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SYMMETRY BREAKING IN A SUPERSYMMETRIC MODEL
OF STRONG AND ELECTROWEAK INTERACTIONS

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ABSTRACT

We describe a supersymmetric model of strong and electroweak interactions based on the gauge group $SU(3) \times SU(2) \times U(1) \times \tilde{U}(1)$. We concentrate on the pattern of the spontaneous symmetry breaking by the tree level scalar potential. It is possible to break the $\tilde{U}(1)$ factor at superlarge energies relative to the simultaneous breaking scale of $SU(2) \times U(1)$ and supersymmetry. The model has $\tilde{U}(1)$ anomalies. Attempts to make an anomaly-free model based on the group E_6 are described. We also comment on possible modifications of the $\tilde{U}(1)$ anomaly problem due to gravitational effects.

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1. - INTRODUCTION

The possibility that supersymmetry is realized in nature to describe the fundamental interactions is, from many points of view, an attractive idea. A most remarkable fact is the softening of quantum divergences in supersymmetric field theory models both with and without gravity¹⁾. This may help²⁾ in giving a natural explanation of the very small ratio M_W/M_{Pl} ($M_{Pl} \equiv$ Planck mass), by avoiding the quadratic divergences associated with the Higgs boson mass of the standard electroweak model³⁾ (the first "hierarchy" problem)⁴⁾, as well as in unifying the known interactions at a very large energy scale M , with M_W/M perhaps as small as 10^{-13} (the second "hierarchy" problem)⁵⁾.

Along these lines of thought, prior to any unification, a viable supersymmetric model of strong and electroweak interactions has to be exhibited. An important contribution to such a programme has been given by Fayet⁶⁾ who has pointed out that, in order to obtain a realistic mass spectrum in a supersymmetric theory spontaneously broken at the tree level, an enlargement of the standard $SU(3) \times SU(2) \times U(1)$ gauge group is unavoidable. A precise statement is that in any model based on the gauge group $G = SU(3) \times SU(2) \times U(1) \times \tilde{G}$

$$\sum_{\alpha} g_{\alpha}^2 D_{\alpha} t_{\alpha} \quad (1)$$

has to be positive for any fermion of given charge and helicity, where t_{α} are the generators of the group, with coupling constants g_{α} , and D_{α} are the corresponding D terms at the minimum of the scalar potential. This excludes $SU(3) \times SU(2) \times U(1)$ itself, since the relevant generators, T_{3W} and the hypercharge Y , are traceless over fermions^{6),7)}. The minimal extension of the gauge group is obtained by adding an extra chiral factor $\tilde{U}(1)$. A model which would obviously work from this point of view is one in which the generator $\tilde{Y}$ associated with this extra factor has a positive value over all fermion supermultiplets and the corresponding $g^2 D$ is positive and large enough to dominate over the other contributions in (1).

A basic problem in this general class of models is to obtain the correct pattern of symmetry breaking, namely to spontaneously break supersymmetry and the gauge group G down to $SU(3)_C \times U(1)_{em}$, keeping the successful neutral current structure of the standard model. A careful discussion of this problem is particularly compelling in view of the large number of variables - among others all superpartners of normal matter - upon which the scalar potential depends, and the strong restrictions that supersymmetry imposes on the scalar potential itself. The danger of not being able to break supersymmetry at all is

the most evident one. This paper is an attempt to consider these problems in some detail. In terms of a definite set of fields other than the normal matter we describe an $SU(3) \times SU(2) \times U(1) \times \tilde{U}(1)$ model with the following properties:

- i) no supersymmetric minimum exists;
- ii) the breaking scale of the additional $\tilde{U}(1)$ can be made arbitrarily large, in spite of the fact that the $SU(2) \times U(1)$ breaking occurs at the much smaller scale M_W ;
- iii) the "low energy" $SU(3) \times SU(2) \times U(1)$ approximation of the model is free of quadratic divergences to the extent that the scale parameter μ , which controls the breaking of $SU(2) \times U(1)$ and supersymmetry, is not renormalized.

The model, or some extension of it, could be a candidate for a consistent "natural"⁵⁾ supersymmetric theory of strong and electroweak interactions.

