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QUARK-SPIN ISOSPIN SUM RULES
AND THE ADLER - WEISBERGER RELATION IN NUCLEI

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A B S T R A C T

We use a quark model to extend the classical Gamow - Teller sum rule for the difference of the β^- and β^+ strengths to excitations of the nucleon (mainly the Δ isobar). A schematic model illustrates the realization of the new sum rule when a particle-hole force is introduced. We discuss the connection of our result with the model-independent Adler-Weisberger sum rule.

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The recent experiments on (p,n) reactions ^{1),2)} have provided a unique tool for the study of the spin-isospin excitations in nuclei. Their success stems from the character of the isovector NN amplitude which is dominated at energies around 200 MeV by the spin flip part ³⁾, favouring Gamow-Teller (G.T.) excitations. The (p,n) process has permitted the measurement of the total G.T. strength, a quantity not accessible from β decay where many excitations are energetically forbidden. It has been possible to make a comparison with the following sum rule which applies to the difference of the β^- and β^+ axial strengths [the β^+ branch would be accessible, e.g., through (n,p) reactions]. The corresponding operators are $g_A \sum_i \sigma_i \tau_i^\pm$ with $g_A = 1.25$, $\tau^\pm = (\tau_1 \pm i\tau_2)/2$. Using closure, one gets the sum rule :

$$S = g_A^2 \sum_I \left\{ \left| \langle I | \sum_i \sigma_i \tau_i^- | 0 \rangle \right|^2 - \left| \langle I | \sum_i \sigma_i \tau_i^+ | 0 \rangle \right|^2 \right\} = g_A^2 (N-Z) \quad (1)$$

The sum depends only on the neutron excess of the nucleus (N,Z), and is totally independent of the structure of the ground state $|0\rangle$.

Experimentally, only the β^- branch (p,n) is measured. By itself it should saturate the sum if the β^+ branch is totally blocked by the Pauli principle. In all cases the sum rule should provide (in the case $N > Z$) a lower limit for the strength of the β^- branch. It has been found instead ²⁾ that the measured strength is a factor ≈ 3 lower than the sum rule in a series of elements ranging from ^{90}Zr to ^{208}Pb .

An explanation commonly invoked to explain this reduction (at least a large part of it) is the Lorentz-Lorenz (L.L.) effect ⁴⁾, which is due to the axial polarization of the nuclear medium. The reduction of the strength results from the fact that the nuclear states do not have a purely nucleonic content but also Δ component which can be admixed ⁵⁾ by a large particle-hole force ⁶⁾.

As previously emphasized, the sum rule (1) is independent of the configuration mixing as long as one restricts to nucleonic degrees of freedom. It breaks down when the basis is enlarged to Δ states and becomes dependent on the particle-hole interaction as is apparent in the L.L. effect. It is then natural to investigate whether one can extend the description and find a sum rule which incorporates both nucleon and Δ degrees of freedom in a broader perspective than the traditional one of nuclear physics.

The frame for such a sum rule is naturally found in the constituent quark model where the nucleon and the Δ are treated on the same ground, being different spin and isospin states of three quarks. In the constituent quark model the weak axial current couples non-relativistically to the spin and isospin of the individual quarks with a unit coupling constant : $A_Z^\pm = \sum_\lambda \sigma_Z(\lambda) \tau^\pm(\lambda)$, where the sum labelled by λ runs now over the 3A quarks. The initial nucleus is described as a multiquark state, with the very complicated structure and correlations implied by its baryonic constituents. The aim is to derive, as previously a sum rule independent of this structure.

Taking the difference between the β^- and β^+ branches and performing closure over the intermediate states one obtains the following sum rule :

$$\mathcal{P} = \sum_I \left\{ \left| \langle I | \sum_\lambda \sigma_Z(\lambda) \tau^+(\lambda) | 0 \rangle \right|^2 - \left| \langle I | \sum_\lambda \sigma_Z(\lambda) \tau^-(\lambda) | 0 \rangle \right|^2 \right\} = N - Z \quad (2)$$

As the previous one (1), the sum is totally independent on the structure of the initial state ; in particular it is not influenced by the amplitude of the Δ admixture. However, the new sum rule has the remarkable feature that it is reduced with respect to the sum (1) by the factor g_A^2 .

