ANTIPROTON PRODUCTION FROM EXTENDED TARGETS
USING A WEIGHTED MONTE CARLO HADRON CASCADE MODEL

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ABSTRACT

We calculate the production of antiprotons generated by high-energy beams on extended targets by a weighted Monte Carlo hadron cascade model. Results are given for beam and target conditions envisaged for the proton-antiproton collider at CERN and the Tevatron project at Fermilab.

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1. INTRODUCTION

At present there are two projects for large proton-antiproton facilities: the $p\bar{p}$ collider now coming into operation at CERN, and the Tevatron under construction at Fermilab. Reliable estimates of the antiproton production rates, taking into account the beam and target parameters envisaged for both machines, are therefore of great importance.

So far such calculations have been done using a weighted Monte Carlo calculation for a special target combined with a magnetic lens [1], by approximated analytical methods [2], as well as by means of specialized Monte Carlo programs [2,3]. These latter Monte Carlo calculations [2,3] also include several approximations; for instance secondary interactions and ionization energy losses are neglected in Ref. 2, elastic and Coulomb scattering of the incident beam in Ref. 3.

The weighted Monte Carlo method is well suited for calculating the particle flux from an extended target irradiated by high-energy hadrons [1,4,5]. The corresponding computer program KASPRO [4] takes into account all essential physical processes governing the interaction of high-energy particles in extended materials. In this paper we apply the method to the production of antiprotons under beam and target conditions corresponding to the ones assumed in the design studies for the CERN $p\bar{p}$ collider and the Fermilab Tevatron.

For a reliable comparison with other results, we have calculated directly the dependence of the particle fluxes on the phase-space emittance of cylindrical targets. Here we limit ourselves to the production of antiprotons: expected numbers of antiprotons in a given phase-space acceptance, influence of the target length, and the choice of different target materials. Our results are compared with the antiproton yields calculated in Refs. 2 and 3.

In Section 2 we briefly characterize the basic features of the model, outline our procedure for the emittance calculation of the generated antiprotons, and discuss one basic input of the model -- the antiproton production formulae applied. The results of the calculations for the CERN and the Fermilab projects are presented in the third section. Section 4 gives a brief summary.

2. CHARACTERISTICS OF THE MODEL FOR THE HADRON CASCADE CALCULATIONS

2.1 Basic features of the cascade model

Our model [1,5] describes the propagation of the hadronic cascade initiated by high-energy particles in extended materials. For the realistic calculation of the particle flux from an extended target, the following physical processes have generally to be taken into account: particle production in inelastic hadron-nucleus collisions, elastic and Coulomb scattering of all cascade particles, as well as ionization energy losses of charged particles.

Particle production processes are described according to empirical formulae for the single-particle inclusive distributions. These formulae are fitted to available experimental data and assume, for most secondaries, approximate scaling properties of the spectra in order to extrapolate into primary and secondary momentum ranges where no data have been taken so far. Actually the particles are generated by a weighted sampling procedure [1], which allows the production of a given particle type to be enhanced within the phase-space region of interest. In this way the statistical significance of the results can be increased considerably, especially for rarely produced particles. This feature of the model makes it efficient for calculating the antiproton fluxes within certain small phase-space regions.

Elastic and Coulomb scattering as well as ionization energy losses within the material are treated for all cascade particles on the basis of approximated formulae discussed in Ref. 5.

2.2 Calculation of the particle flux as a function of the transverse phase-space emittance

For a reliable comparison with design requirements and experimental data, we calculate the number of particles of a given type as a function of the secondary momentum p and the transverse phase-space emittance ϵ of the target (normalized to incident particles). The results are given in form of the distribution $dN/(dp/p)$ (in %) within a given emittance ϵ . The method used for the emittance calculation

was adopted from Ranft and Nelson [6]. The reference plane transverse to the beam direction is positioned at half the target length L . For each secondary particle produced from the target we know the coordinates of the point (x_0, y_0, z_0) where it leaves the target, its direction cosines $(\cos \theta_x, \cos \theta_y, \cos \theta_z)$, and the angle θ_z with respect to the beam axis (z -direction). So we can calculate the coordinates of the particles in the reference plane (x, y) , and the projections of the scattering angle θ_z on to the x - z and y - z planes (ϑ_x, ϑ_y):

$$\begin{aligned} x &= x_0 + (0.5L - z_0) \cos \theta_x \\ y &= y_0 + (0.5L - z_0) \cos \theta_y \\ \vartheta_x &= \theta_z \frac{\cos \theta_x}{\sqrt{\cos^2 \theta_x + \cos^2 \theta_y}} \\ \vartheta_y &= \theta_z \frac{\cos \theta_y}{\sqrt{\cos^2 \theta_x + \cos^2 \theta_y}} \end{aligned} \tag{1}$$

