r^2 - k_i^2 u^2} \end{aligned} \quad (3.6)$$

where

$$r^\alpha \equiv \langle \xi \rangle^{\mu\alpha} p_\mu \quad (3.7)$$

and

$$\langle \phi \rangle = u \mathbb{1} \quad (3.8)$$

as required by global Lorentz invariance.

By using the condition $\sum_i C_i = 0$ we recognize the equality of the last two integrals of Eqs. (3.5) and (3.6). Introducing now the dimensional proportionality constant η by

$$\langle W \rangle_{\alpha\mu} = \eta \langle \xi^{-1} \rangle_{\alpha\mu} \quad (3.9)$$

eqs. (3.2), (3.5) and (3.6) imply

$$uv = \eta = V(v) \quad (3.10)$$

$$I_4 \equiv \sum_i c_i \int \frac{d^4 r}{(2\pi)^4} \frac{r^2}{r^2 h_i^2} = \eta^5, \quad h_i \equiv K_i u$$

where, we remark, the h_i satisfy the same PV conditions (2.10) as the K_i .

Since W and ξ have dimensions, their non-zero constant v.e.v. implies a vacuum generated scale Λ which, taking into account Eq. (3.9), can be defined without loss of generality by

$$\langle \xi \rangle^{\mu\alpha} = \Lambda^{-1} \eta_{\mu\alpha}, \quad \langle W \rangle_{\alpha\mu} = \eta \Lambda \cdot \eta_{\mu\alpha} \quad (3.11)$$

$$\eta_{\mu\alpha} \equiv \text{diag.} (1, -1, -1, -1)$$

Equations (3.11) show explicitly that our vacuum breaks (spontaneously) local Lorentz as well as general co-ordinate transformation invariances leaving a diagonal global Lorentz invariance. The spontaneous breaking of scale invariance generates the scale Λ as a vacuum property⁷⁾. This is the only dimensional scale of the theory, so that all masses (as well as other dimensional quantities) will be proportional to it. In the case of fermions, the masses which appear as poles in the propagator are given by

$$m_i = h_i \Lambda \quad (3.12)$$

We may, of course, choose $h_0 \ll h_i$, $i \neq 0$ in order to have the original fermion arbitrarily light. This mass is protected by a chiral invariance for the light fermion in the limit $K_0 \rightarrow 0$ ($h_0 \rightarrow 0$). Indeed, as is evident from the form (2.9) of the action, K_i measures the strength of the γ_5 non-invariant terms. Furthermore, for $K_0 \rightarrow 0$, the light fermion is excluded from the condensate (3.2).

Let us call h the smallest h_i ($i \neq 0$) so that $M = h\Lambda$ represents the mass of the lightest regulator (or set of regulators). M acts as a cut-off in our theory and will appear explicitly in Green's functions. M cannot be sent to infinity since, from (3.10) and the evaluation of I_4 (cf. Appendix A),

$$V(v)^5 = I_4 \sim h^4 = \left(\frac{M}{\Lambda}\right)^4 \quad (3.13)$$

Let us discuss for a moment the meaning of Eq. (3.13). In order for the homogeneous flat-space solution^{*)} to exist, the parameters h_i and $V(v)$ have to be related by Eq. (3.10). This can be seen either as a condition on the regulator mass M in terms of a given $V(v)$ and Λ or, once M/Λ is arbitrarily chosen, as a fine tuning of an additional constant contribution to $V(H)$. This is not surprising if one realizes that both V and the PV parameters are part of the regularization needed to give meaning to the theory. In this connection, let us remark that we will be able to identify some heavy fields (as, for instance, $\lambda^{\mu, \alpha\beta}$), the integration over which would induce an additional contribution to the effective potential $V(H)$ which depends on the PV parameters and is singular for $H \rightarrow 0$. One may thus consider $V(H)$ of Eq. (2.12) as a driving term. We notice, however, that, had we considered the regularization mass M as the basic scale of the theory, Eq. (3.13) would determine, without fine tuning, Λ and hence the vacuum scale of Eq. (3.11). This could appear to follow from having explicitly broken scale invariance through the introduction of M . Nevertheless, the just stated equivalence with the spontaneous breaking of scale invariance shows that our regularization procedure represents a soft breaking.

The fact that our breaking is really of a spontaneous nature will be substantiated in Section 4 where we will show a typical generation of massive $O(3,1)$ gauge bosons through the eating of Goldstone particles associated with broken generators.

We wish to remark at this point that given the set of PV parameters h_i and $V(v)$ as discussed before, Eq. (3.10) fixes the vacuum parameter η and, finally, a relation between $uv = \eta$ appears, thus showing that the results are independent of the combination $v = \eta$ appears, thus showing that the results are independent of the actual value of v in Eq. (3.2) as long as $v \neq 0$.

We have restricted our discussion to the homogeneous solutions because they are certainly the relevant ones for describing the physical vacuum. Other stationary points could exist (instanton-like, for instance). Finding them implies, as discussed before, the evaluation of the trace log functional which is a far from trivial task. Techniques for evaluating this functional are only known for slowly varying fields⁸⁾ or for fields possessing particular symmetries⁹⁾. We have also avoided so far the explicit introduction of sources for the matter fields ψ_i . Their inclusion implies as usual an extra term $\bar{J}\psi + j\bar{\psi}$ in the action (2.6), which becomes bilinear in $\bar{J}, j$ after integration over $\psi, \bar{\psi}$.

^{*)} This will also imply the absence of an induced cosmological term as discussed in Section 4.

As long as the source is simply a device for calculating Green's functions it is of no relevance to our preceeding (and subsequent) discussions. If, instead, one were interested in studying the theory in the presence of classically treated external matter, one would consider j as a given source which will therefore intervene explicitly in the minimization equations (3.2) - (3.4) leading typically to non-flat classical solutions.

4. - QUADRATIC FLUCTUATIONS AND INDUCED KINETIC TERMS

In this section we shall compute the quadratic part of the action (2.12) in the shifted bosonic fields. Furthermore, we shall expand this quadratic action up to second order in the field momenta q_i . The q independent terms, once diagonalized, will allow us to determine masses and, in particular, to identify the massless modes. Terms quadratic in the momenta will provide induced kinetic terms for the massless fields and will lead to the identification of the induced Newton and fine structure constants.

