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SOFT PION EMISSION IN $p\bar{p}$ RESONANCE FORMATION

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ABSTRACT

The differential cross-section is given for the process $p\bar{p} \rightarrow X + \pi^0$ in the soft pion limit, X being a charmonium state. The result depends only on the partial width for the decay $X \rightarrow p\bar{p}$ and on the spin-orbit wave function of the resonating $p\bar{p}$ system.

There are experimental proposals ¹⁾ to study charmonium formation in $p\bar{p}$ annihilation at resonance mass :

$$p + \bar{p} \rightarrow X \quad (1)$$

In this note we study processes where the resonance is accompanied by a soft pion :

$$p + \bar{p} \rightarrow X + \pi^0 \quad (2)$$

The process (2) represents a potential source of background for the study of radiative modes like

$$p + \bar{p} \rightarrow X \rightarrow \psi + \gamma \quad (3)$$

but is also of interest in its own right. Approximate chiral symmetry ²⁾ fixes the amplitude for (1) to be that given by the Born approximation of Fig. 1, up to terms which vanish with vanishing pion four momentum. The resulting cross-section is proportional to the partial decay width $\Gamma(X \rightarrow p\bar{p})$ and the pion angular distribution depends only on the spin and orbital quantum numbers of X . For states with only one allowed orbital wave the prediction is unique; otherwise there is an unknown parameter which depends on the relative admixture of $L = J \pm 1$ in the wave function of the resonating $p\bar{p}$ system. The process (2) can therefore serve as a probe of resonance quantum numbers.

A straightforward evaluation of the diagrams of Fig. 1 gives, for the invariant matrix element,

$$M_a = \frac{g_A}{\sqrt{2} f_\pi} (\Gamma_a \not{W} \gamma_5 + \not{W} \gamma_5 \Gamma_a) + O(q_\mu) \quad (4)$$

where $\Gamma_a(p, \bar{p})$ is the (generally momentum dependent) Dirac matrix for the effective $p\bar{p}$ coupling :

$$\langle p\bar{p} | X_a \rangle = \bar{u}(\bar{p}) \Gamma_a u(p) \quad (5)$$

with a a spin index; g_A and f_π are the usual nucleon and pion axial current coupling constants ($g_A = 1.25$, $f_\pi = 0.94 m_\pi$). The four vectors W and $\bar{W}$ are defined by

$$\begin{aligned} W_\mu &= \frac{m}{v} q_\mu - \frac{1}{m} P_\mu \\ \bar{W}_\mu &= \frac{m}{\bar{v}} q_\mu - \frac{1}{m} \bar{P}_\mu \end{aligned} \quad (6)$$

and satisfy

$$\begin{aligned} W \cdot P &= \bar{W} \cdot \bar{P} = 0 \\ W^2 &= -1 + m^2 \mu^2 / v^2 \\ \bar{W}^2 &= -1 + m^2 \mu^2 / \bar{v}^2 \end{aligned} \quad (7)$$

where m and μ are the nucleon and pion masses, p_μ , $\bar{p}_\mu$ and q_μ are the four-momenta of p , $\bar{p}$ and π^0 , and

$$\begin{aligned} v &= P \cdot q \\ \bar{v} &= \bar{P} \cdot q \end{aligned} \quad (8)$$

The conditions (7) imply that the Dirac matrices γ_{v_5} and $\bar{\gamma}_{v_5}$ appearing in (4) are proportional to the operators which measure the p and $\bar{p}$ polarization in the direction of the pion three-momentum. Summing over spins, the matrix element (4) gives a differential cross-section in the c.m. frame :

$$\frac{d\sigma(p + \bar{p} \rightarrow X + \pi^0)}{d\cos\theta} = \frac{2J+1}{4v^2} \frac{q}{4E^2} \frac{M}{2E} \frac{q_A^2}{g_\pi^2} \Gamma(X \rightarrow p\bar{p}) \cdot F(W, \bar{W}) \quad (9)$$

where J and M are the X spin and mass, $v = P/E$, and $q = |\vec{q}|$, $p = |\vec{p}|$, $E = P_0$ are c.m. variables. The function

$$F(W, \bar{W}) = -\frac{1}{2}(W^2 + \bar{W}^2) + \frac{\text{Tr} [\Gamma_a \gamma_{v_5} (\not{P} + m) \bar{\Gamma}_a \gamma_{v_5} (\not{\bar{P}} - m)]}{\text{Tr} [\Gamma_a (\not{P} + m) \bar{\Gamma}_a (\not{\bar{P}} - m)]} \quad (10)$$

can be interpreted as a spin-spin correlation function for the resonating $p\bar{p}$ system :

$$F(W, \bar{W}) = -\frac{1}{2}(W^2 + \bar{W}^2) + \frac{1}{\sqrt{W^2 \bar{W}^2}} \langle (\vec{\sigma}_P \cdot \hat{q})(\vec{\sigma}_{\bar{P}} \cdot \hat{q}) \rangle \quad (11)$$

