Ref.TH.3221-CERN
Erratum

WHAT DO WE LEARN FROM PROTON-ANTIPROTON DIFFRACTIVE
SCATTERING AT THE CERN COLLIDERS

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I) There is a confusion for which I apologize in references 1), 2) and 3)

- 1) L. Camilleri, talk at the November 20th 1981 Meeting on Antiprotons at CERN.
- 2) G. Carboni et al., (CERN-Napoli-Pisa-Stony Brook Collaboration), Phys. Lett. 108B (1982) 145.
- 3) D. Favart et al., Phys. Rev. Lett. 47 (1981) 1191 and seminar by C. Vander Velde Wilquet, December 7th 1981.

II) It should be stressed that I am using the most accurate data available, which are the unpublished ones, and are subject to changes. I am assuming that the West-Yennie phases used in the extraction of the real parts are sufficiently accurate to allow an estimation of the difference of the real parts (a point currently investigated by R. Cahn). It is more consistent to use the same experiments to get ρ_{pp} and $\rho_{p\bar{p}}$. Equation (23) becomes then:

$$\begin{aligned}\rho_{p\bar{p}} &= 0.073 \pm 0.002 \pm 0.01 \pm 0.005 \\ \rho_{p\bar{p}} &= 0.111 \pm 0.011 \pm 0.02\end{aligned}\tag{23}$$

Eq. (25) becomes

$$(\text{Re } F_{p\bar{p}} - \text{Re } F_{pp}) / \text{Im } F_{p\bar{p}} = 0.038 \pm 0.035$$

Eq. (26) contains a misprint: $\text{tg } \frac{\alpha\pi}{2} \rightarrow \text{cotg } \frac{\alpha\pi}{2}$

Eq. (27) becomes

$$\Delta\rho = 0.012 \pm 0.017$$

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Erratum

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WHAT DO WE LEARN FROM PROTON-ANTIPROTON DIFFRACTIVE SCATTERING
AT THE CERN COLLIDERS

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A B S T R A C T

We point out that the recent proton-antiproton collision experiments at the ISR allow for the first time a comparison with proton-proton collisions and make it possible to test very general predictions based on axiomatic field theory and also to confirm certain aspects of conventional models and to eliminate certain "unorthodox possibilities".

We also discuss what will be the output of the measurement of total cross-sections and elastic scattering at the SPS $p\bar{p}$ collider, and describe what might happen in the ultra-high energy limit.

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21 December 1981

1. - INTRODUCTION

Recently, an impressive collection of experimental results on proton-antiproton collisions at the CERN ISR has been obtained, in particular measurements of total cross-sections, slopes of the elastic diffraction peak, and the real part of the forward scattering amplitude at 2×26.7 GeV centre-of-mass energy ¹⁾⁻³⁾. The precision of the measurements is becoming comparable to that reached in the proton-proton case at the same energy. On the other hand, preliminary results at the SPS proton-antiproton collider at 2×270 GeV have been reported ⁴⁾, and we expect more of them. Generally speaking, the first impression from all these experimental facts is "no surprise". However, especially in the case of the ISR results, where the precision is sufficient to allow a quantitative analysis, we must try to distinguish the facts which are "no surprise" because they fit with general theorems based on universally accepted axioms, from those which agree with theoretical prejudices inspired by models. As we shall see, the ISR results agree first of all with all known exact asymptotic theorems on the comparison of particle-particle and antiparticle-particle collisions obtained by the followers of Pomeranchuk, but they also confirm some theoretical prejudices which were never tested before experimentally at such high energies, in particular the smallness of the "odd signature" amplitude $F_{pp} - F_{p\bar{p}}$. It is only if one uses this theoretical prejudice that one can predict that the total cross-section will continue rising at least up to 10^5 GeV lab energy ⁵⁾.

In this paper we want first to remind the reader of a few general theorems on the comparison between particle-particle and particle-antiparticle amplitudes and show to what extent these theorems are verified experimentally in Section 2. In Section 3 we want to describe on the one hand the rules of the traditional Regge picture, on the other hand some non-orthodox suggestions which have been made in the past, and confront both with experiment which at present tends to favour orthodoxy. We want also to explain why the presently obtained experimental information is necessary to predict the rise of total cross-sections which, hopefully, will be observed very soon at the SPS collider. Finally, Section 4 contains a discussion of the rigorous results concerning the diffraction peak at extremely high energy, which could be relevant at SPS collider energies only if, as has happened many times in particle physics, we have precocious asymptotics.