We shall denote by a tilded field its fluctuation around its v.e.v. of Section 3, lowering for convenience all its upper indices by the flat η tensor.

a) q Independent Terms

Since the field $W_{\alpha\mu}$ does not appear inside the trlog there is no dependence of A_b on the derivatives of W . One then finds, at all momenta:

$$\frac{\delta A_b}{\delta \tilde{W}_{\alpha\mu} \delta \tilde{W}_{\beta\nu}} = -\eta^3 \Lambda^2 (\eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\mu\beta} \eta_{\nu\alpha}) \quad (4.1)$$

$$\frac{\delta A_b}{\delta \tilde{W}_{\alpha\mu} \delta \tilde{\xi}_{\nu\beta}} = -\eta^4 \Lambda^4 (\eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\alpha\beta} \eta_{\mu\nu}) \quad (4.2)$$

the derivatives being of course evaluated for vanishing fluctuations.

The dependence of A_b on ξ is more complicated. One finds:

$$\frac{\delta A_b}{\delta \tilde{\xi}_{\mu\alpha} \delta \tilde{\xi}_{\nu\beta}} = -\Lambda^6 T_{\alpha\beta,\mu\nu}(q) \quad (4.3)$$

where $T_{\alpha\beta,\mu\nu}(q)$ is given by the one-loop Feynman diagram of Fig. 1, i.e.,

$$T_{\alpha\beta,\mu\nu}(q) = \sum_i C_i \int \frac{d^4 p}{(2\pi)^4} \frac{p_\mu p_\nu}{D_{\pm q} D_{\mp q}} \text{Tr} [\gamma_\alpha (\not{p} + \not{q}/2 + m_i) \gamma_\beta (\not{p} - \not{q}/2 + m_i)]$$

$$D_{\pm q} = (P \pm q/2)^2 - m_i^2 \quad (4.4)$$

with m_i given in Eq. (3.12). At $q = 0$ one finds

$$T_{\alpha\beta,\mu\nu}(0) = \sum_i C_i \int \frac{d^4 p}{(2\pi)^4} \frac{4 p_\mu p_\nu \eta_{\alpha\beta} (m_i^2 - p^2) + 8 p_\mu p_\nu p_\alpha p_\beta}{(p^2 - m_i^2)^2} =$$

$$= -\eta_{\alpha\beta} \eta_{\mu\nu} I_4 + \frac{1}{3} (\eta_{\mu\nu} \eta_{\alpha\beta} + \eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\mu\beta} \eta_{\nu\alpha}) \cdot \sum_i C_i \int \frac{d^4 p}{(2\pi)^4} \frac{p^4}{(p^2 - m_i^2)^2} \quad (4.5)$$

with I_4 defined in Eq. (3.10). Using the relations derived in Appendix A the last integral in (4.5) is equal to $3I_4$ so that:

$$\left. \frac{\delta A_b}{\delta \tilde{\xi}_{\mu\alpha} \delta \tilde{\xi}_{\nu\beta}} \right|_{q=0} = -I_4 \Lambda^6 (\eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\alpha\nu} \eta_{\beta\mu}) =$$

$$= -\eta^5 \Lambda^6 (\eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\alpha\nu} \eta_{\beta\mu}) \quad (4.6)$$

where Eq. (3.10) has been used.

The mass matrix formed by the quadratic terms (4.1), (4.2) and (4.6) can be written as

$$-\Lambda^4 \eta^5 \left(\Lambda \tilde{\xi}_{\alpha\mu} + \frac{1}{\Lambda \eta} \tilde{W}_{\alpha\mu} \right) \left(\Lambda \tilde{\xi}_{\beta\nu} + \frac{1}{\Lambda \eta} \tilde{W}_{\beta\nu} \right) (\delta_{\alpha\mu} \delta_{\beta\nu} + \delta_{\alpha\nu} \delta_{\beta\mu}) \quad (4.7)$$

from which one can recognize the two eigenmodes

$$\tilde{V}_{\alpha\mu} = \frac{1}{2} \left(-\Lambda \tilde{\xi}_{\alpha\mu} + \frac{1}{\Lambda \eta} \tilde{W}_{\alpha\mu} \right) ; \quad \tilde{U}_{\alpha\mu} = \frac{1}{2} \left(\Lambda \tilde{\xi}_{\alpha\mu} + \frac{1}{\Lambda \eta} \tilde{W}_{\alpha\mu} \right) \quad (4.8)$$

the first of which is massless. The mode $\tilde{U}$ is a heavy mode, i.e., a field with mass of $O(\Lambda)$. Later on we shall be able to see that, in a suitable low energy approximation, $\tilde{V}$ of Eq. (4.8) is the fluctuation of a vierbein field V satisfying the equations of the usual Einstein theory.

Let us now turn to the q independent quadratic terms in the connections $A_{\mu,\beta\gamma}$, A_μ and in their corresponding Lagrange multipliers $\lambda^{\mu,\beta\gamma}$, λ^μ .

Straightforward calculations give

$$\frac{\delta A_b}{\delta A_{\mu,\alpha\beta} \delta A_{\nu,\gamma\delta}} = 8 \Lambda^2 I_2 O_{\mu\nu}^{\alpha\beta,\gamma\delta} \quad (4.9)$$

$$\frac{\delta A_b}{\delta \lambda_{\mu,\alpha\beta} \delta A_{\nu,\gamma\delta}} = \frac{1}{2} \Lambda^4 \eta^4 \tilde{O}_{\mu\nu}^{\alpha\beta,\gamma\delta} \quad (4.10)$$

$$\frac{\delta A_b}{\delta \lambda_{\mu,\alpha\beta} \delta \lambda_{\nu,\gamma\delta}} = \Lambda^6 \hat{O}_{\mu\nu}^{\alpha\beta,\gamma\delta} \quad (4.11)$$

where I_2 is a regularized integral defined in Appendix A, whose evaluation gives:

$$I_2 = c \eta^{5/2} = c' (M^2/\Lambda^2) \quad (4.12)$$

with c and c' proportionality constants given in Appendix A for our particular regularization prescription. The tensors appearing in (4.9), (4.10) are:

$$\tilde{O}_{\mu\nu}^{\alpha\beta,\gamma\delta} = \eta_{\nu\alpha} (\eta_{\beta\gamma} \eta_{\mu\delta} - \eta_{\beta\delta} \eta_{\mu\gamma}) - \eta_{\nu\beta} (\eta_{\alpha\gamma} \eta_{\mu\delta} - \eta_{\alpha\delta} \eta_{\mu\gamma}) \quad (4.13)$$

$$O_{\mu\nu}^{\alpha\beta,\gamma\delta} = \tilde{O}_{\mu\nu}^{\alpha\beta,\gamma\delta} + \eta_{\mu\nu} (\eta_{\alpha\gamma} \eta_{\beta\delta} - \eta_{\alpha\delta} \eta_{\beta\gamma}) \quad (4.14)$$

whereas $\hat{O}_{\mu\nu}^{\alpha\beta,\gamma\delta}$ has a similar expression whose exact form will not be needed here.