The model is not safe from triangle anomalies involving the $\tilde{U}(1)$ generator⁸⁾. We shall discuss instructive unsuccessful attempts to make the full model anomaly free. However, it should be noticed that anomalies associated with a generator broken at superlarge energies may not necessarily lead to inconsistencies in the low energy approximation of the model itself. This has to be contrasted with the class of models so far considered⁹⁾, in which the $\tilde{U}(1)$ factor is broken at low energy and in turn the anomaly problem is a compelling one⁸⁾ for the consistency of the theory.

2. - THE MODEL AND ITS SYMMETRY BREAKING PATTERN

The supersymmetric gauge model we will consider is completely defined by the gauge group $SU(3) \times SU(2) \times U(1) \times \tilde{U}(1)$, the transformation properties of the chiral supermultiplets, the two D terms associated with $U(1) \times \tilde{U}(1)$ and the superpotential f . Apart from the 15 matter superfields transforming under G as

$$Q(3, 2, \frac{1}{6}, 1) ; \quad u^c(\bar{3}, 1, -\frac{2}{3}, 1) ; \quad d^c(\bar{3}, 1, \frac{1}{3}, 1)$$

$$L(1, 2, -\frac{1}{2}, 1) ; \quad e^c(1, 1, 1, 1)$$

and the two Higgs $SU(2)$ doublets

$$H(1, 2, -\frac{1}{2}, -2) ; \quad H^c(1, 2, \frac{1}{2}, -2)$$

we introduce three supermultiplets non-trivial only under the $\tilde{U}(1)$ factor

$$S(1,1,0,4) ; R(1,1,0,4) ; R^c(1,1,0,-4)$$

and finally an over-all singlet N . For the D terms associated with the $U(1) \times \tilde{U}(1)$ factors the most general form is

$$D_Y = z_i^* \gamma z_i + \xi_Y \quad (2)$$

$$D = z_i^* \tilde{\gamma} z_i + \xi \quad (3)$$

where ξ_Y and ξ are two parameters of dimension $(\text{mass})^2$. For the D terms of the $SU(3) \times SU(2)$ part we have

$$D_A = z^* \lambda_A z, \quad z \in Q, u^c, d^c \quad (4)$$

$$D_i = z^* \frac{\sigma_i}{2} z, \quad z \in Q, L, H, H^c \quad (5)$$

λ_A ($A=1, \dots, 8$) are the $SU(3)$ generators in the $3(\bar{3})$ representation of $SU(3)$ and σ_i are the Pauli matrices.

The over-all scalar potential is given by the standard formula

$$V = \frac{1}{2} g_3^2 D_A^2 + \frac{1}{2} g_2^2 D_i^2 + \frac{1}{2} g_Y^2 D_Y^2 + \frac{1}{2} g^2 D^2 + \left| \frac{\partial f}{\partial z_i} \right|^2 \quad (6)$$

where f is the superpotential yet to be specified.

As a general strategy for discussing the pattern of symmetry breaking, we separate all the scalar fields into two sets: the superpartners of the standard fields (called σ) from the others (called ϕ). In order for the model to be physically acceptable, all the fields σ must not acquire a vacuum expectation value. A v.e.v. for the neutrino-like state would break lepton number, whereas any other v.e.v. would break colour and/or electromagnetism. In the following we shall show that it is possible to choose a superpotential f such that, in a range of parameters:

- i) the potential $V(0, \phi)$, obtained by putting to zero in Eq. (6) the variables σ , has an absolute minimum at $\phi = \phi_0$ describing the desired pattern of symmetry breaking;

ii) the point $(\sigma, \phi) = (0, \phi_0)$ is a local minimum of the over-all potential $V(\sigma, \phi)$ since the squared mass matrix of the σ fields around the minimum is seen to be positive definite.

For the superpotential we choose

$$f = f_Y + f_H + f_N \quad (7)$$

where

$$\begin{aligned} f_Y &= \alpha Q u^c H^c + \beta Q d^c H + \gamma L e^c H \\ f_H &= \lambda H H^c S + \mu R R^c \\ f_N &= \eta N S R^c + m N (\rho m + N) \end{aligned} \quad (8)$$

with obvious contractions of the group indices. The term f_Y provides the Yukawa couplings to give rise to the various fermion masses and plays no role in the discussion of $V(0, \phi)$.