The fact that the two sum rules give different results is no surprise since they refer to different G.T. strengths. The classical sum rule (1) refers to the operator $\sum_i \sigma_Z(i) \tau^\pm(i)$ acting on the nucleonic variables. Hence, the intermediate states have a purely nucleonic content. The corresponding G.T. strength thus lies essentially in the low energy domain, as measured in the (p,n) experiments.

The quark sum rule instead incorporates all the intermediate states which can be excited with the operator $\sum_\lambda \sigma_Z(\lambda) \tau^\pm(\lambda)$ acting on the quark variables. This operator can transform a nucleon into a Δ so that the G.T. strength in the new sum rule contains also the high energy response ($\omega \approx 300$ MeV) where a real Δ is excited [in the strict SU(6) quark model the Δ is degenerate with the nucleon ; however, our sum rule is based on the algebra of spin and isospin of the constituent quarks and it does not imply this degeneracy].

The origin of the reduction between the quark sum rule (2) and the nucleon one (1) becomes transparent when they are applied to the case of the single neutron. The sum rule (1) is trivially g_A^2 whereas both Δ and nucleon states are necessary to saturate Eq. (2) as shown in Table 1. Here g_A has been taken as 5/3 according to the SU(6) quark model.

In the case of nuclei a similar Table can be obtained under the assumption of decoupling the nucleon and Δ sectors. With total Pauli blocking the contributions of the nucleon and Δ sectors become, respectively, in the β^- branch, $\frac{25}{9}(N-Z)$ and $\frac{8}{9}(N+3Z)$; in the β^+ branch 0 and $\frac{8}{9}(Z+3N)$, in agreement with the value $(N-Z)$ of the sum rule (2).

When the coupling between nucleon and Δ is switched on the different contributions are modified. The low energy nucleon sector is quenched by the admixture with the Δ but the quark sum rule implies that this quenching should be compensated by a corresponding modification in the Δ sector (for the difference $\beta^- - \beta^+$). We illustrate this with the help of a schematic RPA model fully consistent with the SU(6) quark model and specialized to the case of ^{208}Pb . This model, inspired from that of Ref. 2), consists of two sets of degenerate states, N-hole states at 9.5 MeV and Δ -hole states at 300 MeV carrying all the unperturbed strengths (which are those of the constituent quark model; cf. Tables 1 and 2) with separable N-h and Δ -h interactions taken proportional to the axial strengths. The (repulsive) mixing force is adjusted to reproduce the giant G.T. resonance at 19.2 MeV. With total Pauli blocking, the dispersion equations of the β^- and β^+ branches are, respectively :

$$\begin{aligned} \frac{25}{9} \frac{N-Z}{9.5-\epsilon} + \frac{8}{9} \frac{N+3Z}{300-\epsilon} + \frac{8}{9} \frac{Z+3N}{300+\epsilon} &= -10.1 \\ \frac{25}{9} \frac{N-Z}{9.5+\epsilon} + \frac{8}{9} \frac{Z+3N}{300-\epsilon} + \frac{8}{9} \frac{N+3Z}{300+\epsilon} &= -10.1 \end{aligned} \quad (3)$$

The resulting strengths and energies are shown in Table 2 together with their unperturbed values. The nucleon sector is quenched by the interaction [L.L. effect : it is moderate here due to the use of the small SU(6) axial N- Δ coupling constant]. So is the Δ sector for the two $\beta^\pm$ branches separately as well as their difference (which is <0) leaving unchanged the over-all sum rule.

Notice that there is no conservation of the total strength for each β^- or β^+ branch so that the L.L. quenching is not compensated ^{*)}. This feature characterizes the RPA which introduces Δ -h correlations in the ground state. The commonly made statement that the missing strength is shifted from the low energy to the Δ region is therefore misleading. This would occur only in a Tamm-Dancoff treatment.