Since in the approximation being made the q dependence of Γ_a is negligible, $F(W, \bar{W})$ is of the form

$$F(W, \bar{W}) = -\frac{1}{2}(W^2 + \bar{W}^2) - \alpha(W \cdot \bar{P})(\bar{W} \cdot P) - \beta \bar{W} \cdot W \quad (12)$$

The Lorentz invariant functions α and β depend only on

$$t = p \cdot \bar{p} = 2E^2 - m^2 \quad (13)$$

and may be evaluated by considering the functions

$$\begin{aligned} G(W_i, \bar{W}_j) &= -\langle \sigma_{p_i} \sigma_{\bar{p}_j} \rangle = \\ &= \alpha (W_i \cdot \bar{p})(\bar{W}_j \cdot p) + \beta (W_i \cdot \bar{W}_j) \end{aligned} \quad (14)$$

where W_i and $\bar{W}_j$, $i, j = 1, 2, 3$, are an orthogonal set of polarization vectors

$$\begin{aligned} W_i \cdot p = \bar{W}_j \cdot \bar{p} &= 0 & W_i \cdot W_j = \bar{W}_i \cdot \bar{W}_j &= -\delta_{ij} \\ W_i \cdot \bar{W}_j &= 0 \quad i \neq j \end{aligned} \quad (15)$$

Quantizing spin in the direction of the incident proton, we define

$$W_3 = \frac{-m^2 \bar{p} + t p}{m \sqrt{t^2 - m^4}} \quad \bar{W}_3 = \frac{m^2 p - t \bar{p}}{m \sqrt{t^2 - m^4}} \quad (16)$$

$$W_1 = \bar{W}_1, \quad W_2 = \bar{W}_2$$

which satisfy (15) and

$$\begin{aligned} W_{1,2} \cdot \bar{p} = \bar{W}_{1,2} \cdot p &= 0 & W_3 \cdot \bar{W}_3 &= -t/m^2 \\ W_3 \cdot \bar{p} = -\bar{W}_3 \cdot p &= \sqrt{t^2 - m^4}/m \end{aligned} \quad (17)$$

Then, using Eqs. (14)-(17), we obtain for the expectation values of the total spin operator

$$\vec{S} = \frac{1}{2} (\vec{\sigma}_p + \vec{\sigma}_{\bar{p}}) \quad (18)$$

$$\begin{aligned} \langle \vec{S} \cdot \vec{S} \rangle &= S(S+1) = \frac{1}{2} \langle 3 + \vec{\sigma}_p \cdot \vec{\sigma}_{\bar{p}} \rangle = \\ &= \frac{1}{2} [3 - \sum_i G(W_i, \bar{W}_i)] = \\ &= \frac{1}{2} [3 + \frac{t^2 - m^4}{m^2} \alpha + (\frac{t}{m^2} + 2) \beta] \end{aligned} \quad (19)$$

and

$$\begin{aligned}\langle S_3^2 \rangle &= \frac{1}{2} \langle 1 + \sigma_{3p} \sigma_{3\bar{p}} \rangle = \\ &= \frac{1}{2} \left(1 + \frac{t^2 - m^4}{m^2} \alpha + \frac{t}{m^2} \beta \right)\end{aligned}\quad (20)$$

Eliminating α from Eqs. (19) and (20) gives simply

$$S(S+1) - \langle S_3^2 \rangle = 1 + \beta$$

while α is t dependent as well as spin dependent.

