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TOWARDS UNIFICATION WITH JUST FUNDAMENTAL FERMIONS

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ABSTRACT

I discuss a class of fermionic Lagrangians. A classical gauge invariance induces dynamical gauge fields quantum mechanically. The construction requires flavours and a unification of all gauge couplings at a common Landau pole mass. I then discuss a toy model of a 'tumbling' gauge theory in two dimensions and exhibit the difference between an elementary and dynamical Higgs mechanism. Finally, I briefly discuss the possibility of a unified theory with just fermions.

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Fermions are an essential constituent in Nature. Could they be the only fundamental constituents? For instance, could all other particles required by low energy phenomenology, e.g., gauge bosons and Higgs scalars, be bound states of just fermions? Here I show how this appealing suggestion might be realized.

First I discuss a class of four-dimensional fermionic Lagrangians with a local gauge invariance, but no explicit gauge field. In this class of Lagrangians a classical gauge symmetry induces dynamical gauge fields at the quantum level. The resulting effective action is equivalent to a standard gauge theory at energies less than an ultra-violet cut-off Λ . To give an unambiguous meaning to the theory a cut-off is required, and it cannot be removed with our present understanding of the theory. However, to establish the existence of dynamical gauge bosons renormalizability is not crucial due to a decoupling between the massless sector and a spectrum at Λ . A QED-like model, non-Abelian-like model and an Abelian-Higgs-like model are successively discussed. For non-Abelian groups the construction requires: i) flavour (or families) since, ultimately, the theory has to be infra-red free; ii) for non-simple groups a unification of all gauge couplings at a common Landau pole mass of the generated effective theory. This work is the result of a collaboration with D. Amati, R. Barbieri and G. Veneziano and is described in Ref. 1.

To discuss unified field theories it is necessary to consider the mechanism of symmetry breaking. In theories with just fermions an appealing possibility is that of 'tumbling'. That is, if a fermion bilinear, which is not a scalar of the gauge group, acquires a non-zero vacuum expectation value (VEV) the gauge symmetry will

be dynamically broken. I present an explicit example of a 'tumbling' gauge theory in two dimensions. The model is a non-Abelian generalization of the CP^{n-1} model with fermions, having a global $U(n)$ and local $U(1)$ symmetry. This model allows a study of dynamical Higgs mechanisms and shows the difference between dynamical and elementary Higgs bosons. This was done in collaboration with A. D'Adda and P. Di Vecchia². I discuss some interesting features of the model, in particular the problem of vacuum alignment. This is in progress, in collaboration with M. Peskin.

Finally I briefly discuss the possibility and problems of constructing a unified theory in four dimensions with just fermions.

There have been previous attempts to generate gauge bosons from fermions³, trying to convert a global symmetry of the classical theory into a local symmetry of the quantum theory. This approach has met with limited success⁴ and has to circumvent a no-go theorem⁵. Alternatively, in the two-dimensional (bosonic) CP^{n-1} models⁶ a local gauge invariance (but no explicit gauge field) classically is converted into a dynamical gauge field quantum mechanically.

Following this second line of approach, consider the four-dimensional, fermionic Lagrangian

$$\mathcal{L} = i\bar{\psi} \not{\partial} \psi - i\bar{\psi} \gamma_{\mu} \psi \frac{\overleftrightarrow{\partial}_{\mu} \psi}{\bar{\psi} \psi} \quad (1)$$

where ψ is a Dirac field^{*)}. Now $\mathcal{L}$ is invariant under a local $U(1)$ transformation, $\psi(x) \rightarrow e^{i\alpha(x)}\psi(x)$. It is scale invariant and contains no dimensionless parameter. However, it is not obviously renormalizable. (That it contains no dimensionless parameter might, ultimately, make it well-behaved in the ultra-violet, as has recently been suggested for a similar, scalar theory⁸.)