Let us first consider the potential $V(0, \phi)$ neglecting f_N . After insertion of (7) and (8) into (6) we obtain^{*)}

$$\begin{aligned} V(0, \phi) = & \frac{1}{2} g_2^2 \left(H^\dagger \frac{6i}{2} H + H^{c\dagger} \frac{6i}{2} H^c \right)^2 + \\ & + \frac{1}{2} g_Y^2 \left(-\frac{1}{2} |H|^2 + \frac{1}{2} |H^c|^2 \right)^2 + \\ & + \frac{1}{2} g^2 \left(-2 |H|^2 - 2 |H^c|^2 + 4 |S|^2 + 4 |R|^2 - 4 |R^c|^2 + 3 \right)^2 \\ & + \lambda^2 |H H^c|^2 + \lambda^2 |S|^2 (|H|^2 + |H^c|^2) + \mu^2 (|R|^2 + |R^c|^2) \end{aligned} \quad (9)$$

Notice that we have put $\xi_Y = 0$. It is straightforward to show that the above potential, for the range of values of the parameters

^{*)} We use the same notation to denote the chiral multiplets and their scalar components.

$$g^2 > 2\lambda^2 \quad (10)$$

$$4g^2\xi > \mu^2 \left(1 + \frac{8g^2}{\lambda^2}\right)$$

reaches its absolute minimum at

$$H = \begin{pmatrix} \mu/\sqrt{2}\lambda \\ 0 \end{pmatrix}, \quad H^c = \begin{pmatrix} 0 \\ \mu/\sqrt{2}\lambda \end{pmatrix} \quad (11)$$

$$R_{C^2} = \frac{4g^2\lambda^2\xi - \mu^2(8g^2 + \lambda^2)}{16g^2\lambda^2} \quad (12)$$

$$S = R = 0 \quad (13)$$

At this point

$$D_3 = D_Y = 0, \quad D = \frac{\mu^2}{4g^2} \quad (14)$$

The physics of this potential is simple. The ξ parameter in the D term of the $\tilde{U}(1)$ generator controls the breaking of $\tilde{U}(1)$ itself, whereas the mass μ of the fields R, R_C controls the supersymmetry as well as the $SU(2) \times U(1)$ breaking. This gives us the physically appealing possibility of raising the mass of the $\tilde{U}(1)$ boson relative to the standard W, Z masses by taking ξ sufficiently larger than μ^2 .

The D term associated with $\tilde{Y}$ is positive and dominates over D_3, D_Y which in fact vanish as long as $\xi_Y = 0$ ^{*)}. This is the desired situation to raise the masses of the unobserved scalar superpartners of quarks and leptons (the σ fields). For a charged fermion of mass m , the corresponding two-scalar superpartners (one for helicity state) have squared masses

$$M^2(m) = g^2 D + m^2 = \frac{\mu^2}{4} + m^2 \quad (15)$$

whereas the massless neutrino has two neutral scalar partners of squared mass $\mu^2/4$.

^{*)} Incidentally, D_3 and D_Y turn out to be zero at the minimum since we have put $\xi_Y = 0$ to make the discussion simpler. Of course, this has been possible since, for fixed ξ greater than zero, μ controls the supersymmetry breaking. Up to a maximum value we might have switched on ξ_Y too, getting non-vanishing D_3, D_Y and an extra source of supersymmetry breaking.

This shows that the point defined by Eqs. (11) - (13) and all other scalar supermultiplets equal to zero is a local minimum of the over-all potential. This is not the fully story. It is easy to show that, when the Yukawa superpotential f_Y is switched off, the over-all potential $V(\sigma, \phi)$ has supersymmetric minima with $\sigma \neq 0$, breaking colour and/or charge. However, if α, β, γ do not vanish, the supersymmetric solutions disappear. We conjecture that for α, β, γ greater than some minimum values our minimum can be made the absolute one. The potential danger of this situation is the occurrence of lower bounds on the masses of quarks and leptons which could be inconsistent with their actual values. We note that if extra fields and couplings appear in the superpotential it may be possible to avoid this problem.