2. - THE POMERANCHUK-LIKE THEOREMS AND THEIR TEST BY EXPERIMENT

In the old times (around 1968), total cross-sections were believed to approach a finite value at high energies and the Pomeranchuk theorem ⁶⁾⁻⁸⁾ stated that the particle-particle and particle-antiparticle cross-sections should approach the same limit as infinite energy. To prove the theorem one has to postulate that the real part of the odd signature amplitude is growing less fast than the imaginary part of this amplitude times the logarithm of the energy ⁸⁾. Though this assumption is physically reasonable it has never been proved from some more basic principle and it is possible to construct counter-examples to the Pomeranchuk theorem which are not, to the best of our knowledge, in contradiction with the sacred laws of analyticity, unitarity and crossing. However, as they were trying to keep the Pomeranchuk theorem under control with the help of the unitarity condition, Eden ⁹⁾ and Kinoshita ¹⁰⁾ and later on in a more general form Truong and his collaborators ¹¹⁾, discovered a remarkable result.

Theorem I

If one of the two total cross-sections (particle-particle or particle-antiparticle) tends to infinity as the energy approaches infinity, the ratio of the two total cross-sections approaches unity (if this ratio has a limit ; otherwise unity is among the limiting values of this ratio).

This theorem makes use of analyticity and unitarity and more specifically of the fact that the effective maximum angular momentum is ¹²⁾ $L_{\max} = C \sqrt{E} \log E$.

The early version of the theorem ^{9),10)} was more restrictive, postulating $\sigma_{\text{total}} \sim (\log E)^\gamma$, $\gamma > 0$, but more precise, since it stated

$$\frac{\sigma_{\text{tot}}^A - \sigma_{\text{tot}}^P}{\sigma_{\text{tot}}^A + \sigma_{\text{tot}}^P} = O(\log E)^{-8/2}, \quad (1)$$

where $\sigma_A = \sigma_{\text{antiparticle-particle}}$, $\sigma_P = \sigma_{\text{particle-particle}}$.

Now, what does experiment say about this ?

First of all, proton-proton total cross-section experiments ⁵⁾ indicate that σ_{total} increases with energy. A very good fit is given for instance by ¹³⁾

$$\sigma_{+}^{pp} = 41 E^{-0.37} - 24 E^{-0.55} + 27 + 0.17 (\log E)^2 \quad (2)$$

millibarns,

with $\alpha = 2.1 \pm 0.1$; E lab. energy in GeV.

We have therefore good reasons to believe that the total cross-section goes to infinity and that therefore Theorem I applies to the physical situation.

Before the recent ISR measurements, the $p\bar{p}$ total cross-section was seen to be steadily decreasing with energy and it would have been a catastrophe if this had continued. I must say that there existed already an acceptable fit to the total cross-section difference, including the highest Fermilab measurement at $E = 200$ GeV, of the form $CE^{-\alpha}$:

$$\sigma_{\text{tot}}^{p\bar{p}} - \sigma_{\text{tot}}^{pp} = 48 E^{-0.55} \text{ mb} . \quad (3)$$

However, there was a lot of freedom in reproducing the data and Kang and Nicolescu ¹⁴⁾, following a suggestion by Lukaszuk and Nicolescu ¹⁵⁾ proposed a model in which σ_{pp} and $\sigma_{p\bar{p}}$ behave like $(\log E)^2$, in agreement with the Froissart bound ^{12), 16)} and $\sigma_{pp} - \sigma_{p\bar{p}}$ behaves like $\log E$, in agreement with inequality (1), and which fitted the existing data before 1981 as can be seen in Ref. 14).

At this point one can make two obvious remarks about the $p\bar{p}$ ISR data. First of all, they indicate that the $p\bar{p}$ cross-section, after going through a minimum rises again :

$$\begin{aligned} \text{At } E_{\text{lab}} = 200 \text{ GeV} \quad \sigma_E &= 41.5 \text{ mb} \\ \text{At } E_{\text{lab}} = 1500 \text{ GeV} \quad \sigma_E &= \begin{cases} 43.8 \pm 0.4 \text{ mb}^{(1)} \\ 43.4 \pm 0.6 \text{ mb}^{(3)} \end{cases} \quad (4) \end{aligned}$$

Secondly, the difference between the total cross-sections tends to get smaller :

$$\sigma_{\text{tot}}^{p\bar{p}} - \sigma_{\text{tot}}^{pp} = \begin{cases} 1.1 \pm 0.5 \text{ mb} \\ 0.95 \pm 0.6 \text{ mb} \end{cases} , \quad (5)$$

and is compatible with the extrapolation of Eq. (3) which gives :

$$\sigma_t^{h\bar{p}} - \sigma_t^{hp} = 0.85 \text{ mb.}$$

Naturally, the ratio of the total cross-sections is very close to unity :

$$\sigma_t^{h\bar{p}} / \sigma_t^{hp} = \begin{cases} 1.025 \pm 0.01 \\ 1.016 \pm 0.014 \end{cases} \quad (6)$$