Turning now to the A_μ, λ^μ sector, the situation is the same as the one discussed in Ref. 3). The two-by-two mass matrix has only one non-zero entry corresponding to a mass of order Λ for the λ^μ field. The absence of a mass term for A_μ or of a $\lambda^\mu A_\mu$ mixing is of course a consequence of U(1) gauge invariance (the case of $A_{\mu,\beta\gamma}$ is more subtle in this respect, as discussed below).

Finally, the two scalar fields ϕ and M can be shown to be also heavy. We do not give here the explicit expression but just remark that the mass of M depends on the details of $V(M)$ [e.g., on $V''(v)$].

b) q Dependent Terms

We start with terms linear in the momenta. These mix tensors differing by one index, in particular $A_{\mu,\beta\gamma}$ and $\lambda^{\mu,\beta\gamma}$ with $\xi^{\nu\alpha}$. We find

$$\frac{\delta A_b}{\delta \lambda_{\mu,\alpha\beta} \delta \tilde{\xi}_{\gamma\delta}} = \frac{1}{8} \eta^4 \Lambda^5 q_\nu \tilde{O}_{\mu\nu}^{\alpha\beta,\gamma\delta} \quad (4.15)$$

$$\frac{\delta A_b}{\delta A_{\mu,\alpha\beta} \delta \tilde{\xi}_{\gamma\delta}} = 2 \Lambda^3 I_2 q_\nu O_{\mu\nu}^{\alpha\beta,\gamma\delta} \quad (4.16)$$

where the tensors O and $\tilde{O}$ of Eqs. (4.13), (4.14), being antisymmetric in $\gamma \rightarrow \delta$, pick up the antisymmetric part of $\tilde{\xi}$. We shall come back to the above results after discussion of quadratic terms in q for the W, ξ sector which, as mentioned above, contain only ξ . Defining

$$\xi_{s,a}^{\mu\alpha} = \frac{1}{2} (\xi^{\mu\alpha} \pm \xi^{\alpha\mu}) \quad (4.17)$$

we find up to second order in q :

$$\frac{\delta A_b}{\delta \tilde{\xi}_s \delta \tilde{\xi}_a} = 0 ; \quad \frac{\delta A_b}{\delta \tilde{\xi}_{\mu\alpha}^s \delta \tilde{\xi}_{\nu\beta}^s} = O_{\mu\alpha,\nu\beta}^{(+)} ; \quad \frac{\delta A_b}{\delta \tilde{\xi}_{\mu\alpha}^a \delta \tilde{\xi}_{\nu\beta}^a} = O_{\mu\alpha,\nu\beta}^{(-)} \quad (4.18)$$

where

$$O_{\mu\alpha,\nu\beta}^{(+)} = \frac{1}{6} \Lambda^4 I_2 [q^2 (\eta_{\alpha\beta} \eta_{\mu\nu} + \eta_{\alpha\nu} \eta_{\beta\mu} - 2 \eta_{\alpha\mu} \eta_{\beta\nu}) - \quad (4.19)$$

$$-g_\alpha g_\beta \eta_{\mu\nu} - g_\mu g_\beta \eta_{\alpha\nu} - g_\mu g_\nu \eta_{\alpha\beta} - g_\alpha g_\nu \eta_{\beta\mu} + 2g_\beta g_\nu \eta_{\alpha\mu} + 2g_\alpha g_\mu \eta_{\beta\nu} \quad (4.19) \text{ cont.}$$

$$O_{\mu\alpha, \nu\beta}^{(-)} = \frac{1}{2} \Lambda^4 I_2 \left[g^2 (\eta_{\alpha\beta} \eta_{\mu\nu} - \eta_{\alpha\nu} \eta_{\beta\mu}) + g_\mu g_\beta \eta_{\alpha\nu} + g_\alpha g_\nu \eta_{\beta\mu} - g_\alpha g_\beta \eta_{\mu\nu} - g_\mu g_\nu \eta_{\alpha\beta} \right] \quad (4.20)$$

Recalling that $\tilde{\xi}_s$ does not appear in Eqs. (4.15), (4.16) we see that the full kinetic term in $\tilde{\xi}_s$ is given by $O^{(+)}$, implying a contribution to A_b of the form:

$$-\frac{1}{6} \Lambda^4 I_2 \left[\square (\tilde{\xi}_s^{\mu\alpha} \tilde{\xi}_{s,\mu}^\alpha - \tilde{\xi}_{s,\mu}^\mu \tilde{\xi}_{s,\alpha}^\mu) - 2\partial_\alpha \partial_\beta \tilde{\xi}_s^{\mu\alpha} \tilde{\xi}_{s,\mu}^\beta + 2\partial^\alpha \partial^\mu \tilde{\xi}_{\mu\alpha}^s \tilde{\xi}_{s,\nu}^\nu \right] \quad (4.21)$$

The quadratic pieces that contain ξ_a [Eqs. (4.15), (4.16), (4.20)], can be combined with those containing $A_{\mu,\gamma\delta}$ and $\lambda^{\mu,\gamma\delta}$ to give the following contributions to A_b :

$$4\Lambda^2 I_2 B_{\mu,\alpha\beta} B_{\nu,\gamma\delta} O_{\mu\nu}^{\alpha\beta,\gamma\delta} + \frac{1}{2} \Lambda^4 \eta^4 \lambda_{\mu,\alpha\beta} B_{\nu,\gamma\delta} \tilde{O}_{\mu\nu}^{\alpha\beta,\gamma\delta} \quad (4.22)$$

where

$$B_{\mu,\alpha\beta} = A_{\mu,\alpha\beta} + \frac{\Lambda}{4} g_\mu \tilde{\xi}_{\alpha\beta}^a \quad (4.23)$$