For (1) to be well-defined $\bar{\psi}\psi$ should have a non-zero VEV. A correct treatment may well result in $\langle \bar{\psi}\psi \rangle \neq 0$. However, with present techniques we are unable to treat Lagrangians with such a high degree of non-linearity. Thus we introduce auxiliary fields to make (1) quadratic, and to give $\bar{\psi}\psi$ a non-zero VEV. Hence we consider the theory defined by the (Euclidean) generating functional

$$Z = \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}A_{\mu} \mathcal{D}\lambda_{\mu} \mathcal{D}\lambda \exp - \int d^4x [\bar{\psi}(\not{\partial} + i A) \psi + \quad (2)$$

*) This Lagrangian was first considered by Sugawara⁷ in a similar context. We have only recently become aware of this paper. I wish to thank a member of a seminar audience in Copenhagen for informing me of it.

$$+ i\lambda(\bar{\psi}\psi - v) + i\lambda_{\mu}(v A_{\mu} - \bar{\psi} i \overleftrightarrow{\partial}_{\mu} \psi) \quad (2)$$

cont

After elimination of the auxiliary gauge field, A_{μ} , and Lagrange multipliers, λ_{μ} and λ , this Lagrangian reduces to (1), supplemented by the condition $\bar{\psi}\psi = v$. (The VEV for $\bar{\psi}\psi$ could be introduced through a potential without modifying our conclusions.) We introduce an ultra-violet cut-off, Λ , and, to preserve gauge invariance, use the Pauli-Villars regularization.

Integrating over fermion fields gives

$$Z = \int \mathcal{D}A_{\mu} \mathcal{D}\lambda_{\mu} \mathcal{D}\lambda \exp - S_{\text{eff}} \quad (3)$$

where

$$S_{\text{eff}} = \int d^4x (i \lambda_{\mu} A_{\mu} v - i \lambda v) - \text{tr} \log (\not{D} + \lambda_{\mu} \partial_{\mu} + i\lambda) \quad (4)$$

Solving by saddle-point approximation in the remaining fields yields the Lorentz invariant solution

$$\langle \lambda_{\mu} \rangle = \langle A_{\mu} \rangle = 0; \quad \langle i\lambda \rangle = m \quad (5)$$

where m is the generated fermion mass given by

$$v = -\text{tr} \int \frac{d^4p}{(2\pi)^4} \frac{1}{(i\not{p} + m)} \quad (6)$$

Expanding around the saddle-point solution we find the Green functions. These are:

$$\frac{\delta^2 S_{\text{eff}}}{\delta A_{\mu} \delta A_{\nu}} = \langle A_{\mu} A_{\nu} \rangle = (q_{\mu} q_{\nu} - q^2 g_{\mu\nu}) \frac{1}{12\pi^2} \log \left(\frac{\Lambda^2}{q^2 + 4m^2} \right) \quad (7)$$

This is just the standard QED vacuum polarization with $e = 1$. To determine whether or not A_{μ} has acquired dynamics we have also to consider $\langle \lambda_{\mu} \lambda_{\nu} \rangle$ and $\langle \lambda_{\mu} A_{\nu} \rangle$. These are:

$$\frac{\delta^2 S_{\text{eff}}}{\delta \lambda_{\mu} \delta \lambda_{\nu}} = \langle \lambda_{\mu} \lambda_{\nu} \rangle = g_{\mu\nu} O(\Lambda^4) + (q_{\mu} q_{\nu} - q^2 g_{\mu\nu}) O(\Lambda^2) \quad (8)$$

showing, upon rescaling $\ell_{\mu} = \lambda_{\mu} \Lambda$, that it is a non-propagating field for all $q^2 \ll \Lambda^2$. Finally

$$\frac{\delta^2 S_{\text{eff}}}{\delta \ell_{\mu} \delta A_{\nu}} = \langle \ell_{\mu} A_{\nu} \rangle = O\left(\frac{m}{\Lambda}\right) (q_{\mu} q_{\nu} - q^2 g_{\mu\nu}) \quad (9)$$

because of a cancellation between the tree and one-loop contributions. Whence, because of the invariant form of (9), A_{μ} rigorously describes a massless photon in interaction with fermions of mass m . Finally,

$$\frac{\delta^2 S_{\text{eff}}}{\delta \lambda \delta \lambda} = \langle \lambda \lambda \rangle \sim O(\Lambda^2) \quad (10)$$

Thus, for all $q^2 \ll \Lambda^2$, $\lambda(x)$ is a non-propagating field. From gauge invariance there is no λ - A_μ mixing.