This should be compared to

$$\sigma_t^{h\bar{p}} / \sigma_t^{hp} = 1.065 \text{ at } E_{\text{lab}} = 200 \text{ GeV.} \quad (7)$$

In conclusion, Theorem I is satisfied beyond our hopes, i.e., not only does the ratio of the cross-sections seem to approach unity, but also the difference seems to go to zero. This latter fact is not required by very general consequences of axiomatic field theory but agrees with familiar models as will be discussed in Section 3.

Another quantity which has been measured in the slope of the diffraction peak. The slope is in fact a local quantity depending not only on energy but also on the value of transfer at which you measure it :

$$b(s, t) = \frac{d}{dt} \left[\log \left(\frac{d\sigma}{ds_2}(s, t) \right) \right], \quad (8)$$

where s is the square of the centre-of-mass energy.

In the Louvain-Northwestern experiment ³⁾, the slope was measured at very small $|t|$, $|t| < 0.03 \text{ GeV}^2$, allowing a comparison with pp measurements at the same energy (53 GeV centre-of-mass). In the CERN-Pisa-Stony Brook experiment ²⁾, measurements at larger values of t were made, allowing a comparison with $p\bar{p}$ slopes at lower energies.

Here the relevant theorem by Cornille and Martin ¹⁷⁾⁻¹⁹⁾ is the following.

Theorem II

Inside the diffraction peak, the ratio

$$\frac{d\sigma^A}{dt}(s, t(s)) / \frac{d\sigma^P}{dt}(s, t(s))$$

approaches unity as $s \rightarrow \infty$, for any sufficient smooth $t(s)$ such that

$$\frac{d\sigma}{dt}(s, 0) / \frac{d\sigma}{dt}(s, t(s))$$

stays finite as $s \rightarrow \infty$. If the limit does not exist, unity naturally belongs to the set of limiting values.

A corollary of Theorem II is :

$$\lim_{s \rightarrow \infty} b_A(s, t(s)) / b_P(s, t(s)) = 1, \quad (9)$$

as $s \rightarrow \infty$, if $t(s)$ is chosen as above. In particular

$$\lim_{s \rightarrow \infty} b_A(s, 0) / b_P(s, 0) = 1. \quad (10)$$

I would like (immodestly!) to draw the attention of the reader to the non-trivial character of this theorem.

Properties of that type have been stated many times in many places in the physics literature, but, however, the starting point was usually a very specific assumption on the structure of the amplitudes^{20),21)}. Without the positivity properties of the absorptive parts of elastic amplitudes it is possible to violate Theorem II. Suppose that the amplitude is dominated by a complex Regge Pole :

$$F_{rp} = i f(t) s^{1+i\alpha t} \quad (11)$$

One would predict

$$\begin{aligned} d\sigma_{rp}/d\sigma_{rp} &= \exp - 2\alpha\pi t \\ \text{and } b_{rp} - b_{rp} &= - 2\alpha\pi. \end{aligned} \quad (12)$$

However, as was shown in Ref. 18), Eq. (11) is incompatible with the fact that the expansion coefficients of the absorptive part of an elastic amplitude are positive, which allows one to prove that inside the diffraction peak the phase of the amplitude is bounded by a quantity independent of energy.

Again, experimental data indicate that already at energies accessible by ISR the ratio of the slopes is very close to unity. From the Louvain-Northwestern experiment ³⁾ one gets

$$\frac{b_{p\bar{p}}}{b_{pp}} = 1.015 \pm 0.04 \text{ at } E_{\text{lab}} = 1500 \text{ GeV} \quad (13)$$

$$|t| < 0.03 \text{ GeV}^2.$$

However, the lack of experimental measurement on $p\bar{p}$ at very small t makes it impossible to compare with lower energies. On the contrary, the CERN-Pisa-Stony Brook measures a slope at $\langle |t| \rangle = 0.2 \text{ GeV}^2$ and a comparison is possible. One gets

$$\frac{b_{p\bar{p}}}{b_{pp}} (|t| \sim 0.2 \text{ GeV}^2) = 1.12 \text{ at } E_{\text{lab}} = 30 \text{ GeV}$$

$$1.05 \text{ at } E_{\text{lab}} = 100 \text{ GeV}$$

$$1.00 \pm 0.06 \text{ at } E_{\text{lab}} = 1500 \text{ GeV} \quad (14)$$

Within large errors, the trend is the right one. It will be very interesting to be able to make the same type of comparison for very small t in a few months when the energy of the ISR can be moved up and down.