The fact that $A_{\mu,\alpha\beta}$ and $\tilde{\xi}_{\mu\alpha}^a$ appear only in the combination $B_{\mu,\alpha\beta}$ of Eq. (4.23) is due to the $O(3, 1)$ gauge invariance of the theory. $A_{\mu,\alpha\beta}$, being a connection, transforms inhomogeneously under the transformation (2.16):

$$A_{\mu,\alpha\beta} \rightarrow A'_{\mu,\alpha\beta} + \frac{1}{4} (\sigma^{\gamma\delta})_{\alpha\beta} \partial_\mu \omega_{\gamma\delta}(x) \quad (4.24)$$

Naively this would seem to forbid terms in the action containing $A_{\mu,\alpha\beta}$ without derivatives as is the case for the electromagnetic field A_μ . On the other hand, in our case we can construct another field transforming as a connection by taking derivatives of $\tilde{\xi}_{\alpha\beta}^a$. Thus the combination $B_{\mu,\alpha\beta}$ of Eq. (4.23) is again a good tensor. The first term in Eq. (4.22) shows that $B_{\mu,\alpha\beta}$ is a massive field which has "eaten up" à la Higgs the six would-be Goldstone bosons $\tilde{\xi}_{\alpha\beta}^a$ of the spontaneously broken $O(3, 1)$ invariance.

This fact confirms the validity of our statement that the flat-metric expectation values of W and ξ represent a spontaneous rather than explicit breaking of the symmetries of the original action, dilatation symmetry included.

Turning finally to the q dependent terms involving the λ^μ and A_μ electromagnetic fields we find again, as in Ref. 3), that they involve A_μ only through the field strength tensor $F_{\mu\nu}$. This ensures that A_μ is an exactly massless field contributing to the action with a term

$$-\frac{1}{4} I_0 F_{\mu\nu}^2 \quad ; \quad I_0 \Big|_{q=0} = \frac{1}{12\pi^2} \log(M^2/m^2) + O(1) \quad (4.25)$$

I_0 here is the usual QED vacuum polarization integral whose dependence on q is well known.

c) Summary

Summarizing the results obtained so far, we have identified in our spectrum the following light particles:

- i) one or several light fermions (according to the number of parameters K_i set to be much smaller than one) protected by chiral invariance from obtaining masses of $O(\Lambda)$;
- ii) one massless spin one particle associated with the exact $U(1)$ gauge symmetry, i.e., a photon. In a similar way, one would obtain eight gluons from requiring local $SU(3)$ invariance;
- iii) a massless excitation with the structure of the vierbein, associated with general relativistic invariance.

Besides these, we have obtained a number of "heavy" excitations, i.e., fields with mass of $O(\Lambda)$. As long as these heavy degrees of freedom carry energies $E_i \ll \Lambda$, they will not be excited away from their v.e.v.'s. In this limit we have, to a very good approximation, the following relations

$$\eta \wedge \tilde{\xi} = -\frac{1}{\Lambda} \tilde{\omega} \Rightarrow \xi = \eta \omega^{-1} \quad (4.26)$$

$$H = v \mathbb{1} \quad , \quad \phi = u \mathbb{1} \quad (4.27)$$

$$B_{\mu}^{\alpha\beta} = 0 \quad \Rightarrow \quad A_{\mu}^{\alpha\beta} = \frac{1}{4} \xi_{\alpha\nu}^{-1} \partial_{\mu} \xi^{\nu\beta} \quad (4.28)$$

$$\lambda^{\mu} = \lambda^{\mu, \alpha\beta} = 0 \quad (4.29)$$

In this regime Eq. (4.26) allows, as discussed before, the definition of a metric after identification of $\Lambda \xi$ ($W/\eta\Lambda$) with the usual vierbein fields with upper (lower) indices. In that case we recognize in Eq. (4.28) the usual definition of the Lorentz connection in terms of the vierbein. Looking at the momentum-dependent part of the fluctuations, we then see that Eq. (4.21) is nothing but the quadratic expansion around the flat vierbein solution of

$$\frac{1}{6} \Lambda^2 I_2 \sqrt{-g} R \quad (4.30)$$

with g and R the usual metric determinant and scalar curvature expressed in terms of the vierbein.

At fourth order in the derivatives, a term¹⁰⁾ $(\log M/m) (R^2 - 3 R_{\mu\nu} R^{\mu\nu})$ appears. Higher order terms compatible with general relativistic invariance will also be generated, their dimensions being compensated by appropriate powers of M^{-1} . All these R^2 and higher order terms are therefore negligible in the low energy regime under discussion.

We notice at this point the absence of an induced $\sqrt{-g}$ cosmological term that, if present, would have appeared with an M^4 scale factor. This is an expected consequence of the fact that in our approach the flat metric is a classical solution and that a cosmological term would contain linear vierbein fluctuations.

Hence, in the low energy regime under discussion, we recover the Einstein-Dirac action for gravity plus spin 1/2 matter, with a vanishing cosmological constant and an induced Newton constant given by

$$(16\pi G_N)^{-1} = \frac{1}{6} \Lambda^2 I_2 = \frac{1}{6} c' M^2 \quad (4.31)$$

where the numerical constant c' of $O(1)$ is given in Appendix A for our explicit regularization. Equation (4.31) shows that the Newton constant is related to the vacuum generated scale Λ by precisely the co-efficient that makes it equal to the momentum cut-off M .