We should now do a systematic study of higher point functions. An inspection of diagrams with insertions of 'heavies' (λ_μ and λ) suggests that the effective action is equivalent to that of standard QED for $q^2 \ll \Lambda^2$, deviations being of order q/Λ . The only quantities for which 'heavies' give renormalizations of $O(1)$ are the fermion mass and the vacuum polarization, (7). Thus there is a decoupling between the 'heavies' and the light spectrum. Upon rescaling we obtain the standard $-(1/4)F_{\mu\nu}F^{\mu\nu}$ kinetic energy term for A_μ , with renormalized charge

$$e_R^2|_{q^2=0} = \left[\frac{1}{12\pi^2} \log \left(\frac{\Lambda^2}{m_R^2} \right) + O(1) \right]^{-1} \quad (11)$$

at the one-loop level. $O(1)$ includes the 'heavy' insertions. Similarly, $m_R = O(m)$. Equation (11) shows that the cut-off, Λ , should be identified with the position of the Landau pole for the effective QED Lagrangian.

We could also consider (1) with many fermion flavours and do a $1/N_f$ expansion. The saddle-point approximation is more justified in this case. 'Heavy' insertions are further suppressed in this case by powers of $1/N_f$.

Finally we mention that Lagrangians such as (1) have no Noether current - the standard construction leads to $0 = 0$, as in the CP^{n-1} model. This is due to having an auxiliary gauge field initially. The quantum mechanical generation of kinetic terms for A_μ gives rise to a non-trivial conserved current.

Generalizing our construction to non-Abelian groups we consider a Lagrangian with a local $U(n)$ invariance,

$$\mathcal{L} = i\bar{\psi}^{i\alpha} \not{\partial} \psi_{i,\alpha} + \bar{\psi}^{i,\alpha} W_\alpha^\beta \psi_{i,\beta} \quad (12)$$

where

$$W_\mu = \frac{1}{2} M_\mu M_\mu^{-1} + \text{h.c.} \quad (13)$$

$$(M_\mu)^\beta_\alpha = -\psi^{Ti\beta} C^{-1} i\partial_\mu \psi_{i\alpha}, \quad M^\beta_\alpha = \psi^{Ti\beta} C^{-1} \psi_{i\alpha} \quad (14)$$

$\psi_{i\alpha}$ is a (left-handed) Weyl spinor; $i = 1, \dots, f$ is the flavour index and the gauge index is $\alpha = 1, \dots, n$. Equation (12) is invariant under

$$\psi_{i\alpha}(x) \rightarrow U_{\alpha}^{\beta}(x) \psi_{i\beta}(x); \quad U(x) U^{\dagger}(x) = U^{\dagger}(x) U(x) = \mathbb{1} \quad (15)$$

To gauge a subgroup G of $U(n)$ then

$$W_{\mu} = \sum_{a \in U(n)} \lambda^a \text{Tr}(W_{\mu} \lambda^a) \rightarrow \sum_{b \in G} \lambda^b \text{Tr}(W_{\mu} \lambda^b) \quad (16)$$

where λ^a are the generators of $U(n)$ in the fundamental representation and λ^b span the subgroup, G .

For a vector-like theory we can proceed by introducing

$$\lambda^{\alpha}_{\beta} (M^{\beta}_{\alpha} - v \delta^{\beta}_{\alpha}) \quad (17)$$

and use Pauli-Villars regularization. For a theory with chiral fermions (17) would break the gauge group down to a vector subgroup. We can get around this problem by being more subtle. Also, there are indications from the Abelian case that we did not really need a VEV as long as we were off-shell. However, there are problems regularizing chiral fermions⁹.

For vector-like theories we can establish the dynamical generation of massless gauge bosons. At low energies their interactions with fermions will be the same as in standard gauge theories. Thus,

$$\alpha(q^2)^{-1} = \left(\frac{f}{6\pi} - \frac{11n}{12\pi} \right) \left[\log \frac{\Lambda^2}{q^2} + O(1) \right], \quad (18)$$

whence the effective coupling constant will be $O(1)$ for $q^2 \ll O(\Lambda^2)$. Hence, for the theory to be meaningful at low energies ($q^2 \ll \Lambda^2$), it must be infra-red free such that α decreases with decreasing q^2 . This requires the number of flavours to be greater than the number of colours so that the fermionic contribution to the β function dominates over that of gluons. Furthermore, for non-simple groups the couplings have to be unified at Λ . This plays the role of a universal Landau pole for the effective theory. At low energies couplings develop according to their respective β functions. Our 'Landau pole unification' scheme is similar to that suggested in Ref. 10 on the basis of different motivations.