Since the target is defined to be cylindrically symmetric, we bin each produced particle into all emittance ellipses larger than

$$\begin{aligned} E_x &= \pi |x| |\vartheta_x| , \\ E_y &= \pi |y| |\vartheta_y| , \end{aligned} \tag{2}$$

finally correcting the resulting distributions by a factor of 0.5.

2.3 Discussion of the empirical antiproton production formulae

Although available data have been used to adjust the parameter of the empirical particle production formulae in our model, there is little reliable experimental information on antiproton production, especially in the interesting primary and secondary momentum ranges. Therefore, to be definitive and to allow reasonable comparisons with other estimates of antiproton cross-sections, we now describe the input used for our calculations.

The antiproton momenta are sampled according to the following distribution in the c.m.s.:

$$E \frac{d^3\sigma}{d^3p} \propto f(\vec{p}) = \exp \left[-8.2m_T - 2.2 \left(\frac{y}{\frac{1}{2}\ln\sqrt{s}} \right)^2 \right] , \quad (3)$$

$$m_T^2 = m_p^2 + p_T^2$$

$$y = \frac{1}{2} \ln \frac{E + p_L}{E - p_L} ,$$

where m_T and y are the transverse mass and the rapidity, respectively. The current method uses this distribution normalized to one. As additional input fixing the average multiplicity, we extracted the inelasticities

$$K_{\bar{p}} = \langle n_{\bar{p}} \rangle \langle E_{\bar{p}} \rangle / \sqrt{s} \quad (4)$$

from the data. In this formula the average c.m.s. energy of a generated antiproton $\langle E_{\bar{p}} \rangle$ is determined by Eq. (2):

$$\langle E_{\bar{p}} \rangle = \int E_{\bar{p}} f(\vec{p}) \frac{d^3p}{E} / \int f(\vec{p}) \frac{d^3p}{E} . \quad (5)$$

These inelasticities, shown in Table 1, reflect the strong non-scaling behaviour of antiproton cross-sections.

We have reproduced the inclusive distributions, as generated in the model, by sampling events with our Monte Carlo method. The results, shown in Figs. 1a,b for different momenta of the incident protons and the interesting momenta of generated antiprotons, compare reasonably with the data from Dekkers et al. [7] and Allaby et al. [8], and from other estimates based on experimental information. So for $p_{lab} = 80$ GeV/c, Cronin [9] gives a value of

$$\frac{d^2\sigma}{d\Omega dp/p} (p_T = 0, p = 6.5 \text{ GeV/c}) \simeq 30 \text{ mb} \cdot \text{sr}^{-1} \text{ (per nucleon)}$$

quoting a possible range from 15 to 45 mb/sr. Our curve corresponds to about 15 mb/sr.

Another estimate for $p_{lab} = 26$ GeV/c gives the range

$$1.3 \times 10^{-2} < \frac{d^2N}{d\Omega dp} < 2.1 \times 10^{-2} \frac{\bar{p}}{\text{GeV/c} \cdot \text{sr}}$$

for heavy targets such as lead [10], to be compared with our value of about 1.5×10^{-2} deduced from the curve in Fig. 1a.

The large uncertainties in these estimates reflect the unsatisfactory experimental situation and can cause deviations of different calculations for the antiproton yield by about a factor of two.

The value taken in Ref. 3 for the CERN collider design,

$$\frac{d^2N}{d\Omega dp} (p_T = 0, p = 3.5 \text{ GeV/c}) = 2.46 \times 10^{-2} \text{ sr}^{-1} (\text{GeV/c})^{-1}$$

per interacting proton at $p_{\text{lab}} = 26 \text{ GeV/c}$, is larger than our value by a factor of about 1.7; for the invariant cross-section per nucleon at $p_{\text{lab}} = 80 \text{ GeV/c}$, Ref. 2 assumes

$$E \frac{d^3\sigma}{d^3p} (p_T = 0, p = 5.4 \text{ GeV/c}) = 0.8 \text{ mb} \cdot (\text{GeV/c})^{-2},$$

which is again about 1.6 times larger than our value found from Fig. 1b.