At this point it is useful to specify the quantum numbers of the state X . Charge conjugation and parity are related to total spin and orbital momentum by

$$P = (-1)^{L+1} ; \quad C = (-1)^{L+S} ; \quad CP = (-1)^{S+1} \quad (21)$$

So $S=0$ for CP odd states, giving

$$\beta = -1 \quad \alpha = 1/(t+m^2) \quad (22)$$

For CP even states we have $S=1$, giving

$$\begin{aligned}\beta &= 1 - \langle S_3^2 \rangle \\ \alpha &= \frac{m^2}{t^2 - m^4} - \beta \frac{t + 2m^2}{t^2 - m^4}\end{aligned}\quad (23)$$

Using these results together with the definitions (6), which give

$$\begin{aligned}(W \cdot \bar{p})(\bar{W} \cdot p) &= m^2 - \left(\frac{v}{\bar{v}} + \frac{\bar{v}}{v} \right) t + t^2/m^2 \\ (W \cdot \bar{W}) &= \frac{m^2 \mu^2}{v \bar{v}} - \left(\frac{v}{\bar{v}} + \frac{\bar{v}}{v} \right) + t/m^2\end{aligned}\quad (24)$$

we obtain :

$$\underline{CP = -1, S = 0}$$

$$F(W, \bar{W}) = \sigma_{00} = 1 - \frac{m^2 \mu^2}{2} \left(\frac{1}{v} - \frac{1}{\bar{v}} \right)^2 + \frac{t - m^2}{t + m^2} - \frac{m^2}{t + m^2} \left(\frac{v}{\bar{v}} + \frac{\bar{v}}{v} \right) \quad (25)$$

$$\underline{CP = +1, S = 1}$$

$$F(W, \bar{W}) = \beta \sigma_{1,0} + (1-\beta) \sigma_{1,1} \quad ; \quad 0 \leq \beta \leq 1 \quad (26)$$

$$\sigma_{1,0} = 1 - \frac{m^2 \mu^2}{2} \left(\frac{1}{\bar{v}} + \frac{1}{v} \right)^2 + \frac{t+m^2}{t-m^2} - \frac{m^2}{t-m^2} \left(\frac{v}{\bar{v}} + \frac{\bar{v}}{v} \right) \quad (27)$$

$$\begin{aligned} \sigma_{1,1} = 1 - \frac{m^2 \mu^2}{2} \left(\frac{1}{v^2} + \frac{1}{\bar{v}^2} \right) - \frac{t^2+m^4}{t^2-m^2} + \\ + \frac{m^2 t}{t^2-m^4} \left(\frac{v}{\bar{v}} + \frac{\bar{v}}{v} \right) \end{aligned} \quad (28)$$

CP even states may further be separated into "natural" and "unnatural" spin-parity states. Since $L_3=0$, $J_3=S_3$, so that the operator $(S^3)^2$ commutes both with parity and with the total angular momentum. For unnatural spin-parity $P=(-1)^{J+1}$, there is a single orbital wave $L=J$, so the wave function must be an eigenstate of $(S_3)^2$. Parity conservation implies furthermore that $S_3^2=0$ is forbidden. For natural spin-parity $P=(-1)^J$, the operator $(S_3)^2$ will mix the allowed orbital waves $L=J\pm 1$. Explicitly, if $\psi_{J,L}$ is a state of unit normalization :

$$S_3^2 |\psi_{J,J}\rangle = |\psi_{J,J}\rangle \quad (29)$$

$$\langle S_3^2 \rangle = 1 \quad ; \quad \beta_{J=L} = 0$$

and

$$S_3^2 |\psi_{J,J+1}\rangle = \frac{J}{2J+1} |\psi_{J,J+1}\rangle - \frac{\sqrt{J(J+1)}}{2J+1} |\psi_{J,J-1}\rangle$$

$$S_3^2 |\psi_{J,J-1}\rangle = -\frac{\sqrt{J(J+1)}}{2J+1} |\psi_{J,J+1}\rangle + \frac{J+1}{2J+1} |\psi_{J,J-1}\rangle$$

so that for a normalized wave function

$$\psi_J = a_+ \psi_{J,J+1} + a_- \psi_{J,J-1}$$

$$|a_+|^2 + |a_-|^2 = 1$$

we get

$$\langle S_z^2 \rangle_{J=L\pm 1} = 1 - \beta = \frac{1}{2J+1} |a + \sqrt{J} - a - \sqrt{J+1}|^2 \quad (30)$$

The results of Eqs. (25)-(30) have been checked by explicit calculations for simple cases, e.g.,

$$0^{-+} : CP = -1 ; J=L=S=0 ; \Gamma = i\gamma_5$$

$$1^{++} : CP = +1 ; J=L=S=1 ; \Gamma = \gamma_\mu \gamma_5, \beta=0$$

$$0^{++} : CP = +1 ; L=S=J+1 ; \Gamma = 1, \beta=1$$

Up to now we have not specified the resonance X beyond its J^{PC} quantum numbers. We are more specifically interested in charmonium states, and, for definiteness, we will assume :

$$M \equiv M_\psi = 3.1 \text{ GeV} \quad (31)$$

in the following discussion.