A realistic theory necessitates the treatment of broken gauge theories. This can be done within our construction. For instance, consider the Abelian Lagrangian

$$\begin{aligned} \mathcal{L} = & i\bar{\psi}\not{\partial}\psi + i\bar{\chi}\not{\partial}\chi + \lambda(\bar{\psi}\chi\bar{\chi}\psi - v^2) - \\ & - \left(\frac{e_1\bar{\psi}\gamma_{\mu}\psi + e_2\bar{\chi}\gamma_{\mu}\chi}{e_1 + e_2} \right) \left(\frac{\bar{\psi}\overleftrightarrow{\partial}_{\mu}\chi}{\phi\chi} + \frac{\bar{\chi}\overleftrightarrow{\partial}_{\mu}\psi}{\bar{\chi}\psi} \right) \end{aligned} \quad (19)$$

where ψ, χ are Weyl spinors, one left- and one right-handed. Equation (19) has a local $U(1)$ invariance

$$\psi \rightarrow e^{ie_1\alpha(x)} \psi; \quad \chi \rightarrow e^{ie_2\alpha(x)} \chi$$

and a global $U(1)$ symmetry

$$\psi \rightarrow e^{ic} \psi, \quad \chi \rightarrow e^{-ic} \chi$$

When $\bar{\psi}\chi, \bar{\chi}\psi$ acquire non-zero VEV, two possible breaking patterns are possible:

$$\left. \begin{array}{l} e_1 = e_2 \\ e_1 \neq e_2 \end{array} \right\} U(1)_{\text{local}} \otimes U(1)_{\text{global}} \left\{ \begin{array}{l} \supset U(1)_{\text{local}} \\ \supset U(1)'_{\text{global}} \end{array} \right.$$

where $U(1)_{\text{global}}$ is a combination of the original $U(1)_{\text{local}}$ and $U(1)_{\text{global}}$ symmetries.

To proceed we make (19) quadratic in the fields as previously. When $e_1 = e_2$, A_μ again describes a massless photon and the 'heavies' decouple. In addition we obtain a Goldstone boson from chiral symmetry breaking.

For $e_1 \neq e_2$ we obtain

$$\begin{aligned} \frac{\delta^2 S_{\text{eff}}}{\delta A_\mu \delta A_\nu} &= \langle A_\mu A_\nu \rangle = \left(\frac{e_1^2 + e_2^2}{12\pi^2} \right) (q_\mu q_\nu - q^2 g_{\mu\nu}) \log \left(\frac{\Lambda^2}{q^2 + 4m^2} \right) \\ &+ \frac{1}{4\pi^2} g_{\mu\nu} m^2 (e_1 - e_2)^2 \log \left(\frac{\Lambda^2}{q^2 + 4m^2} \right) \end{aligned} \quad (20)$$

Thus the 'photon' acquires a mass of

$$m_Y^2 = 3m^2 \frac{(e_1 - e_2)^2}{e_1^2 + e_2^2} \left[1 + O \left[\left(\log \left(\frac{\Lambda^2}{q^2} \right) \right)^{-1} \right] \right] \quad (21)$$

i.e., $m_Y^2 \propto$ generated fermion mass.

Here, the Goldstone boson has been eaten to give rise to the longitudinal component of the photon. This is just the Higgs mechanism. All left-over scalars are of $O(\Lambda)$. We do not find a light Higgs scalar, though this is not required for renormalizability in this Abelian example.

So what next? There are several natural extensions to the scheme discussed here. Gravity has already been included in such a construction by Amati and Veneziano¹¹ and will be discussed here by Veneziano¹². When gravity is included the cut-off, Λ , naturally becomes the Planck mass. Alternatively, we could investigate a non-linear realization of supersymmetry in such a scheme, in a manner similar to that of Volkov and Akulov¹³. Arguably the most natural, and maybe the most ambitious, extension is to try to construct a phenomenologically realistic model based on our 'Landau pole unification' scheme. For fermionic Lagrangians such as ours, we have to discuss the breaking of the (non-Abelian) gauge symmetry. Now, if a fermion bilinear, which is not a singlet of the gauge group, acquires a non-zero VEV the gauge group will be broken^{14,15}. If several energy scales result then this is known as tumbling¹⁵. It would be natural to try to merge our scheme with that of tumbling. However, this would be plagued with the usual technical problem of trying to find a gauge invariant regularization for chiral fermions⁹.

