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TOPONIUM AT HADRONIC COLLIDERS ?

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A B S T R A C T

We calculate hadronic toponium production by specific diagrams obeying colour conservation and charge conjugation. The resulting rates, though lower than those calculated using semi-local duality arguments are encouraging and may allow for toponium discovery at hadronic colliders currently in development.

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The $p\bar{p}$ collider at CERN is now under operation and also the pp machine at Brookhaven will be ready for experiments before the CERN projected e^+e^- ring, LEP. There is then hope that hadronic colliders may first reveal expected new objects like the weak bosons and the bound states of the, up to now unseen, top quark if quarks are to come into weak doublets for the third time. Historically the T was first seen in hadronic collisions and then its family was thoroughly studied with e^+e^- machines.

With this in mind, we present a calculation of some lowest order contributions to hadronic production of the next quarkonium, which we assume to be a bound state of a quark of charge $2/3 e$ with its antiquark, i.e. toponium. We consider the two processes which were found¹⁻³⁾ to be dominant in the production of J/ψ and T at small (fig. 1a,b) and large (fig. 1b) transverse momentum. We also calculated the production of toponium via quark-antiquark annihilation (fig. 2a) having in mind the possible mixing between toponium and the neutral weak boson Z^0 ⁴⁾. The higher order diagram (fig. 2b) in $q\bar{q}$ annihilation was shown to be negligible¹⁾.

There already exist estimates for hadronic production of the next quarkonium based on scaling from lower resonances⁵⁾, as well as semi local duality (SLD) arguments⁶⁾, which imply that the production rate for $Q\bar{Q}$ bound state is just obtained from unbound $Q\bar{Q}$ production⁷⁾. In particular ref. 6 gives a thorough analysis of the usual Drell Yan background as well as the associated production of $(Q\bar{q}) - (q\bar{Q})$ mesons with their subsequent semileptonic decays which compete with the lepton pair signal of toponium. The conclusion was that toponium should be observable even at a $p\bar{p}$ collider with a low luminosity as $10^{29} \text{ cm}^{-2} \text{ s}^{-1}$ in the leptonic channel. There are problems with the SLD approach, however, notably that color conservation and also charge-conjugation invariance are not respected, and the results are therefore not understandable in terms of perturbation theory. There is a priori no reason to expect that the results based on a proper diagram approach should coincide with those from SLD, not even concerning the type of diagrams which is found to be important. Therefore, both approaches should confront experiment also for toponium. For charmonium the diagrammatic approach gave excellent agreement²⁾, for upsilon the predictions came out a factor four below experimental data²⁻⁴⁾. The discrepancy was explained away^{2),3)} by a "fudge factor" based on the calculation of α_s at different Λ values for quarkonium production and decay. For toponium the scale dependence will be less important because of the high mass involved, and the predictions should therefore be more absolute and give a crucial test of our approach.

To anticipate our conclusions: although our cross-sections are smaller than those of Ref. 6, our calculations indicate that the toponium could be marginally produced at the CERN $p\bar{p}$ collider ($\sqrt{s} = 540 \text{ GeV}$) if it is not too heavy. In

that respect the Fermilab machine ($\sqrt{s} = 2000$ GeV) and mainly the Isabelle pp machine ($\sqrt{s} = 800$ GeV) are in a better position to detect it. This is obvious in our approach: since toponium production comes essentially from gluon-gluon diagrams, cross-sections will be equal for pp and $p\bar{p}$ collisions, and so luminosity, which is expected at Isabelle to be 100 times that of $p\bar{p}$ colliders, plays a crucial role.