There is another consequence of Theorem II. If there is no "pathology" one can just integrate

$$\frac{d\sigma_A}{dt}(s, t(s)) = \frac{d\sigma_P}{dt}(s, t(s)) [1 + \varepsilon(s, t)]$$

and show that the ratio of the integrated elastic cross-sections approaches unity for $s \rightarrow \infty$. Pathologies are for instance a small background contribution expanding beyond a fixed t range, for instance a contribution

$$\frac{d\sigma}{dt} (\text{background}) = s^{-1/2} \exp\left(-\frac{|t|}{s^{1/2}}\right) \quad (15)$$

This is a negligible contribution to $d\sigma/dt$ inside the diffraction peak but contributes a finite amount to σ_{elastic} and could upset the equality of elastic cross-sections. However, experience shows that $d\sigma/dt$ decreases very fast with t and we have good reasons to believe that such a pathology does not occur. We conclude that we expect

$$\sigma_{\text{elastic}}^{p\bar{p}} / \sigma_{\text{elastic}}^{pp} \rightarrow 1, \text{ for } s \rightarrow \infty. \quad (16)$$

And because of Theorem I on total cross-sections, by subtraction :

$$\sigma_{\text{inelastic}}^{p\bar{p}} / \sigma_{\text{inelastic}}^{pp} \rightarrow 1, \text{ for } s \rightarrow \infty. \quad (17)$$

Cornille and I ²²⁾ pointed out a long time ago that the only natural way to understand the asymptotic equality of inelastic cross-sections was to assume the dominance of fragmentation at high energies as proposed by Yang ²³⁾. It happens that this has also been tested at the ISR by the CERN-Los Angeles-Saclay collaboration ²⁴⁾, which showed that the fragments of the proton are the same, whether they are produced by the impact of a proton or of an antiproton.

Finally let me indicate another consequence of Theorem II to be tested. If it is true that the minimum of the elastic proton-proton scattering around $t = 1.4 \text{ GeV}^2$ belongs to the diffraction peak this minimum should occur in $p\bar{p}$ collisions at about the same t value (this occurs naturally in the Chou-Yang theory because the minimum is controlled by the total and elastic cross-sections ²⁵⁾ but our claim is that it is a more general feature). A minimum in $p\bar{p}$ has been seen at much lower energies ^{26), 27)}, but we predict that it should be present also at the ISR.

3. - CONFRONTATION WITH ORTHODOX AND UNORTHODOX MODELS

The orthodoxy is that the "Pomeron", which is the leading contribution to high energy diffraction scattering, is an even signature object, i.e.,

$$F_{\text{Pomeron}}(s, t, u) = F_{\text{Pomeron}}(u, t, s) \quad (18)$$

where $s+t+u = 4M^2$, M : nucleon mass. What is the Pomeron? Nobody really knows (some people will disagree with me because they have in mind a specific model). In the past (when total cross-sections remained finite at infinite energy and cuts were neglected), it was contributing the scattering amplitude by a term of the form

$$f(t) s^{\alpha(t)} e^{i\alpha(t)\pi/2} \quad (19)$$

with $\alpha(0) = 1$, $\alpha'(0)$ finite. The phase factor in (19) is characteristic of the even signature.

Now we have indications that cross-sections are rising. They may even saturate the Froissart bound ^{12),16)} as in the fit (2) and, in that case, the Pomeron contribution for $t = 0$ should behave like $s(\log s)^2$ or $E(\log E)^2$. If the Froissart bound is saturated qualitatively one cannot just multiply the familiar expression (19) by $(\log s)^2$. Otherwise one gets into conflict with analyticity and unitarity. This conflict, which would become acute only at extremely high energies, is discussed in Section 4. Here we do not need to know in detail what is the Pomeron, but rather to know that what is not the Pomeron is made of Regge poles and cuts with intercept strictly less than unity at $t = 0$. In addition, in the traditional picture, there is a link between odd and even signature due to "duality". The absence of sharp resonances expected in the pp channel, made of six quarks, indicates a cancellation between odd and even signature poles in the total cross-section. In practice this cancellation is very imperfect as seen from the fit (2) of σ_{total} , where the first two terms have respectively even and odd signature. Anyway, what matters for us is that in the orthodox view the odd signature amplitude contains contributions which all have an intercept less than unity, not very far from 0.5. This means that one can write an unsubtracted dispersion relation for the odd signature amplitude, which is therefore completely fixed by the knowledge of the total cross-section difference, in the case of pion-nucleon scattering at least. In the nucleon-nucleon case we also need the unphysical cut contribution for $p\bar{p}$ centre-of-mass energies less than two-nucleon mass. Most of dispersion relation studies of hadron scattering make the implicit assumption of no subtraction in the odd signature amplitude ^{5),28)}.