As already said, the quadratic terms in λ^μ and A_μ provide the usual kinetic term $F_{\mu\nu}^2$. As in Ref. 3), in the low energy regime where we can set $\lambda^\mu = 0$, we find nothing but the ordinary QED action with an effective fine structure constant given by the inverse of the quantity I_0 of Eq. (4.25). For $m^2 \ll q^2 \ll M^2$ one finds:

$$\alpha(q^2) = 3\pi \left(\log(M^2/q^2) + O(1) \right)^{-1} \quad (4.32)$$

In the case of N light fermions of $U(1)$ charges e_i we would have obtained for the i^{th} fermion the effective electromagnetic coupling

$$\begin{aligned} \alpha_i(q^2) &= 3\pi e_i^2 \left(\sum_{j=1}^N e_j^2 \log(M^2/q^2) + O(1) \right)^{-1} \\ &= 3\pi e_i^2 \left(\sum_{j=1}^N e_j^2 \log\left(\frac{1}{G_N N} q^2\right) + O(1) \right)^{-1} \end{aligned} \quad (4.33)$$

where we have used Eq. (4.31) with the appropriate factor coming from the multiplicity of fermions. Equation (4.33) expresses the fact that, up to some factors $O(N)$, the Landau pole of QED and the Planck mass have to coincide in our approach. Proceeding as in Ref. 3) a similar statement can be made for other unbroken gauge symmetries (as for instance QCD) provided that enough fermionic thresholds make them eventually asymptotically non-free. This typically implies, for non-Abelian gauge theories, a certain number of massive fundamental fermions. In the case of QCD $[SU(3)]$ the equivalent of Eq. (4.33) becomes, at $m_1^2 \ll q^2 \ll 1/G_N$,

$$\alpha_c(q^2) = 12\pi \left[(2N_f^c - 33) \log\left(\frac{1}{N G_N} q^2\right) + O(1) \right]^{-1} \quad (4.34)$$

for N_f^c colour triplet Dirac fermions.

The phenomenological viability of schemes of this type for unification at the Planck mass has been demonstrated in Refs. 5) and 6). We want to recall the difficulty of including in the scheme³⁾ gauge symmetries with non-real fermionic content due to the well-known problems met in regularizing chiral theories.

We close this section by recalling that the regime in which we have seen ordinary gauge theories arise as effective interactions is the one in which all fields carry, by decree, small momenta. This regime, however, does not necessarily coincide with that of the full quantum theory at low energy. It is only so at the tree level or if high momentum quantum fluctuations (as appearing through ultra-violet behaviours of loops) are strongly damped. We do not know yet if our approach will lead to such behaviour. In the next section we shall discuss some reasons why we can nourish some hopes on this crucial issue.

5. - DISCUSSION AND OUTLOOK

a) In the previous sections we have seen how, after integration over the fermions, certain massless excitations are generated in the auxiliary bosonic fields. The massless excitations appeared in one-to-one correspondence with the generators of the local (gauge) symmetries of the starting Lagrangian. In particular we have generated a spin one photon associated with a $U(1)$ internal symmetry and a spin two graviton associated with local Lorentz invariance.

Before turning to a short discussion of higher order corrections (i.e., to bosonic loops) we want to comment briefly on the way our contribution has avoided certain no-go theorems, in particular that of Weinberg and Witten¹¹⁾. A basic ingredient in the proof of Ref. 11) is the existence of a conserved current J_μ ($T_{\mu\nu}$ for gravity) which transforms as a four vector under Lorentz transformations. In theories with a global symmetry at the classical level, the corresponding Noether current is such an object. In that case, the results of Ref. 11) say that the charge associated with J_μ cannot connect massless states of spin larger than $1/2$. Furthermore, J_μ cannot connect a Lorentz invariant vacuum to such a one particle massless state. In our case, the Noether currents J_μ and $T_{\mu\nu}$ vanish^{*)} and therefore the fact that they do not connect massless

*) This is due to the fact that the action contains $\partial_\mu \psi$ only through $D_\mu \psi$ with D_μ given by Eqs. (2.2) - (2.4). In performing the derivative with respect to $\partial_\mu \psi$ and multiplying by a $\delta\psi$ corresponding to a global transformation, we shall always find two terms (one from ∂_μ and the other from the corresponding connection) that cancel exactly.

spin 1 or spin 2 states is trivial. What would create a one particle boson state from the vacuum is rather a composite operator, such as $A_\mu \sim \psi \delta_\mu \psi (\psi\psi)^{-1}$, which is not gauge invariant. This is the property that here, as well as in the case of the Noether current of conventional gauge theories, allows the evasion of the no-go theorem. Indeed, if one insists on a simple particle Hilbert space of positive definite norm then one has to fix a non-covariant gauge for A_μ and loses the four vector nature of A_μ . Otherwise, A_μ creates extra states of indefinite metric which do not belong to the massless spin 1 (or 2) representation of the Lorentz group and the theorem is again not applicable.

b) The next point we would like to discuss here is the connection stated in Section 2 between our semiclassical development and a $1/N$ expansion. As discussed in Section 2 the introduction of N fermion species introduces a factor N in front of the trlog term of Eq. (2.12). The minimization conditions (3.10) would now read

$$\begin{aligned} \eta^5 &= N I_4 \simeq N (M/\Lambda)^4; \\ \eta &= \sqrt{v} \quad ; \quad uv = \eta \\ \langle \xi^{-1} \rangle &= \eta^{-1} \langle W \rangle = \Lambda \end{aligned} \tag{5.1}$$

Clearly, the Newton constant which was given in Eq. (4.31) now becomes

$$G_N = \frac{3}{8\pi c' N M^2} = O(1/N) \tag{5.2}$$

Choosing the graviton propagator to be N independent each vertex carries a $1/\sqrt{N}$ factor and each graviton loop is suppressed by $1/N$. In other words, if M is fixed as $N \rightarrow \infty$ then $G_N \rightarrow 0$ and graviton loops become more and more suppressed as in a $1/N$ expansion. Examples of $1/N$ counting are shown in Fig. 2 where the wavy lines represent any one of the effective bosonic fields.

c) The $1/N$ expansion discussed above is just a useful book-keeping of all higher order loops, i.e., integrals over the fluctuation of auxiliary fields. The crucial problem, however, is to make sure that these integrals are actually defined. This is related to the ultra-violet behaviour of the theory, a regime in which our approach differs essentially from the conventional one. This difference is of course welcomed due to the ultra-violet sickness of the conventional general relativistic theory where, as is well known, the strong cut-off dependence

cannot be reabsorbed by a renormalization procedure. On the other hand, our theory has a single mass scale which acts as a cut-off and has a physical meaning (the Planck mass). Therefore no further reabsorption of infinities is possible so that in order for the theory to be meaningful the integral over the fluctuations (i.e., boson loops) should be finite. In other words, no further regularization or renormalization is possible, the only freedom left could be to increase the gauge symmetries (or supersymmetries?) the theory should satisfy.