The mass of the 3S_1 state for toponium (T) will be denoted by M ; while the $^3P_{0,2}$ states also to be considered have different masses, it is legitimate to neglect the mass differences except of course in their radiative decays into 3S_1 and a photon. It will furthermore be instructive to formulate the results in a way valid for the bound states of any heavy quark by introducing the quark charge e_Q explicitly. The squared CMS energy of a subprocess and of the total reaction will be denoted by $\hat{s}$ and s , respectively, and we define the dimensionless quantities

$$\tau = \frac{\hat{s}}{s}, \quad \tau_0 = \frac{M^2}{s}. \quad (1)$$

The hadronic cross-section is obtained from the cross-section for a subprocess via:

$$\sigma_{ab}^{\text{had.}}(s) = \int_{\tau_0}^1 \frac{d\tau}{\tau} \hat{\sigma}_{ab}(\tau s) \mathcal{F}_{ab}(\tau) \quad \mathcal{F}_{ab}(\tau) = \int_{\tau}^1 \frac{dx}{x} f_a(x) f_b(\tau/x) \quad (2)$$

where the f 's are momentum distributions of constituents a and b . A similar relation holds for a distribution with respect to a boost invariant variable, for instance the transverse momentum p_T .

The differential cross-section¹⁻³⁾ corresponding to the subprocess of fig. 1b where two gluons fuse into a 3S_1 ($Q\bar{Q}$) state and a gluon can be obtained from a calculation of $\gamma g \rightarrow ^3S_1 g$ ⁹⁾. Noting that the singularity in $\left. \frac{d\sigma}{dp_T^2} \right|_{p_T=0}$ as $s \rightarrow M^2$ is an integrable one, we calculate the total rate of production $\hat{\sigma}_b(gg \rightarrow gT)$ at the constituent level and find:

$$\hat{\sigma}(gg \rightarrow gT) = \frac{5\pi}{9e_Q^2} \frac{\alpha_s^3}{\alpha^2} \frac{M^3 \Gamma_e}{\hat{s}} \frac{1}{(\hat{s}-M^2)^2} K_0(\hat{s}, M^2) \quad (3)$$

where

$$K_0(\hat{s}, M^2) = \frac{\hat{s}-M^2}{2\hat{s}} + \frac{\hat{s}^2+M^4}{M^2} \frac{\hat{s}-M^2}{(\hat{s}+M^2)^2} - \frac{M^2(5\hat{s}^2+M^4+2\hat{s}M^2)}{(\hat{s}+M^2)^3} \ln \frac{\hat{s}}{M^2} \quad (4)$$

The strong coupling constant is calculated at the toponium mass with six flavours and $\Lambda = .5$ GeV

$$\alpha_s = \frac{12\pi}{21 \ln(M^2/\Lambda^2)}$$

For the electronic width of T we chose the Sakurai rule

$$\Gamma_e = 12 \cdot 10^{-6} e_Q^2 \text{ GeV}$$

which is roughly justified by dynamical calculations¹⁰⁾.

The cross-section for the production of a T associated with a transverse jet in pp and $p\bar{p}$ reactions is therefore

$$\sigma_b(p(\bar{p}) \rightarrow TgX) = \frac{5\pi}{9e_Q^2} \frac{\alpha_s^3}{\alpha^2} \frac{\Gamma_e}{M^3} I(\tau_o) \quad (5)$$

$$I(\tau_o) = \tau_o^3 \int_{\tau_o}^1 \frac{d\tau}{\tau^2} \frac{K_o(\hat{s}, M^2)}{(\tau - \tau_o)^2} \mathcal{F}_{gg}(\tau) \quad (6)$$

One can check that $I(\tau_o)$ and therefore σ_b behaves as $\ln s/M^2$ for large s using a scaling gluon distribution $f_g(x) = 3(1-x)^5$. Even with scale breaking effects¹¹⁾ this behaviour should be maintained since the dominant contribution comes from the threshold vicinity due to the $1/\hat{s}^2$ behaviour of $\hat{\sigma}_b$ given by (3) and (4).