However, there is no really fundamental principle which forbids important odd signature terms. Difficulties occur only when one wants to give them a physical interpretation in the t channel, but even in the case of the ordinary Pomeron there is no consensus on its t channel interpretation.

Three kinds of proposals have been made. We have already mentioned in Section 2 the idea of Kang, Łukaszuk and Nicolescu ^{14),15)} :

$$\bar{F}(E, t=0) = C E (\log E - i\pi/2)^2 \quad (20)$$

which we might call an odd-Froissarton. On the other hand, on the basis of an analysis of early pion-nucleon charge exchange experiments ²⁹⁾, Joynson, Leader,

Lopez and Nicolescu ³⁰⁾ proposed to introduce an "odderon", i.e., an amplitude

$$F^- = C f(t) E^{\alpha(t)} e^{i(\alpha-1)\frac{\pi}{2}}$$

$$\alpha = 1, C \text{ real.} \quad (21)$$

Such an "odderon" does not contribute to the total cross-sections which can approach each other at infinite energy. More recent pion-nucleon charge-exchange experiments ³¹⁾ do not seem to require any more this term, but it could be necessary according to Bouquet and Diu ³²⁾ to explain neutron-proton charge exchange. Anyway it remains a possibility.

Finally we could have the situation corresponding to the old counter-example to the original Pomeranchuk theorem

$$F^- = C E (\log E - i\frac{\pi}{2}), \quad (22)$$

which leads to a finite, non-zero, difference between the total cross-sections at high energy.

What we want to say is that the Louvain-Northwestern experiment ³⁾, with simultaneous measurements of total cross-sections and real parts gives indications against the present of such "anomalous" terms. The cross-section measurements alone disfavour possibility (20) and constrain the coefficient in (22) to be very small since, as we have seen, the cross-section difference is compatible with the extrapolation from lower energies.

In fact, one can give other arguments in favour of $\sigma_T^{p\bar{p}} - \sigma_T^{pp} \rightarrow 0$, using the real part measurements. These arguments, at the present time, are weakened by the systematic errors of the experiment, but let us present them nevertheless. A theorem by Fischer and collaborators ^{33),34)}, also proved independently by Vinh Mau ³⁵⁾, states :

Theorem III

If $\text{Re } F^-$ and $\text{Im } F^-$ have the same sign above a certain energy the integral

$$\int_0^\infty \frac{\sigma_t^A(E) - \sigma_t^P(E)}{E} dE$$

converges.

This implies that if $\sigma^A - \sigma^P$ is monotonous beyond a certain energy it goes to zero. A simplified proof of this theorem is given in the Appendix [the original proof in Ref. 34) is rather involved].

We already know from Eq. (5) that experimentally $\sigma_t^{p\bar{p}} - \sigma_t^{pp}$ has a high probability of being positive. From the real part measurements at 2×26.7 GeV, we get

$$\rho_{pp} = \frac{\text{Re} F_{pp}}{\text{Im} F_{pp}} = 0.078 \pm 0.01, \quad (5)$$

$$\rho_{p\bar{p}} = \frac{\text{Re} F_{p\bar{p}}}{\text{Im} F_{p\bar{p}}} = 0.111 \pm 0.011 \pm 0.02, \quad (3) \quad (23)$$

which, combined with the total cross-section measurements

$$\sigma_t^{pp} = 42.7 \pm 0.2 \text{ mb} \quad (5)$$

$$\sigma_t^{p\bar{p}} = 43.4 \pm 0.6 \text{ mb} \quad (3), \quad (24)$$

give

$$\frac{\text{Re} F_{p\bar{p}} - \text{Re} F_{pp}}{\text{Im} F_{pp}} = 0.035 \pm 0.035 \quad (25)$$

It is therefore probable that $\text{Re} F^-$ is also positive, and if this continues at higher energies the conditions of the Fischer theorem will be fulfilled.

In fact, the conditions of the Fischer theorem are too strong. If $\Delta\sigma$ approaches a non-zero limit not only does $\text{Re} F^-$ have an opposite sign to $\text{Im} F^-$ but it is large, according to Eq. (22), and if we use the GeV as a typical scale and assume that the asymptotic regime is reached at 53 GeV centre-of-mass we get $\text{Re} F^-/\text{Im} F^- \sim -5$, which seems to be excluded by experiment.