Let us recall that the ultra-violet behaviour we would like to explore does not appear only as very high energy virtual fluctuations in low energy processes (i.e., intermediate loops). It also appears, of course, in processes in which energies and transverse momenta start approaching the Planck mass or, also, on conditions of extremely high matter density and temperature as in the very early universe. We are planning to explore, in some simple case, the initial effects of these departures of our theory from the conventional one, looking for signs of a softer ultra-violet behaviour.

We must nevertheless acknowledge that for the time being we have no firm evidence that our approach is indeed ultra-violet meaningful. We can only give a few qualitative hints of why we can nourish the hope that its short distance behaviour is milder than the conventional one. We shall therefore first underline the modifications implied by our theory on a conventional calculation of gravity effects at short distances. Sticking to the bosonic language that proved useful in recognizing the low-lying spectrum, we find differences with the usual treatment of gauge and gravity theories at various levels.

- i) The rich heavy sector (masses of order M , the Planck mass) we have found will contribute as much as the light one as soon as the momenta they transfer approach M . Cancellations may of course occur. We recall nevertheless that among these heavy particles with well-defined masses and couplings, we find also the PV ghosts. These represent a nuisance (we have perhaps some ideas on how to cope) even if the meaning of tree-unitarity or its violation at a level in which not even a space-time metric is definable is far from clear.
- ii) Both the light and heavy sectors have quadratic terms in the fields with higher derivatives rescaled, of course, by powers of $1/M$. These higher derivatives may be resummed and lead to propagators which are calculable

functions of q^2 . Only for $q^2 \ll M^2$ do they coincide, for the light sector, with the conventional propagators. A calculation at this level to test the short-distance behaviour of the theory is in progress. A behaviour milder than the conventional one would already be a gratifying signal for our alternative approach to gravity.

- iii) Besides the aforementioned contributions that count as soon as $q^2 \sim M^2$, we also meet an infinite series of induced many field couplings. They would, of course, contribute to any given process at higher and higher loop levels and therefore should be depressed by higher and higher powers of $1/N$. It is nevertheless very unclear whether or not we have any right to expect an ultra-violet behaviour which is uniform in $1/N$.

All the high frequency modifications to the conventional theory, i.e., extra particles, higher derivative interactions and extra couplings, are there to recall the basic fact that the graviton and gauge bosons were composite objects. At short distances we should indeed see the effects of their structure through some sort of form factor which softens their contribution as one approaches momentum transfers of order M .

These considerations show perhaps that the language of auxiliary fields, so convenient for studying the spectrum and the low energy structure of the theory, is not the correct language for studying the short-distance behaviour. The complexity of this language here is to remind us of its duality with respect to the language of components which, in our theory, is the one of the original fermions. This is perhaps better suited to short distances and our hope for a mild behaviour in this limit can be recomforted by the high derivative structure of our original action which should penalize very high frequencies. In this regime we should also lose the scale Λ provided by the vacuum condensate and recuperate therefore all the local symmetries of the theory. This could suggest a deep ultra-violet behaviour which might be not worse than that of ordinary re-normalizable theories. All these arguments are only hints, but they nevertheless support our hope that the pregeometric gravity we propose could be a valid alternative for overcoming the pathologies of the conventional theory.

APPENDIX A

In this Appendix we will discuss in some detail our regularization procedure and thus obtain some relations which have been used in the text.

The action whose integral over the fermions is to be regularized is the one of Eq. (2.9). As discussed in the text we have introduced a set of PV regulators in order to go over to Eq. (2.12). The parameters C_i and K_i appearing in (2.12) should be chosen in such a way as to make finite the trlog which has the form

$$T \equiv \sum_i C_i \text{trlog} (A + K_i B) \quad (\text{A.1})$$

where A and B are operators containing fields, derivatives and Dirac matrices.

Introducing a real meromorphic function $\rho(z)$ with poles at $z = K_i$ and residues C_i , we can rewrite (A.1) as

$$T = \frac{1}{2\pi i} \int_C dz \rho(z) \text{trlog} (A + zB) \quad (\text{A.2})$$

where the contour C goes around the poles of $\rho(z)$. In order to understand the properties that $\rho(z)$ must satisfy, let us consider the expansion of the trlog around the classical (constant) field values. The expressions to be evaluated take the form

$$\int_C dz \rho(z) \int d^4p P_1/P_2 \quad (\text{A.3})$$

where P_1 and P_2 are polynomials in p, z and the external momenta q_i . Expanding P_1/P_2 for large p we encounter terms of order 1 , $1/p^2$ and $1/p^4$ which lead to divergent integrals unless their coefficients vanish upon integration over z . One easily finds that these coefficients involve arbitrarily high even powers of z if we expand trlog to arbitrary order in the fields (in particular in the field $\lambda^{\mu, \alpha\beta}$). Hence, in order to regularize the divergences that appear in all Green's functions, we must choose $\rho(z)$ such that all the superconvergence equations

$$\int dz \text{disc. } \rho(z) \cdot z^{2n} = 0 \quad ; \quad n=0, 1, 2, \dots \quad (\text{A.4})$$

are satisfied. As a simple example, we choose

$$\rho(z) = \frac{a}{\sin a(z - K_0)} \quad ; \quad \frac{\text{disc } \rho(z)}{2i\pi} = \sum_{j=-\infty}^{+\infty} (-1)^j \delta(z - K_0 - j\pi/a) \quad (\text{A.5})$$

so that

$$C_i = (-1)^i \quad ; \quad K_i = K_0 + i\pi/a \quad (\text{A.6})$$

Since $\rho(z)$ goes to zero exponentially for $|z| \rightarrow \infty$ ($\arg z \neq 0, \pi$) Eqs. (A.4) are satisfied. Also, the above behaviour of $\rho(z)$ allows us to deform the contour C in Eq. (A.2) and to pick up contributions from the singularities of $\text{tr log}(A + zB)$.