In fig. 3 we have plotted in solid line the lepton pair production $B_{ee} \sigma_b(p(\bar{p}) \rightarrow TgX)$ versus M for M between 40 GeV and 90 GeV, and for $\sqrt{s} = 540, 800$ and 2000 GeV. B_{ee} represents the branching ratio for the decay of toponium into a lepton pair. For pp collisions at $\sqrt{s} = 800$ GeV, we have checked that the cross-section is insensitive to scaling violations, while for $p\bar{p}$ the production rate is reduced by about 30% at $\sqrt{s} = 540$ GeV, but could be enhanced by a factor 2-3 at $\sqrt{s} = 2000$ GeV.

In fig. 4, we show the differential cross-section $B_{ee} d\sigma/dp_T(p(\bar{p}) \rightarrow TgX)$ for a toponium mass $M = 40$ GeV. We note that for the three considered energies $\sqrt{s}$, the cross-section is maximal at around $p_T = 5$ GeV. Since one infers from eq. (4) that asymptotically for the subprocess, $\langle p_T^2 \rangle \sim M^2 \ln \hat{s}^2/M^2$, this is a demonstration of the dominance of threshold contributions.

In Table 1, we give the average values $\langle p_T \rangle$ and $\langle p_T^2 \rangle$ for $M_T = 40, 60$ and 80 GeV.

All our calculations have been done using the value $\Lambda = 0.5$ GeV as in ref. 2. Taking $\Lambda = 0.2$ would decrease all the results by about 40%.

We now turn to another mechanism which contributes at small transverse momentum, depicted in fig. 1a. The two gluons fuse into 3P_0 and 3P_2 states which subsequently decay radiatively into $^3S_1 + \text{photon}$. Obviously the transverse momentum of the toponium is limited by the primordial motion of the gluons and by the energy of the photon, related to the mass difference between the P wave and the S wave states. We shall assume in the following that $E_\gamma = 500 \text{ MeV}$. The production rate^{2),12)} given by this cascade is

$$\sigma_a(p \bar{p} \rightarrow ^3P_{0,2} X \rightarrow ^3S_1 \gamma X) = XB_{0,2} r \frac{\pi^2 \alpha_s^2}{\alpha^2 e_Q^2} \frac{\Gamma_e}{M^3} \mathcal{F}(\tau_0) \quad (7)$$

with

$$XB_0 = \frac{3}{4} \frac{\Gamma(^3P_0 \rightarrow ^3S_1 \gamma)}{\Gamma_{\text{Tot}}(^3P_0)} \quad \text{and} \quad XB_2 = \frac{\Gamma(^3P_2 \rightarrow ^3S_1 \gamma)}{\Gamma_{\text{Tot}}(^3P_2)} \quad (8)$$

r is a dimensionless quantity which relates the radial wave functions at the origin of the P-wave and the S-wave state for the $Q\bar{Q}$ system

$$r = \frac{4}{M^2} \left| \frac{R'_p(0)}{R_s(0)} \right|^2$$

The relative importance of this process with respect to the direct production is given by

$$\frac{\sigma_a(p \bar{p} \rightarrow ^3P_{0,2} X \rightarrow T \gamma X)}{\sigma_b(p \bar{p} \rightarrow gTX)} = \frac{9}{20} \frac{\pi}{\alpha_s} \frac{\mathcal{F}(\tau_0)}{I(\tau_0)} r [3B_0 + 4B_2] \quad (10)$$

In order to evaluate the quantity $r(3B_0 + 4B_2)$ we consider two different potential models: the harmonic oscillator and the Coulomb potential. Due to the high mass of the top quark the Coulomb potential is likely¹⁰⁾ to give a good description of the actual physics. We include the harmonic oscillator potential as an opposite extreme for the sake of comparison only. The wave functions $R_s(r)$ and $R_p(r)$ ¹³⁾ in both cases depend on a scale parameter which we determine using the Sakurai rule

$$\Gamma(^3S_1 \rightarrow e^+ e^-) = \alpha^2 e_Q^2 |R(0)|^2 \frac{4}{M^2} = 12 \cdot 10^{-6} e_Q^2 \text{ GeV} \quad (11)$$