^{*)} This has also been noted for symmetric nuclear matter ⁷⁾.

We now discuss the conditions of validity of the (N-Z) sum rule (2). In our simple model where only Δ and N states are accessible by the action of the G.T. operator on the nucleon, the axial NN and NA coupling constants should be those of the SU(6) constituent quark model. It is interesting to observe that any model which satisfies the equal time commutation relation of the axial current $A_Z^\pm$ (current algebra) :

$$[A_Z^+(\vec{x}, t), A_Z^-(\vec{y}, t)] = 2 \delta(\vec{x} - \vec{y}) V_Z^3(\vec{x}, t) \quad (4)$$

[$V_Z^3(\vec{x})$ being the total isospin density], respects the sum rule (2). In particular there exist improvements of the constituent quark model such as the inclusion of D state mixing proposed by Glashow⁸⁾ where other states than N and Δ become accessible but the sum rule still holds. Of course, the respective N and Δ contributions are no longer those given in Table 1. It would be natural to extend our schematic treatment to this model since it gives (by construction) the right value of g_A which is overestimated in the case of SU(6) (5/3 vs. 1.25).

We also remark that schematic RPA models like the present one satisfy generally some sort of G.T. sum rule provided the Δ -h and N-h interactions in the spin-isospin channel are derived consistently from the axial N- Δ and N-N couplings which do not need to be those of a quark model. For instance, with the Chew-Low ratio for NA/NN coupling, the sum rule is $(N-Z)g_A^2/9$ in considerable reduction with respect to the nucleon one (1). However, this result needs not be taken seriously because strict application of the Chew-Low model to the Adler-Weisberger (A.W.) dispersion relation leads to a very unrealistic value of g_A (≈ 3).

The quark G.T. sum rule (2) has instead deeper foundations and deserves more attention though it can be derived only from models satisfying the algebra of currents. It has the advantage of connecting the low (nucleon sector) to high energy (Δ sector) part of the G.T. strengths. It is amusing in this context to make the following observation : the quark sum rule (2) is smaller by a factor $\frac{25}{9}(\approx 3)$ than the nucleon sum rule (1) ; this factor coincides with the observed reduction of the nucleon sector suggesting a saturation of the sum (2) by the low energy part alone and hence a blocking of the Δ sector. In our schematic model where a partial blocking occurs, it is realized by a different modification of the intensities in the β^- and β^+ branches of the Δ sector which strongly reduces

the difference. The quantitative reliability of the result is in principle doubtful since it is based on the crude SU(6) quark model. However, it is remarkable that it subsists in a realistic treatment based on the A.W. relation which is discussed below.

The algebra obeyed by the isospin charges, though originally suggested by the underlying quark structure, has a validity which extends far beyond the constituent quark model, as a consequence of the chiral symmetry of strong interactions. In particular, the equal time commutator of two axial charges, together with PCAC leads to the A.W. sum rule ^{9),10)} for the nucleon :

$$1 = g_A^2 + \frac{2f_\pi^2}{\pi} \int_{m_\pi}^{\infty} d\omega \frac{M+\omega}{\omega(2M+\omega)} [\sigma_0^+(\omega) - \sigma_0^-(\omega)] \quad (5)$$

where f_π is the π decay constant ($f_\pi = 0.94 m_\pi$) and $\sigma_0^\pm$ the $\pi^\pm$ total cross-sections on the neutron for soft pions. The 33 resonance makes the largest part of the dispersive integral. If only the Δ is retained and its width neglected the integral is proportional to the square of the $\pi N\Delta$ coupling constant independent of the position of the resonance. Through the Goldberger-Treiman relation, the A.W. relation reduces then to our relation between the axial strengths, as given in Table 1. However, with the Δ alone g_A is overestimated : 1.44 instead of 1.24, close to the constituent quark value ⁹⁾. With the dispersive integral correctly estimated, the A.W. sum rule gives the right value of g_A . It takes into account properly the width of the Δ and, through the high energy part of the dispersive integral, the coupling to higher resonances due for instance to deformation as in the Glashow model.