We show how the method works by computing the contribution of T to the effective potential when we set to zero the fields $A_{\mu, \alpha\beta}$ and $\lambda^{\mu, \alpha\beta}$. In this limit T_4 becomes $J_4(\xi^{\mu\alpha}, \phi)$ with

$$\begin{aligned} J_4 &= \frac{1}{2\pi i} \int_C dz \rho(z) \int \frac{d^4 \varphi}{(2\pi)^4} \text{tr log}(\xi^{\mu\alpha} \gamma_\alpha \varphi_\mu - z\phi) = \\ &= \frac{2}{\det \xi} \frac{1}{2i\pi} \int_C dz \rho(z) \int \frac{d^4 q}{(2\pi)^4} \log(q^2 - z^2 \phi^2) \end{aligned} \quad (\text{A.7})$$

By rescaling q and using (A.4) one finds that J_4 is homogeneous of degree +4 in ϕ . Then

$$\phi \frac{\partial J_4}{\partial \phi} = 4 J_4 = \frac{2}{\det \xi} \int \frac{dz}{2i\pi} \rho(z) \int \frac{d^4 q}{(2\pi)^4} \frac{(-2z^2 \phi^2)}{q^2 - z^2 \phi^2} = -\frac{4 I_4}{\det \xi} \quad (\text{A.8})$$

where I_4 is the integral defined in Eq. (3.10).

After a Wick rotation (allowed by the $i\epsilon$ prescription) and change of variable to $v = \sqrt{-q^2/\phi^2}$ one gets, in the limit $K_0 \rightarrow 0$:

$$\begin{aligned} 4 I_4 &= \frac{4\phi^4}{16\pi^2} \int_0^\infty dv v^4 [\rho(iv) - \rho(-iv)] = \frac{\phi^4 a}{2\pi^2} \int_0^\infty \frac{dv v^4}{\sinh av} = \\ &= \frac{\phi^4/a^4}{2\pi^2} \int_0^\infty \frac{dx x^4}{\sinh x} = \frac{\phi^4/a^4}{2\pi^2} \frac{31}{16} 4! \zeta(5) \end{aligned} \quad (\text{A.9})$$

Using Eqs. (3.6), (A.6) we see that the lightest PV fermions have mass

$$M = k_1 \phi \Lambda = \Lambda \pi (\phi/a) \quad (\text{A.10})$$

The result (A.9) can be rewritten as

$$I_4 = \left(\frac{M}{\pi \Lambda} \right)^4 \frac{93}{16 \pi^2} \zeta(5) \quad (\text{A.11})$$

Hence the constants c, c' defined in Eq. (4.12) are related by

$$(c'/c)^2 = \frac{93}{16 \pi^6} \zeta(5) \quad , \quad (\zeta(5) \simeq 1.037)$$

In the text we had encountered other quartically divergent integrals, i.e.,

$$\begin{aligned} I_4' &= \int \frac{d^4 q}{(2\pi)^4} \frac{q^2}{q^2 - k^2 \phi^2} \quad ; \quad I_4'' = \int \frac{d^4 q}{(2\pi)^4} \frac{q^4}{(q^2 - k^2 \phi^2)^2} \\ I_4''' &= \int \frac{d^4 q}{(2\pi)^4} \frac{q^2 k^2 \phi^2}{(q^2 - k^2 \phi^2)^2} \quad ; \quad I_4'''' = \int \frac{d^4 q}{(2\pi)^4} \frac{(k^2 \phi^2)^2}{(q^2 - k^2 \phi^2)^2} \end{aligned} \quad (\text{A.12})$$

Obviously, with our regularization, one has $I_4' = I_4$. Using

$$\int \frac{d^4 q}{(2\pi)^4} \frac{(q^2 + z^2 \phi^2)(q^2 - z^2 \phi^2)}{(q^2 - z^2 \phi^2)^2} = 2 I_4 \quad (\text{A.13})$$

one gets

$$I_4'' - I_4'''' = 2 I_4 \quad (\text{A.14})$$

Similarly one proves that

$$I_4''' = I_4 + I_4'''' \quad (\text{A.15})$$

On the other hand, from the homogeneity of I_4 one has

$$\phi^2 \frac{\partial I_4'}{\partial \phi^2} = 2 I_4 = I_4''' \quad (\text{A.16})$$

hence

$$I_4''' = I_4 \quad , \quad I_4'' = 2 I_4 \quad , \quad I_4' = 3 I_4 \quad (\text{A.17})$$

which are the relations used in the text.

The standard type of quadratically divergent integral we have encountered is

$$I_2 = \int_C \frac{dz}{2i\pi} \rho(z) \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - z^2 \phi^2} \quad (\text{A.18})$$

Proceeding as for I_4 one finds:

$$I_2 = \frac{\phi^2}{2\pi^2 a^2} \int_0^\infty dx \frac{x^2}{\sinh x} = \frac{\phi^2}{2\pi^2 q^2} \frac{7}{2} \zeta(3) = \left(\frac{M}{\pi \Lambda} \right)^2 \frac{7}{4\pi^2} \zeta(3) \quad (\text{A.19})$$

Comparing with Eq. (4.12) this gives (A.19)

$$C' = \frac{7}{4\pi^4} \zeta(3) \quad , \quad \left(\zeta(3) \simeq 1.202 \right)$$

It is also easy to derive the relation

$$I_2' \equiv \int \frac{d^4 q}{(2\pi)^4} \frac{q^2}{(q^2 - k^2 \phi^2)^2} = 2 I_2 \quad (\text{A.20})$$

APPENDIX B

In this Appendix we summarize the results of Sections 3 and 4 by deriving a simple, general covariant form of the effective action A_b defined by Eq. (2.12). The expression we shall write has to be seen as an approximation to Eq. (2.12) in the sense that it contains only the dependence of A_b on the "relevant" fields and, for them, it contains at most two derivatives. This is enough however to obtain and discuss in a simple way the classical solutions discussed in Section 3 and the quadratic fluctuations obtained in Section 4.