Then we find:

$$r(3B_0 + 4B_2) = \frac{\alpha E_Y^3}{18 \alpha_s^2} \left(\frac{4}{3} \frac{\alpha^2}{M^2} \right)^{5/3} 10^{10} \times \begin{cases} \frac{2^{16}}{3^{10}} & \text{Coulomb} \\ \pi^{-1/3} & \text{harmonic oscillator} \end{cases} \quad (12)$$

The resulting value of r , for the Coulomb potential, is in very good agreement with the value $r = \alpha_s^2/216$ as expected from positronium results¹⁴⁾. The contribution of these radiative transition to the lepton pair yield is presented in Fig. 3 versus the quark mass, by dashed lines. The upper curve always corresponds to the Coulomb potential and the lower one to the harmonic oscillator model. The actual cross-section should be somewhere in the shaded area between the two dashed curves although closest to the Coulombic case; it is however interesting to note that the probably unphysical harmonic oscillator potential gives results which differ at most by a factor two with respect to the idealized Coulombic situation. We note that σ_a and σ_b appear to be of the same importance. Finally, in order to take into account the effect of off-shell gluons, we evaluated the contribution of quark gluon²⁾ subprocesses which produce ${}^3P_{0,1,2}$ which in turn decay into $T + \gamma$, they come out to be less than 15% of the total expected yield.

A potentially interesting contribution is given by the $q\bar{q}$ annihilations which produce a photon converting next into T (fig. 2a). Evaluation of this reaction in the narrow width approximation gives

$$\sigma_{q\bar{q}}(p_{\bar{p}}^{(-)} \rightarrow TX) = \frac{4\pi}{M^3} \Gamma_e \sum_Q e_Q^2 \int_{\tau}^1 \frac{dx}{x} \left\{ f_Q^P(x) f_{\bar{Q}}^{\bar{P}}(\tau/x) + (Q \leftrightarrow \bar{Q}) \right\} \quad (13)$$

As expected this cross-section represents only a small percentage of the total expected yield ($\sigma_a + \sigma_b + \sigma_{q\bar{q}}$) as shown in Table 2. Such a channel could however be important if T was mixed with the Z^0 which precisely populates this channel although it seems phenomenologically unlikely¹⁵⁾. Mixing of these two states proceeds via fig. 5, and was shown to give effects just on the limit of being measurable in e^+e^- physics⁴⁾ provided the two masses are close enough.

Mixing could affect the transition rates of the toponium states. In the case of nearly degeneracy toponium will decay like a weak boson that is the 3 gluon signal could be less important than naively expected; the "bare" widths being very different would make the physical toponium have the decay characteristics of Z^0 even for a small Z^0 mixture. This we checked to happen in fact in a mass range of ~ 10 - 15 GeV to both sides of the Z^0 where the branching ratio into lepton pairs falls by a factor ~ 2.5 to the Z^0 value. Of course this can be expected to modify even the jet properties of T decay in e^+e^- reactions¹⁶⁾.

In the production cross-sections, mixing effects are less drastic, except for rather close degeneracy : with the T width being dominated by its Z^0 component, the ratio between T and Z^0 production is in the narrow width approximation given by⁴⁾

$$\frac{\Gamma(T \rightarrow \text{all})}{\Gamma(Z \rightarrow \text{all})} \approx \frac{1}{4N^2+1} \left[\frac{e}{\sin\theta_W \cos\theta_W} \frac{f_{T_0}}{\Gamma(Z^0 \rightarrow \text{all})} \left(\frac{1}{4} - \frac{2}{3} \sin^2\theta_W \right) \right]^2 \quad (14)$$

where N is the mass difference between T and Z^0 in units of the Z^0 width, and we insert the Coulombic value

$$f_{T_0} = \frac{2}{3} \pi \left(\frac{\alpha_s}{\pi} \right)^{3/2} M$$

for the bare toponium decay constant f_{T_0} . Comparing (14) with our previous results, it is clear that for $N \leq 1$ the $q\bar{q}$ annihilation channel would give a completely dominant contribution to T production. For $N = 1$, we get from (14) a ratio $\Gamma_T/\Gamma_{Z^0} = 6.10^{-4}$ which means that toponium width is enlarged up to 1.8 MeV.