Furthermore, the real part difference (25) is compatible with the prediction of dispersion theory without subtraction using extrapolation (3) for the cross-section difference :

$$\frac{\rho_{\text{ref}}^{\text{pp}} - \rho_{\text{ref}}^{\text{p}\bar{\text{p}}}}{\rho_{\text{ref}}^{\text{p}\bar{\text{p}}}} = \frac{\rho_{\text{ref}}^{\text{pp}} - \rho_{\text{ref}}^{\text{p}\bar{\text{p}}}}{\rho_{\text{ref}}^{\text{p}\bar{\text{p}}}} \tan \frac{\alpha\pi}{2} = 0.014. \quad (26)$$

We see that there is very little room for an odderon contribution of the type (21). If we call $\pm\Delta\rho$ the odderon contribution to ρ_{pp} and $\rho_{\text{p}\bar{\text{p}}}$ respectively, we get

$$\Delta\rho = 0.010 \pm 0.017 \quad (27)$$

If the accuracy on the measurement of this quantity is improved it will be very useful to confirm that total cross-sections must be rising much beyond the ISR energy. The standard argument, based on dispersion relations and on a measurement of ρ_{pp} alone ⁵⁾, necessitates the absence of an odderon contribution (21). Since the total pp cross-section is slowly varying, an odderon term will in first approximation shift ρ_{pp} by a fixed amount at all energies. If total cross-sections stabilize at $E = 10^4$ GeV, the theoretical ρ_{pp} is too small compared to experiment but can be corrected by adding a term of the form $C(E-E_0)$, i.e., an odderon combined with a modification of the subtraction constant in the even signature term. This term is just barely compatible with (27), but we hope that, in the future, the experimental situation will be improved when the systematic errors are reduced (the statistical errors are already remarkably small!).

To summarize, let us say that present measurements on $\text{p}\bar{\text{p}}$ are necessary to eliminate unorthodox possibilities. Everything is compatible with the standard models, and the experiments make life difficult but not yet impossible for heresy. The credibility of the arguments according to which total cross-sections must rise up to very high energies is increased, but this will not replace a direct $\text{p}\bar{\text{p}}$ total cross-section measurement at 150 000 equivalent lab. energy.

4. - THE VERY HIGH ENERGY LIMIT

We just said that we are expecting to have very soon at our disposal a total cross-section and differential cross-section measurements at 150 000 GeV equivalent lab. energy (like the French, who prefer to use old francs for large amounts of money, we think it more impressive to use lab. energies!). The first thing to do is to compare the total cross-section with the existing fits like (2) which is compatible with the qualitative behaviour given by the Froissart bound ^{12),16)} but far below the upper bound of the coefficient in front of the

logarithm squared ³⁶⁾. If it is so it will be an argument in favour of the theories reproducing these features like for instance the Cheng-Wu-Walker model ³⁷⁾, or the Nussinov model of the Pomeron ³⁸⁾. It would be abusive to say that the saturation of the Froissart bound is a confirmation of QCD because an Abelian gauge theory might as well produce the saturation ³⁷⁾. There is also the possibility that the bound will not be saturated and this would please the adepts of the "critical Pomeron" ³⁹⁾, who, incidentally, have prepared a position of retreat by saying that the distinction between the saturation of the Froissart bound and their model, $\sigma_{\text{total}} \sim (\log E)^{0.3}$, might occur only at much higher energies.

Here I would like to investigate further the scenario in which we have saturation of the Froissart bound. From the bounds on the slope of the diffractive peak

$$b > \frac{1}{18\pi} \frac{(\sigma_{\text{total}})^2}{\sigma_{\text{elastic}}},$$

$$b > \frac{1}{16\pi} \sigma_{\text{total}}, \quad (28)$$

from unitarity ^{40),41)},

$$\left| \frac{d}{dt} F(s, t) \right| \leq s (\log s)^4, \quad t \leq 0 \quad (29)$$

from analyticity and polynomial boundedness, we conclude that b behaves like $(\log s)^2$. This clearly excludes a naïve exponential peak $\exp b(s)t$, because for $0 < t < 4 m_\pi^2$, outside the physical region, this would lead to an increase of the amplitude faster than any power of the energy while we know that the amplitude cannot grow faster than s^2 ⁴²⁾. The solution of this conflict was found long ago ⁴³⁾. The limit behaviour of the scattering amplitude is given by

$$F \sim s (\log s)^2 f(t (\log s)^2) \quad (30)$$

where $f(x)$ is an entire function of x of order $\frac{1}{2}$. An example of such an entire function, $f(x) = J_1(\sqrt{-x})/\sqrt{-x}$, is precisely what is obtained in the Cheng-Wu-Walker model ³⁷⁾, but there are also examples of non-oscillating functions.