We start by computing the dependence of A_b on the fields which have non-vanishing v.e.v. The calculation of A_b is very easy when these fields are constant and gives:

$$A_b \Big|_{q=0}^{\lambda^\mu = A_\mu = \lambda^{\mu\beta\gamma} = A_{\mu\beta\gamma} = 0} = \det W [V(H) - \xi^{\mu\alpha} \omega_{\alpha\mu} + \text{Tr}(\phi H)] + \det \xi^{-1} J_4(\phi) \quad (\text{B.1})$$

where:

$$J_4(\phi) \equiv \sum_i c_i \int \frac{d^4 p}{(2\pi)^4} \text{tr} \log (\xi^{\mu\alpha} \delta_{\alpha\mu} p_\mu - k_i \phi) \quad (\text{B.2})$$

As discussed in Appendix A, $J_4(\phi)$ satisfies the equations

$$\phi \frac{\partial}{\partial \phi} J_4 = 4 J_4 = -4 I_4 = -4 I_4' \quad (\text{B.3})$$

relating it to the other regularized integrals I_4 and I_4' . Using Eqs. (B.1) and (B.3) it is immediate to obtain both the classical solutions (3.8) - (3.10) and the q independent quadratic fluctuations (4.1), (4.2) and (4.6). The absence of an induced cosmological term can be related here to a cancellation between a term proportional to $\det W$ and one proportional to $\det \xi^{-1}$, which is true both at a classical and at the quantum level at low energies. In other words, given a potential $V(M)$, the two vierbein fields can always arrange their v.e.v.s in such a way that any possible cosmological term is cancelled at low energy.

The quadratic part in W and ξ of Eq. (B.1) can be diagonalized in terms of the combinations

$$V_{\alpha\mu} = \frac{1}{2\Lambda} \left(\xi_{\alpha\mu}^{-1} + \frac{1}{2} \omega_{\alpha\mu} \right) \quad (\text{B.4})$$

$$U_{\alpha\mu} = \frac{1}{2\Lambda} \left(-\xi_{\alpha\mu}^{-1} + \frac{1}{\eta} W_{\alpha\mu} \right) \quad \text{(B.4) cont.}$$

whose fluctuations are just the ones defined in Eq. (4.8). Unlike $\tilde{V}$ and $\tilde{U}$, $V_{\alpha\mu}$, $U_{\alpha\beta}$ are good covariant four vectors. One can also define contravariant vectors by

$$\begin{aligned} \hat{V}^{\mu\alpha} &= \frac{\Lambda}{2} \left(\xi^{\mu\alpha} + \eta W^{-1\mu\alpha} \right) \\ \hat{U}^{\mu\alpha} &= \frac{\Lambda}{2} \left(-\xi^{\mu\alpha} + \eta W^{-1\mu\alpha} \right) \end{aligned} \quad \text{(B.5)}$$

It is clear that, only for $U, \tilde{U} = 0$ one has $\hat{V} = V^{-1}$ allowing the identification of V with the usual vierbein. Terms with derivatives of the vierbein appear in A_b as extra terms added to $J_4(\phi)$ in Eq. (B.1). Actually they would appear as homogeneous functions of ∂_μ and $\xi^{\mu\alpha}$ so as to give a dimensionless quantity with the same number of upper and lower space-time indices. In particular, the term quadratic in the derivatives of the symmetric vierbein will have the invariant form:

$$\frac{1}{6} I_2 \det \xi^{-1} R(\xi) \xrightarrow{U \rightarrow 0} \frac{1}{6} \Lambda^2 I_2 \det V \cdot R(V) \quad \text{(B.6)}$$

to be identified, in the low energy limit $U \rightarrow 0$, with the usual Einstein action [see Eq. (4.30) and following discussion]. Since (B.6) had dimensions m^4 the coefficient is a pure number, $1/6 I_2$, with I_2 defined by Eq. (A.18).

As soon as the dependence of A_b on $A_{\mu,\alpha\beta}$ and $\lambda^{\mu,\alpha\beta}$ is also considered, new types of terms appear. For instance, Eq. (4.11) will originate from a term in A_b of the type

$$\frac{1}{2} \det \xi^{-1} \left[\lambda^{\mu,\alpha\beta} \lambda^{\nu,\gamma\delta} \xi_{\alpha'\mu}^{-1} \xi_{\beta'\nu}^{-1} \hat{O}_{\alpha'\beta'}^{\alpha\beta,\gamma\delta} \right] \quad \text{(B.7)}$$

The quadratic term in $A_{\mu,\alpha\beta}$ [Eq. (4.9)] can be combined with those containing $\partial_\nu \xi_a^{\mu\alpha}$ to give the invariant term:

$$4 I_2 \det \xi^{-1} \left[B_{\mu,\alpha\beta} B_{\nu,\gamma\delta} \xi^{\mu\alpha'} \xi^{\nu\beta'} \hat{O}_{\alpha'\beta'}^{\alpha\beta,\gamma\delta} \right] \quad \text{(B.8)}$$

where:

$$B_{\mu, \alpha\beta} = A_{\mu, \alpha\beta} + \frac{1}{8} \left(\xi_{\nu\alpha}^{-1} \partial_{\mu} \xi_{\beta}^{\nu} - (\alpha \leftrightarrow \beta) \right) \quad (B.9)$$

Finally, there will be also a term of the form

$$\frac{1}{2} \eta^4 \det \xi^{-1} B_{\nu, \alpha\beta} \lambda^{\mu, \alpha\beta} \xi^{\nu\delta'} \xi_{\delta'\mu}^{-1} \tilde{O}_{\delta'\delta'}^{\alpha\beta, \delta\delta'} \quad (B.10)$$

Equations (B.8) and (B.9) clearly display in an invariant form the fact that the Goldstone field $\xi_a^{\mu\alpha}$ is eaten up in a Higgs-like mechanism, which provides an invariant mass for the B field.

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FIGURE CAPTIONS

- Figure 1 : A typical fermion loop needed to evaluate the trlog term of A_b .
This loop gives $\delta A_b / \delta \xi^{\mu\alpha} \delta \xi^{\nu\beta}$.
- Figure 2 : Examples of $1/N$ counting for bosonic loops. Wavy lines with a dot refer to (fermion loop) induced propagators of a generic auxiliary field.

FIG. 1

$$\xi \times \text{[bubble]} \times \xi = 0 \quad (N)$$

$$\xi \times \text{[bubble with vertical line]} \times \xi = 0 \left(N \left(\frac{1}{\sqrt{N}} \right)^2 \right) = 0 \quad (1)$$

$$\xi \times \text{[bubble with wavy lines]} \times \xi = 0 \left(N^2 \left(\frac{1}{\sqrt{N}} \right)^4 \right) = 0 \quad (1)$$

$$\xi \times \text{[bubble with wavy lines and internal structure]} \times \xi = 0 \left(N^4 \left(\frac{1}{\sqrt{N}} \right)^{10} \right) = 0 \quad (1/N)$$

FIG. 2