In conclusion we give the total expected number of lepton pairs ($e^+e^- \rightarrow \mu^+\mu^-$) from toponium production $\sigma_{\text{Tot}} = \sigma_a + \sigma_b + \sigma_{q\bar{q}}$ where we assume an integrated luminosity of $310^{36} \text{ cm}^{-2} \text{ S}^{-1}$ for $p\bar{p}$ machines and 10^2 times more for pp rings. For the $p\bar{p}$ colliders the obtained numbers of events is lower than those given in ref. 6. Our results make the detection of toponium difficult, unless impossible, at the CERN collider. It might be found at the Fermilab tevatron if toponium is not too heavy (M less than 60 GeV). In any case, the yields are very encouraging for the discovery of toponium at Isabelle. Hopefully the statistics might be good enough to enable one to put our approach as well as the one with semi-local duality to a decisive test.

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Table 1 : Average values of p_T and p_T^2 as a function of M and $\sqrt{s}$ given by process of fig. 1b

$\sqrt{s}(\text{GeV})$	$M(\text{GeV})$	$\langle p_T \rangle (\text{GeV})$	$\langle p_T^2 \rangle (\text{GeV}^2)$
540	40	10	155
	60	13.3	276
	80	16	395
800	40	11	184
	60	15	347
	80	18	525
2000	40	12	227
	60	17	478
	80	22	792

Table 2 : Percentage of lepton pairs produced by $q\bar{q}$ annihilation with respect to the total yield

$M(\text{GeV})$	40	60	80
$\sqrt{s}(\text{GeV})$			
540	3	5	8
800	2	2	3
2000	2	3	5

Table 3 : Total rate of e^+e^- pairs and the total number of $(e^+e^- + \mu^+\mu^-)$ events expected at different energies and T masses with the indicated integrated luminosities $\mathcal{L}_{\text{int}}$.

$\sqrt{s}(\text{GeV})$	540($\mathcal{L}_{\text{int}} = 3.10^{36}\text{cm}^{-2}\text{s}^{-1}$)			800($\mathcal{L}_{\text{int}} = 3.10^{38}\text{cm}^{-2}\text{s}^{-1}$)			2000($\mathcal{L}_{\text{int}} = 3.10^{36}\text{cm}^{-2}\text{s}^{-1}$)		
M(GeV)	40	60	80	40	60	80	40	60	80
$B_{ee}(\sigma_a + \sigma_b)(\text{pb})$	2	.2	0.033	3.4	.4	0.08	8	1.2	.3
# pairs($e^+e^- + \mu^+\mu^-$)	12	1	-	2000	240	50	48	7	2

FIGURE CAPTIONS

- Fig. 1a Gluon-gluon fusion producing a P-wave state which subsequently decays into 3S_1 and a photon.
- Fig. 1b Gluon-gluon fusion producing an S-wave state and a gluon.
- Fig. 2a,b Quark-antiquark annihilations producing the toponium.
- Fig. 3 Production of lepton pairs from T decay versus M. Solid lines refer to the Fig. 1b diagram. Dashed curves delimit the plausible range of Fig. 1a contribution.
- Fig. 4 Transverse momentum distribution of the lepton pair as expected from diagram 1-b.
- Fig. 5 Z_0 -T mixing diagram.

FIG. 1

FIG. 2

FIG. 5

FIG. 3

FIG. 4