Due to the breaking of supersymmetry, the model has a Goldstone fermion which, in general, up to a normalization factor is given by

$$\psi_g = \frac{1}{2} g_\alpha D_\alpha \lambda^\alpha + \frac{\partial f}{\partial z_\alpha} \chi_\alpha \quad (16)$$

where λ^α and χ_a are the fermionic partners of the vector bosons and of the scalar fields, respectively. For $\xi/\mu^2 \gg 1$ the λ^α component is negligible and the main component among the χ_a 's - enhanced by a relative factor $\sqrt{\xi}/\mu$ - is the fermionic partner of R .

The model discussed so far, with the exclusion of the interactions of the neutral field N by putting $f_N = 0$, has, however, too many global symmetries. They are four independent phase transformations, namely

$$B: \quad Q \rightarrow e^{i\alpha} Q, \quad u^c \rightarrow e^{-i\alpha} u^c, \quad d^c \rightarrow e^{-i\alpha} d^c$$

$$L: \quad L \rightarrow e^{i\beta} L, \quad e^c \rightarrow e^{-i\beta} e^c$$

$$X: \quad R \rightarrow e^{i\gamma} R, \quad R^c \rightarrow e^{-i\gamma} R^c$$

commuting with supersymmetry, and an R type symmetry Z which is the product of the two transformations

$$\begin{aligned} \lambda^\alpha &\rightarrow e^{i\theta} \lambda^\alpha \\ \chi &\rightarrow e^{-i\theta} \chi \end{aligned}$$

and $Q \rightarrow e^{2i\theta} Q$, $L \rightarrow e^{2i\theta} L$, $S \rightarrow e^{2i\theta} S$, $R \rightarrow e^{2i\theta} R$. The second transformation, like B, L, X , acts on the physical components of the corresponding chiral multiplets Q, L, S, R . Obviously, Z does not commute with supersymmetry. After spontaneous symmetry breaking B, L and Z are still conserved, whereas X is spontaneously broken. This in turn has the following consequences:

- i) due to Z conservation, the gluinos are massless to all orders of perturbation theory;
- ii) there exists a massless Goldstone boson due to the breaking of X .

Both facts give rise to apparent phenomenological problems. A possible way out consists of switching on the superpotential contribution f_N , explicitly breaking both X and Z . On the other hand the effect of f_N on the scalar potential does not modify the main features of the minimum, as it can be realized by the following argument.

The field N is decoupled in the limit $\eta = 0$, so its only fate in this situation is that of getting a v.e.v.

$$N = -\frac{\beta m}{2}$$

leaving the other fields untouched, as in Eqs. (11) - (13). On the other hand, by switching on the coupling NSR^C , the minimum of the over-all potential is displaced, in particular by forcing S and R to acquire a non-vanishing v.e.v., as is easily seen from the minimum equations. No other minima arise with fields going to infinity as $\eta \rightarrow 0$.

The tree level spectrum of the model is straightforwardly calculable. The main feature we want to emphasize is that all scalars but R_C have a mass squared of order μ^2 , whereas R_C has a mass squared of order ξ , this in the limit $\mu^2/\xi \ll 1$ and neglecting small mixing angles of order $\mu/\sqrt{\xi}$. Analogously, the standard weak vector boson squared masses are of order μ^2 , whereas the $\tilde{U}(1)$ boson has mass squared of order ξ . It is not inconceivable that the parameter ξ , which receives a quadratically divergent renormalization^{2),10)}, can be taken as super large relative to μ (M_{Pl} ?) without destroying the consistency of the low energy model. The reason to be cautious is because the lowest order situation described above might be modified by radiative corrections. Barring this possibility, that would also take care of the anomaly problem associated with the $\tilde{U}(1)$. Needless to say, the successful neutral current phenomenology of the standard model is preserved without any modification. We believe that our suggestion (ξ large) is the only way to get a reasonable model if $\tilde{U}(1)$ anomalies cannot be avoided. If the particular model considered here undergoes unwanted renormalization effects which destroy the good features of the tree-level spectrum, it is still possible to conceive that some more elaborated version of it will work.