At present energies there is no indication that $b(s)$ behaves like $(\log s)^2$. The existing best fits in terms of the lab. energy $E \sim s/2M$, give ⁴⁴⁾

$$b(E) = 10.5 + 0.30 \log E \quad (31)$$

for small t .

Confrontation of this fit with the fit of the total cross-section (2) leads to a contradiction if one uses inequalities (28), but at what energy ?

If one requires only that the total cross-section be larger than the elastic cross-section, the conflict occurs at the fantastic energy of $E_{\text{lab}} = 3 \times 10^{23}$ GeV or $E_{\text{cm}} = 7 \times 10^{11}$ GeV. If, on the other hand, you believe that ultimately one cannot go beyond the "black disk" case, i.e., $\sigma_{\text{elastic}} \leq \frac{1}{2} \sigma_{\text{total}}$ the conflict occurs at

$$E_{\text{lab}} = 2 \times 10^{12} \text{ GeV} \text{ or } E_{\text{cm}} = 2 \times 10^6 \text{ GeV}.$$

These energies are very remote and will never be reached by man made accelerators. However, there may be some as yet unknown law saying that the elastic cross-section cannot exceed some fixed fraction of the total cross-section, for instance by equipartition between various diffraction dissociation channels. Then it could be that the conflict occurs at much lower energies not too far from what we are reaching now at the SPS collider. An optimist would see in the t dependence of the slope of the diffraction peak, the break, the dip at $t \sim 1.4 \text{ GeV}^2$, precursors of this phenomenon. It would be great to observe at the SPS collider deviations from linearity in $\log E$ of the slope parameter. We have been accustomed to see so many times in particle physics "precocious asymptotics" that this might happen once again.

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APPENDIX - SIMPLIFIED SIMPLIFIED PROOF OF THE THEOREM OF FISCHER

Let $F(E) = F_{pp} - F_{pp}$. F is odd in E . Introducing $Z = E^2$ we define

$$\frac{F(E)}{E} = g(z)$$

Let us assume that for $z > M$ $\text{Im } g(z) \geq 0$, $\text{Re } g(z) \geq 0$. Because of the Froissart bound $|F| < \sqrt{z}(\log z)^2$ we can write an unsubtracted dispersion relation for $g(z)/\sqrt{z_0 - z}$:

$$g(z) = \frac{\sqrt{z_0 - z}}{\pi} \int_{z_0}^{\infty} \frac{\text{Re } g(z') dz'}{\sqrt{z' - z_0} (z' - z)}$$

For $z = -t$, $t > 0$, we get from the assumption made

$$g(-t) > -\frac{\sqrt{z_0 + t}}{t} \frac{1}{\pi} \int_{z_0}^M \frac{|\text{Re } g(z')| dz'}{\sqrt{z' - z_0}}$$

Hence $\liminf_{t \rightarrow \infty} g(-t) \geq 0$, and hence $g(-t) > -K$, for $t > T$.

Combining this result with the "ordinary" dispersion relation

$$g(z) = \frac{z}{\pi} \int_{z_0}^{\infty} \frac{\text{Im } g(z') dz'}{z' (z' - z)} + C,$$

and using the positivity of $\text{Im } g$ for $z > M$, we get

$$\frac{t}{\pi} \int_M^{M'} \frac{\text{Im } g(z') dz'}{z' (z' + t)} < \frac{t}{\pi} \int_M^{\infty} \frac{\text{Im } g(z') dz'}{z' (z' + t)} < K + C + \frac{1}{\pi} \int_{z_0}^M \frac{|\text{Im } g(z')| dz'}{z'}$$

for any $M' > M$, and hence, taking the limit $t \rightarrow \infty$,

$$\frac{1}{\pi} \int_M^{M'} \frac{\text{Im } g(z') dz'}{z'} < \text{const}$$

for any $M' > M$. The integral

$$\int_M^{\infty} \frac{\text{Im } g(z') dz'}{z'}$$

is therefore absolutely convergent and

$$\int (\sigma_t^+ F - \sigma_t^- F) \frac{dE}{E}$$

converges.