I am going to be more modest and discuss a two-dimensional toy model of a 'tumbling' gauge theory^{*}). There are several motivations for going over to two dimensions. (This is demonstrated by the fact that several speakers at this meeting have ventured into the playground of two dimensions - or lower). In two dimensions one can do explicit calculations. This can help gain insight and confidence for the four-dimensional world [e.g., a study of the CP^{n-1} model with fermions⁶ helped elucidate the $U(1)$ problem¹⁶]. Recently it has been shown how to regularize chiral fermions in two dimensions¹⁷. In our case we can write down an explicit Lagrangian which undergoes 'tumbling' and display the difference between dynamical and elementary Higgs mechanisms. (Tumbling in two dimensions has also been studied by Banks et al.¹⁷, though the tumbling picture was never realized in Ref. 17. We differ from Ref. 17 in that our model has a quartic interaction).

The model I am going to discuss is a non-Abelian generalization of the CP^{n-1} model with fermions. It has a global $U(n)$ symmetry, represented by the index $i = 1, \dots, n$, and a local $U(\ell)$ symmetry, represented by the index $\alpha = 1, \dots, \ell$. In this model the scalar Z fields take their values on a Grassmannian manifold, i.e.,

$$\bar{Z}^{i\alpha} Z_{i\beta} = \frac{n}{2f} \delta^\alpha_\beta \quad (22)$$

where f is a coupling constant. The Lagrangian is

$$\mathcal{L} = \bar{D}_\mu^\alpha \bar{Z}^{i\beta} D_{\mu\alpha} \gamma_{Z_{i\gamma}} + i\lambda_\alpha^\beta (\bar{Z}^{i\alpha} Z_{i\beta} - \frac{n}{2f} \delta^\alpha_\beta) + \quad (23)$$

^{*}) I use the word 'tumbling' rather loosely in what follows for want of a better word.

$$\begin{aligned}
& + \bar{\psi}_{L\alpha}^i D_{-\beta}^\alpha \psi_{Li}^\beta + \bar{\psi}_R^{i\alpha} D_{+\alpha}^\beta \psi_{Ri\beta} \\
& - \frac{1}{\sqrt{n}} [\bar{\psi}_{L\alpha}^i \psi_{Ri\beta} \bar{\phi}^{\alpha\beta} + \bar{\psi}_R^{i\alpha} \psi_{Li}^\beta \phi_{\alpha\beta}] + \frac{1}{g} \phi_{\alpha\beta} \bar{\phi}^{\beta\alpha}
\end{aligned} \tag{23}$$

cont.

where

$$D_{\mu\alpha}^\beta = \partial_\mu \delta_\alpha^\beta + \frac{i}{\sqrt{n}} A_{\mu\alpha}^\beta \tag{24}$$

λ_α^β imposes the constraint (22). ϕ , $\bar{\phi}$ and A_μ are auxiliary fields. [Equation (23) has already been linearized and written in a form convenient for a $1/n$ expansion],

$$\phi_{\alpha\beta} = \frac{g}{\sqrt{n}} \bar{\psi}_{L\alpha}^i \psi_{Ri\beta} \tag{25}$$

and

$$D_\pm = D_0 \pm i D_1 \tag{26}$$

In (23) the left- and right moving fermions are in different representations of the gauge group - ψ_R in (ℓ) , ψ_L in $(\bar{\ell})$. (We could have taken them in the same representations. This is considered in Ref. 2.) The left- (right-) mover couples only to $D_-(D_+)$.

The field $\phi_{\alpha\beta}$ is a Lorentz scalar and a scalar of $U(n)$ allowing a $1/n$ expansion. However, it is not a scalar under the $U(\ell)$ gauge group. Thus, if it acquires a non-zero VEV the gauge group will be broken.

Now (23) has a hidden symmetry. Since ψ_L couples only to D_- and ψ_R to D_+ , A_- and A_+ can be two independent fields as far as the fermionic sector is concerned. Thus, (23) has a $U(\ell) \otimes U(\ell)$ symmetry, which is broken by the gauge field in the bosonic sector. It is also broken by a non-Abelian anomaly. This anomaly plays a crucial role in the properties of the model, as will be shown later.