As a typical uncertainty of the soft pion approximation we may take the amplitude of diagrams involving the exchange of low-lying nucleon resonances, in place of the nucleon pole of Fig. 1. In comparison to Eq. (4) such contributions are suppressed by a factor :

$$\delta = \frac{2\gamma}{2\gamma + \Delta M^2} \quad (32)$$

(in the region $v > \bar{v}$ and assuming similar coupling constants). As expected, δ vanishes with q_μ , and it can be minimized by tuning on pions with velocity equal to the nucleon velocity, v .

The optimal pion c.m. energy is :

$$E_\pi = \frac{M_\mu}{2m} \simeq 230 \text{ MeV} \quad (33)$$

in the case (31) and, for this energy, v obtains its minimum value, m_μ , in the forward direction.

The lightest relevant nucleon resonance is the $N(1470)$; correspondingly :

$$0.20 \leq \delta \leq 0.40 \quad (34)$$

in the full angular range. In comparison to Eq. (32), other possible terms, of order :

$$\frac{q P_X}{M^2} = \frac{E_\pi}{M} = \frac{k}{2m} \quad (35)$$

such as the ambiguity in the definition of t , can be safely neglected. In conclusion, the error to be expected is similar to that of standard soft-pion calculations for the case considered, but the detailed $v, \bar{v}$ dependence of σ_{s,s_3} may not be too reliable. If we were to consider heavier onia, such as those of the Y family, the approximation would be considerably worse, δ being of order unity over most of the angular range, corresponding to very relativistic pions.

Figure 2 shows a numerical evaluation of the cross-sections, Eqs. (25), (27) and (28), for M and E given by Eqs. (31) and (33).

Relative to the cross-section for the process (1) at the maximum of the Breit-Wigner curve :

$$\sigma(p+\bar{p} \rightarrow X)_{\max} = (2J+1) \frac{\pi}{P^2} \frac{\Gamma(X \rightarrow p\bar{p})}{\Gamma_X} \quad (36)$$

where Γ_X is the total X width and the cross-section for soft pion emission (2) is :

$$\frac{\sigma(p+\bar{p} \rightarrow X+\pi^0)}{\sigma(p+\bar{p} \rightarrow X)_{\max}} = \frac{g_A^2}{f_\pi^2} \frac{q \Gamma_X}{4\pi} \sigma_{CP} \simeq 10^{-3} \left(\frac{q}{m_\pi}\right) \left(\frac{\Gamma_X}{1 \text{ MeV}}\right) \cdot \sigma_{CP} \quad (37)$$

with

$$\sigma_{CP} = \frac{1}{4} \int d\cos\theta F(w, \bar{w}) \quad (38)$$

$$\begin{aligned} \sigma_- &\simeq 0.50 \\ \sigma_+ &\simeq 0.70 \cdot (1 - \langle S_3^2 \rangle) + 0.15 \cdot \langle S_3^2 \rangle \end{aligned} \quad (39)$$

The numerical values correspond to the case in Fig. 2, and

$$\begin{aligned} \langle S_3^2 \rangle &= 0 & ; J &= 0 \\ \langle S_3^2 \rangle &= 1 & ; P &= (-1)^{J+1} \\ 0 \leq \langle S_3^2 \rangle &\leq 1 & ; P &= (-1)^J \end{aligned} \quad (40)$$

with the extremum values corresponding to roughly equal mixtures of $L=J\pm 1$:

$$\frac{a_-}{a_+} = \begin{cases} \sqrt{J/(J+1)} & S_3^2 = 0 \\ -\sqrt{(J+1)/J} & S_3^2 = 1 \end{cases} \quad (41)$$

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G. Auriemma et al., Proposal to SPSC, P-157, CERN-SPSC-81-11, 5.2.1981.
- 2) See, e.g., S.L. Adler and R.F. Dashen, "Current Algebras, and Applications to Particle Physics", (Benjamin, New York, Amsterdam, 1968).

FIGURE CAPTIONS

Figure 1 : Born diagrams for process (2).

Figure 2 : $\sigma_{0,0}$, $\sigma_{1,0}$ and $\sigma_{1,1}$ as functions of the c.m. pion angle. The curves are computed for $M=3.1$ GeV and $E=M_\mu/2m=230$ MeV.

fig.1

fig. 2