3. - E₆ AND POSSIBLE EXTENSIONS OF THE MODEL

The illustrative model presented in the previous section is not safe from triangle anomalies involving the $\tilde{U}(1)$ generator, which is also not traceless. There is a minimal extension of the previous set of chiral multiplets to an automatically anomaly-free and traceless set which is suggested by the group E_6 of which $SU(3) \times SU(2) \times U(1) \times \tilde{U}(1)$ is a subgroup and its fundamental 27 dimensional representation. The extra fields to be added are

$$T(3, 1, -\frac{1}{3}, -2) ; T^C(\bar{3}, 1, \frac{1}{3}, -2) ; P(1, 1, 0, 1)$$

By a straightforward extension of the previous analysis the following general properties emerge. Without modifying the f superpotential of Section 2 the fields T and T^C would both have a negative squared mass due to the D contribution. One may try to stabilize them by adding possible new superpotential terms of the form $\lambda_S T T^C S$, $\lambda_R T T^C R$, which would give positive squared mass contributions since S and R have non-zero v.e.v. In this way one gets a local minimum with the desired properties.

Unfortunately, this local minimum can never become the absolute minimum of the model as well since, no matter what is chosen for the superpotential, the over-all potential with the inclusion of the fields T, T^C, P always possesses supersymmetric minima. One easily finds at least one of them by putting to zero all matter fields as well as other fields (S, P, R) with positive $\tilde{Y}$ charge and minimizing at zero the over-all potential by properly choosing the values of H, H^C, T, T^C, R^C and N . The search for the supersymmetric solution is not in itself especially illuminating, except for realizing that the number of fields with negative $\tilde{Y}$ charge, which should all receive a stabilizing term around zero if one wants to break supersymmetry by D , is too large. This is the unacceptably high price to be paid in order to cancel the anomalies associated with the $\tilde{U}(1)$ generator. This appears to be a general trend of all possible attempts to take care of the anomaly problem: certainly we have not been able to find a solution by suitably extending the multiplet content of our model. For instance, the introduction of a traceless anomaly-free set of fields carrying only $\tilde{U}(1)$ charge seems not to work because of a conflict between the traceless condition and the structure of the superpotential required to stabilize all the negative $\tilde{Y}$ charge components around zero. A further possibility not yet explored is to extend the gauge group by adding for example another $U(1)$ factor. Here again the E_6 group may help, since it also contains a subgroup $SU(3) \times SU(2) \times U(1) \times \tilde{U}(1) \times \tilde{U}(1)'$. A new D term is now allowed, given by

$$D' = -|Q|^2 - |U|^2 + 3|d|^2 + 3|L|^2 - |e|^2 - 5|P|^2 - 2|T|^2 + 2|H|^2 + 2|H'|^2 + 3'$$

which in turn gives a new constraint for possible supersymmetric solutions without enlarging the multiplet structure.

In any case, whatever the global supersymmetric model may be, one has to consider whether a consistent coupling with gravity exists and to explore its possible effects. It is a consequence of local supersymmetry that extra triangle anomalies are generally introduced in the case of D terms with a ξ parameter different from zero, because of a modification of the charges proportional to ξ/M_{Pl}^2 (11). Coming back to our model of Section 2, this seems to indicate that its ultra-violet behaviour is no worse than that of a fully anomaly-free theory, since the latter may ultimately possess gravitationally induced anomalies.

As a final comment it is worth mentioning that the vacuum energy of the model described in Section 2 is given for $\eta = 0$ by

$$V_0 = \frac{\mu^2 f}{4} - \mu^4 \frac{(\lambda^2 + 8g^2)}{32g^2\lambda^2} \simeq \frac{\mu^2 f}{4}$$

In spite of the fact that μ is the scale parameter which controls the supersymmetry breaking, V_0 is much larger than μ^4 . Correspondingly⁽¹²⁾, the gravitino squared mass m_g^2 is

$$m_g^2 = \frac{V_0}{3M_{Pl}^2} \simeq \frac{\mu^2 f}{12M_{Pl}^2}$$

which can be as large as the weak interaction boson mass squared if $\xi \sim M_{Pl}^2$. Furthermore, the gravitino is essentially neutral under $SU(3) \times SU(2) \times U(1)$, since the Goldstone fermion is given by the fermionic partner of R.

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