Integrating over ψ and Z we obtain the effective action

$$\begin{aligned}
S_{\text{eff}} = & \text{ntrlog}(-D^2 + \frac{i}{\sqrt{n}} \lambda_\alpha^\beta) - \text{ntrlog} \begin{pmatrix} D_{+\alpha}^\beta & -\frac{1}{\sqrt{n}} \phi_{\alpha\beta} \\ -\frac{1}{\sqrt{n}} \bar{\phi}^{\alpha\beta} & D_{-\beta}^\alpha \end{pmatrix} \\
& + \frac{1}{g} \int d^2x \phi_{\alpha\beta} \bar{\phi}^{\beta\alpha} + i \int d^2x \lambda_\alpha^\beta \delta_a^\beta \sqrt{n}/2f
\end{aligned} \tag{27}$$

Solving in a $1/n$ expansion gives

$$\langle \phi_{\alpha\beta} \rangle = \sqrt{n} M_{\alpha\beta} \quad (28)$$

and

$$M_{\alpha\beta} M^{\beta\gamma} = \delta_{\alpha}^{\gamma} M^2 \quad (29)$$

where M is the generated fermion mass, obtained by dimensional transmutation of g . From (28) and (29) we see that ϕ has acquired a non-zero VEV. Thus, the gauge group is broken. However, (29) does not tell us the direction of symmetry breaking. There is a manifold of vacuum states, related by a chiral transformation, and determine the alignment of the vacuum we have to go beyond leading order. This is in progress¹⁸.

Expanding in $1/n$ we obtain

$$S_{\text{eff}}^{(2)} = \int d^2x \left\{ (\partial_{\mu} A_{\nu\alpha}^{\beta} - \partial_{\nu} A_{\mu\alpha}^{\beta})^2 + (\partial_{\mu} B_{\nu\alpha}^{\beta} - \partial_{\nu} B_{\mu\alpha}^{\beta})^2 \right. \\ + 4M^2 B_{\mu\alpha}^{\beta} B_{\mu\beta}^{\alpha} + |\partial_{\mu} \phi_{-\alpha}^{\beta}|^2 + |\partial_{\mu} \phi_{+\alpha}^{\beta}|^2 + 4M^2 (\phi_{+\alpha}^{\beta})^2 \\ \left. - 4M \partial_{\mu} \phi_{-\alpha}^{\beta} B_{\mu\beta}^{\alpha} + 4iM \epsilon_{\mu\nu} \partial_{\mu} A_{\nu\alpha}^{\beta} \phi_{+\beta}^{\alpha} \right\} \quad (30)$$

where the fields diagonalizing the spectrum are:

$$B_{\mu\alpha}^{\beta} = \frac{1}{2} (A_{\mu\alpha}^{\beta} - M_{\alpha\gamma} \frac{A_{\mu\gamma}^{\beta}}{M^2} M^{\gamma\beta}) \quad (31)$$

and we redefine

$$A_{\mu\alpha}^{\beta} = \frac{1}{2} (A_{\mu\alpha}^{\beta} + M_{\alpha\gamma} \frac{A_{\mu\gamma}^{\beta}}{M^2} M^{\gamma\beta}) \quad (32)$$

$$\phi_{-\alpha}^{\beta} = \frac{1}{2M} (\phi_{\alpha\gamma} M^{\gamma\beta} - M_{\alpha\gamma} \bar{\phi}^{\gamma\beta}) \quad (33)$$

these are would-be Goldstone bosons from chiral symmetry breaking.

$$\phi_{+\alpha}^{\beta} = \frac{1}{2M} (\phi_{\alpha\gamma} M^{\gamma\beta} + M_{\alpha\gamma} \phi^{\gamma\beta}) \quad (34)$$

The gauge bosons B_{μ} have acquired a mass, whilst A_{μ} remain massless. Since the unitary matrix $M_{\alpha\beta}$ does not appear in $S^{(2)}$ we do not know the unbroken subgroup.