3. RESULTS FOR THE ANTIPROTON FLUXES

We calculated the antiproton flux for two sets of beam and target parameters chosen in such a way that we can compare our results with the design requirements of both the $p\bar{p}$ collider at CERN and the Tevatron project at Fermilab. Let us stress once more that we considered only the production from the target; effects of scattering in the lithium lenses are not accounted for.

According to the present choice for the CERN $p\bar{p}$ collider, a tungsten target, with a length $L = 10 \text{ cm}$ and a diameter of about 3 mm , is hit by 10^{13} protons per pulse extracted from the CERN Proton Synchrotron (PS) with 26 GeV/c . The proton beam is assumed to have a Gaussian radial distribution with a standard deviation of 0.75 mm , and a Gaussian angular spread with a standard deviation of 0.5 mrad . The apparatus is adjusted for antiprotons with an optimum momentum of 3.5 GeV/c , to be accepted within a momentum spread $\Delta p/p = 1.5\%$, and a transverse phase-space emittance ϵ of $100 \pi \cdot \text{mm} \cdot \text{mrad}$ (the maximum production angle being 50 mrad) [3]. Under these conditions we obtain from Fig. 2a about 1.4×10^7 antiprotons per pulse within the given emittance, to be compared with 2.5×10^7 given in Ref. 3. The difference of a factor of 1.8 is about the same as the one in the antiproton production spectra used. So, within this uncertainty, both results agree.

In Fig. 3 we demonstrate the dependence of the accepted antiproton number on the spatial dimension of the phase-space ellipse [cf. Eq. (2)]. As expected, the $\bar{p}$ number (within a fixed range of production angles $\theta \leq 50$ mrad) tends to become saturated if the spatial dimension becomes larger than the target radius.

The current design of the Tevatron [2] starts from an 80 GeV/c proton beam focused to an r.m.s. radius of 0.2 mm, with an angular spread of about 0.5 mrad. The antiprotons produced on a tungsten target of length $L = 3$ cm are accepted within a transverse phase-space $\varepsilon = 5 \pi \cdot \text{mm} \cdot \text{mrad}$, with production angles restricted to less than 50 mrad.

For a target length $L = 5.7$ cm, Ref. 2 gives an antiproton number per primary proton of 0.8×10^{-6} , obtained by a Monte Carlo procedure. From Fig. 4a, for a 6 cm tungsten target we find 0.95×10^{-6} , although we use a somewhat smaller production cross-section. In Ref. 2, however, the effect of the lithium lenses is included. Nevertheless there seems to be an inconsistency. Furthermore, from our results in Fig. 4a we expect that the shorter tungsten target of $L = 3$ cm, as envisaged in the present project, gives about the same $\bar{p}$ number. Under the same conditions a beryllium target, even for a smaller beam radius of 0.1 mm, generates only about 60% of the antiprotons compared with the tungsten target. This result is found from a comparison of Figs. 4a and 4b and is in agreement with Ref. 2. However, there is considerable uncertainty regarding the nuclear effects on $\bar{p}$ production in beryllium and tungsten targets. In contrast to Ref. 2 we find that this tendency remains valid also for larger beam radii: viz. a tungsten target irradiated by a proton beam of an r.m.s. radius of 1 mm produces more antiprotons than a beryllium target hit by a beam of 0.5 mm radius. These beam radii were considered in Ref. 2 for the case where the full number of 2.7×10^{13} protons reaches the target in each pulse.

We also find from Figs. 4a,b that production targets shorter than the absorption length (about 10 cm for tungsten and 40 cm for beryllium) are definitely more efficient under these conditions, in agreement with analytical estimates in Ref. 2. We do not find this tendency in the calculations for the $p\bar{p}$ collider at

$p_{lab} = 26 \text{ GeV/c}$ (cf. Figs. 2a,b). In this calculation, however, the r.m.s. beam radius is almost comparable to the target radius. Consequently the number of antiprotons within the given emittance, escaping sideways from the target and covering a smaller distance within the material, is expected to be larger. Therefore the total increase of the generated $\bar{p}$ number with increasing target length could be more effective than the destructive influence of Coulomb and elastic scattering, at least up to target lengths of about one absorption length. (We have verified that for a smaller beam radius of 0.1 mm at this energy of 26 GeV/c, targets shorter than one interaction length are more efficient.)