REFERENCES

- 1) L. Camilleri et al. - CERN-Rome Collaboration, to be published, presented by L. Camilleri at the November 20, 1981 Meeting on "Antiprotons at CERN".
- 2) G. Carboni et al. - CERN-Pisa-Stony Brook Collaboration, CERN Preprint EP/81-141 (1981).
- 3) N. Amos et al. - Louvain-Northwestern Collaboration, CERN Preprint EP/81-108 (1981).
More recent results have been given at the meeting "Antiprotons at CERN" and at the seminar by M. Van Der Welde Wilquet, December 7, 1981.
- 4) UA4 Collaboration - Results presented at the meeting "Antiprotons at CERN".
- 5) U. Amaldi et al. - Phys.Letters 62B (1976) 460.
- 6) Y.Ya. Pomeranchuk - JETP 7 (1958) 499.
- 7) N.N. Meiman - JETP 16 (1963) 1609.
- 8) A. Martin - Nuovo Cimento 39 (1965) 704.
- 9) R.J. Eden - Phys.Rev.Letters 16 (1966) 39.
- 10) T. Kinoshita - in Perspectives in Modern Physics, R.E. Marshak Ed., Wiley and Sons, New York (1966), p. 211.
- 11) G. Grunberg and Tran N. Truong - Phys.Rev.Letters 31 (1973) 63 ; Phys.Rev. D9 (1974) 2874.
- 12) A. Martin - Nuovo Cimento 42 (1966) 930.
- 13) A.N. Diddens - in Proceedings of the XVII International Conference on High Energy Physics, London 1974, J.R. Smith Ed., Rutherford Laboratory, Chilton, Didcot (1974), p. I-41.
- 14) K. Kang and B. Nicolescu - Phys.Rev. D11 (1975) 2461.
- 15) L. Lukaszuk and B. Nicolescu - Nuovo Cimento Letters 8 (1973) 405.
- 16) M. Froissart - Phys.Rev. 123 (1961) 1053.
- 17) H. Cornille and A. Martin - Phys.Letters 40B (1972) 671.
- 18) H. Cornille and A. Martin - Nuclear Phys. B49 (1972) 413 ; Nuclear Phys. B77 (1974) 141.
- 19) H. Cornille and A. Martin - Nuclear Phys. B48 (1972) 104.
- 20) A. Logunov, N. Van Hieu, I.T. Todorov and O.A. Krustalev - Phys.Letters 7 (1963) 69.
- 21) L. Van Hove - Phys.Letters 7 (1963) 69.
- 22) H. Cornille and A. Martin - Phys.Letters 39B (1972) 223.
- 23) T.T. Chou and C.N. Yang - Phys.Rev.Letters 25 (1970) 1072.

- 24) CERN-Los Angeles-Saclay Collaboration, to be published, and private communication from Peter Schlein.
- 25) T.T. Chou and C.N. Yang - Phys.Rev. 170 (1968) 1591 ; Preprint, State University of New York, Stony Brook (1981).
- 26) Z. Asa'd et al. - CERN Preprint EP/81-26 (1981) ;
M. Poulet - Proceedings of the XX International Conference on High Energy Physics, Madison 1980, L. Durand and L.G. Pondrom Eds., American Institute of Physics, New York (1981), p. 48.
- 27) The dip has also been observed at Fermilab with 100 GeV $\bar{p}$. M. Poulet - private communication.
- 28) J.P. Burq et al. - CERN Preprint EP/81-78 (1981).
- 29) A.V. Barnes et al. - Phys.Rev.Letters 37 (1976) 76.
- 30) E. Joynson, E. Leader, B. Nicolescu and C. Lopez - Nuovo Cimento 30A (1975) 345.
- 31) W.D. Apel et al. - Nuclear Phys. B154 (1979) 189.
- 32) A. Bouquet and B. Diu - Nuovo Cimento 43A (1978) 53.
- 33) J. Fischer - Physics Reports 76 (1981) 157.
- 34) J. Fischer et al. - Phys.Rev. D18 (1978) 4271.
- 35) R. Vinh Mau - private communication (1976).
- 36) L. Lukaszuk and A. Martin - Nuovo Cimento 52A (1967) 122, Appendix.
- 37) H. Cheng, J. Walker and T.T. Wu - Phys.Letters 44B (1973) 97.
- 38) S. Nussinov - Phys.Rev.Letters 34 (1975) 1286 ;
F.E.Low - Phys.Rev. D12 (1976) 163
- 39) See, for instance, M. Moshe - Physics Reports 37 (1978) 255 ;
C. Bourrely and J. Dash - Preprint CPT-81-1280, Marseille (1981).
- 40) A. Martin - Phys.Rev. 129 (1963) 1432 ;
S.W. McDowell and A. Martin - Phys.Rev. B135 (1964) 960.
- 41) G. Auberson, A. Martin and G. Mennessier - Phys.Letters 67B (1977) 75.
- 42) Y.S. Jin and A. Martin - Phys.Rev. B135 (1964) 1375.
- 43) G. Auberson, T. Kinoshita and A. Martin - Phys.Rev. D3 (1971) 3185.
- 44) J.P. Burq et al. - CERN Preprint EP/81-142 (1981).