To analyze the spectrum of states we may take

$$M_{\alpha\beta} = M \delta_{\alpha\beta} \quad (35)$$

The unbroken subgroup is then $O(l)$ [similar considerations can be made for the other subgroups of $SU(l)$, e.g., $S_p(l)$]. Here the components of ϕ_{-} not in $O(l)$ are eaten by B_{μ} in a dynamical Higgs mechanism. Defining

$$B_{\mu\alpha}^{\beta} = B_{\mu}^i(t^i)_{\alpha}^{\beta}, \quad A_{\mu\alpha}^{\beta} = A_{\mu}^a(\tau^a)_{\alpha}^{\beta} \quad (36)$$

$$\phi_{-\alpha}^{\beta} = B^i(t^i)_{\alpha}^{\beta} + A^a(\tau^a)_{\alpha}^{\beta}, \quad \phi_{+\alpha}^{\beta} = C^s(T^s)_{\alpha}^{\beta} \quad (37)$$

where T^s are the generators of $SU(\ell)$ and

$$\tau^a = (T^a - T^{a*}) \frac{1}{2}, \quad t^i = (T^i + T^{i*}) \frac{1}{2}, \quad (38)$$

τ^a spanning the $O(\ell)$ algebra. The index a (i) runs over the unbroken $O(\ell)$ subgroup [broken part of $U(\ell)$]. Finally we obtain

$$S^{(2)} = \int d^2x \left\{ F_{\mu\nu}^a{}^2 + G_{\mu\nu}^i{}^2 + 4M^2 W_{\mu}^{i2} + (\partial_{\mu} A^a)^2 \right. \\ \left. + (\partial_{\mu} C^s)^2 + 4M^2 C^{s2} + 4iMA^a \epsilon_{\mu\nu} \partial_{\mu} A_{\nu}^a \right\} \quad (39)$$

where

$$W_{\mu}^i = B_{\mu}^i - 2M\partial_{\mu} B^i \quad (40)$$

and $F_{\mu\nu}^a$, $G_{\mu\nu}^i$ are the kinetic terms for A^a and W_{μ}^i respectively. From (39) we see that the B^i fields have been eaten by W_{μ}^i and disappear from the physical spectrum. Thus, we have a massive W_{μ}^i and massive C^s field. The pseudo-scalar A^a couples to the unbroken gauge field A^a via a non-Abelian anomaly. This is a generalization of the $U(1)$ anomaly and here gives mass to the entire A^a multiplet. In the absence of this anomaly term the A^a would be Goldstone bosons from chiral symmetry breaking. This term would be absent if we had elementary Higgs bosons. The presence of such an anomaly term signals the difference between dynamical and elementary Higgs mechanisms. The unbroken gauge fields also acquire a mass. This screens the long-range force, as in the massless Schwinger and CP^{n-1} models. It is a consequence of there being massless fermions in the original Lagrangian. The implications of our results are being investigated and could be generalized to more realistic models.

Perhaps the most interesting, and bizarre, feature of the model is that, to $O(1)$ in the $1/n$ expansion, we are unable to determine the unbroken subgroup. There is a manifold of vacuum states related by a chiral transformation. To determine the alignment of the vacuum we have to calculate the vacuum energy to more orders in $1/n$. It is interesting to speculate that there may be some hidden symmetry protecting the vacuum state to this order. This is in progress¹⁸.

We can consider other representations for the left- and right-moving fields. The representations have to be chosen such that anomalies in the gauged current are cancelled between the left- and right-handed representations. There will be an anomaly

in the ungauged current analogous to the one found here, which will give mass to would-be Goldstone bosons. Such models could also provide insight into the role of the anomaly in tumbling. This is currently under investigation.

Finally, I wish to consider constructing a realistic model with just fermions. For instance, considering the 5 and $\overline{10}$ representations of SU(5) we can generalize (12) - (14),

$$M_\mu = -\chi^{\text{Ta}\beta} C^{-1} i \partial_\mu \psi_\beta \qquad M = \chi^{\text{Ta}\beta} C^{-1} \psi_\beta$$

and

$$W_\mu = M_\mu M^{-1} + \text{h.c.}$$

where ψ and χ are left-handed Weyl spinors in the 5 and $\overline{10}$ representation of SU(5). If M acquires non-zero VEV then the SU(5) group will be broken. Problems here arise with the regularization of chiral fermions. Clearly it would be interesting if we could construct unification models with just fermions. This is a very ambitious programme. However, the models I have presented indicate that it is not completely impossible. Much work is needed in this direction, but the results could be rewarding.

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