4. SUMMARY

We calculated antiproton fluxes from beryllium and tungsten targets into a given phase-space emittance by means of our weighted Monte Carlo method for simulating hadronic cascades [1,4,5], using beam and target conditions as applied at the CERN $p\bar{p}$ collider and envisaged at Fermilab for the Tevatron. The influence of the bending lithium lenses was not taken into account. In general we find a reasonable quantitative agreement of our results with the numbers used in design studies [2,3].

Under conditions applied at the CERN facility, our antiproton yield is lower by a factor of about 1.8 compared with the figure given in Ref. 3. However, this difference can be traced back to the input used for the antiproton production spectra: because of the experimental uncertainties, our input spectrum is lower by about this factor than the one used in Ref. 3. For the combination of beam and target parameters chosen at CERN, our results indicate that the optimum target length should be about one interaction length.

Although we start from lower production spectra also at $p_{lab} = 80 \text{ GeV/c}$ (the incident proton momentum for the Tevatron project), our results for the antiproton flux are even slightly higher (10-20%) compared with the numbers given in Ref. 1 for the standard conditions at present envisaged at Fermilab. We agree with the result of Ref. 2 that under these beam and target conditions the optimum target length is less than one absorption length, roughly $L \sim 0.5\lambda_{abs}$.

In contrast to Ref. 1, our calculations indicate that the use of a tungsten target is even preferable for the extreme case where the full number of 2.7×10^{13} protons hits the target in each pulse, and therefore a larger beam radius has to be chosen for a tungsten target than for a beryllium one.

Considering the large uncertainties in the antiproton production cross-sections on nuclei, new data are needed to improve the reliability of calculations for the antiproton yield from extended targets. Such measurements should give information about spectra in the secondary momentum range of about 2-6 GeV/c and especially about its dependence on the atomic number of the target nuclei when going from light materials such as beryllium to heavy ones, for instance tungsten or lead.

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Table 1

Inelasticities for antiproton production in
pp collisions applied in our weighted Monte Carlo method

$\sqrt{s}$ (GeV)	$K_{\bar{p}}$
2.9	0
4.5	0.00011
5.5	0.00022
6.15	0.00042
8.2	0.00071
9	0.0011
10	0.0014
20	0.003
34.6	0.0042
63.4	0.008

Figure captions

Fig. 1 : Comparison of antiproton spectra in pW collisions at several proton momenta, as sampled by means of KASPRO [4] (using a very small target), with data from pPb collisions [7,8]:

a) $p_{\bar{p}} = 4 \text{ GeV/c}$; b) $p_{\bar{p}} = 6 \text{ GeV/c}$.

Within the statistical fluctuations our calculations give the same distributions for Pb and W targets.

Fig. 2 : Number of antiprotons produced by protons, $p_{\text{lab}} = 26 \text{ GeV/c}$, on targets of different lengths for a transverse phase-space emittance of $\epsilon = 100 \pi \cdot \text{mm} \cdot \text{mrad}$ ($\theta_{\text{max}} = 50 \text{ mrad}$) and normalized to incident protons; the r.m.s. beam radius and angular spread are 0.75 mm and 0.5 mrad, respectively:

a) tungsten target, diameter 3 mm; b) beryllium target, diameter 4 mm.

Fig. 3 : Variation of the antiproton number with the spatial dimension of the phase-space ellipse for a tungsten target and the same beam parameter as given in Fig. 2.

Fig. 4 : Number of antiprotons generated by protons of $p_{\text{lab}} = 80 \text{ GeV/c}$ on targets of different lengths, normalized to incident protons, for a phase-space emittance $\epsilon = 5 \pi \cdot \text{mm} \cdot \text{mrad}$ ($\theta_{\text{max}} = 50 \text{ mrad}$):

a) tungsten target, r.m.s. beam radius 0.2 mm and angular spread 0.5 mrad;

b) beryllium target, r.m.s. beam radius 0.1 mm.

Fig. 1a

Fig. 1b

Fig. 2a

Fig. 2b

Fig. 3

Fig. 4a

Fig. 4b