The generalization of the A.W. sum rule to the nuclear case performed by M. Ericson ¹¹⁾ writes :

$$N-Z = (N-Z)(g_A)_{\text{eff}}^2 + \frac{f_\pi^2}{\pi} \int_{m_\pi}^{\infty} \frac{d\omega}{\omega} [\sigma_A^+(\omega) - \sigma_A^-(\omega)] \quad (6)$$

where $(N-Z)(g_A)_{\text{eff}}^2$ represents the contribution of the low energy nucleon sector. In principle the dispersive integral should run in a nucleus from zero to infinity due to two-nucleon absorption but this process is usually neglected ¹²⁾ and the integral taken to start at $\omega = m_\pi$. The sum rule (6) provides the desired link between the low and high energy axial responses. Notice that it is taken along the soft pion line $|\vec{q}| = \omega$ and not along $|\vec{q}| = 0$ as the G.T. sum rule (2). For the low energy sector ω is small and the distinction of little practical

importance, hence the first term on the right-hand side of Eq. (6) is practically the difference between the measured β^- and β^+ G.T. strengths. For the evaluation of the dispersive integral an off-shell extrapolation of the cross-section from physical to soft pions is required. In principle this extrapolation is a mild one and the integral can be derived from the physical cross-sections. The A.W. relation has thus a predictive power for $(g_A)_{\text{eff}}^2$, contrary to the G.T. sum rules where the Δ sector is unknown.

It is well known that the physical cross-sections undergo a shadowing in the Δ resonance region where the nucleus becomes black. This applies as well to the difference $\Delta\sigma = (\sigma_A^+ - \sigma_A^-)$, which becomes A dependent. This was proved by experiments on ${}^7\text{Li}$ and ${}^9\text{Be}$ ¹³⁾ and a theoretical calculation with an optical potential ^{14),15)}, which shows that $\Delta\sigma$ is totally suppressed in heavy nuclei. Since $\Delta\sigma$ is negative for neutron excess, its shadowing quenches $(g_A)_{\text{eff}}^2$.

The amount of quenching depends on the energy domain where the shadow applies. Indeed the respective contributions to g_A^2 of the Δ resonance and the high energy region are in the nucleon case ⁹⁾ : $g_A^2 = 1 + 0.80(\Delta) - 0.26$ (high energy). In nuclei if we assume a total shadowing of the whole dispersive integral we obtain $(g_A)_{\text{eff}}^2 = 1$; the quenching of the strength $((g_A)_{\text{eff}}^2/g_A^2 = 0.64)$ is less pronounced than the observed one. Under the assumption that the shadowing is effective only for the Δ region, one gets $(g_A)_{\text{eff}}^2/g_A^2 = 0.48$, which is closer to experiment.

The quenching of the G.T. strength thus seems strongly related to the shadow phenomenon. However, the problem of the off-shell extrapolation is delicate in the nucleus and it is not a priori obvious that the shadowing persists for soft pions. An off-shell pion is sustained by a source which penetrates the nuclear medium. The amount of shadowing depends on the competition between absorption and the off-shell propagation as discussed by Bell ¹⁶⁾. In the Δ resonance region the mean free path is so small that the off-shell propagation is negligible and the shadow persists. This is not necessarily the case at high energy where the charge averaged cross-section is typically smaller by a factor ~ 4 , even though some shadow persists for physical pions up to 1 GeV . A detailed theoretical study of this problem is highly desirable, including the possible influence of pion absorption below threshold.

The above discussion strongly suggests a relation between the shadow phenomenon and the L.L. effect. This is plausible since both phenomena are associated with Δ -hole polarization at different frequencies. Consistency between the two approaches requires a connection between the repulsion in the spin-isospin channel and the shadow effect, which may provide a constraint for the corresponding Landau-Migdal parameter.

In summary we have derived a new type of sum rules based on quark models for the difference of the β^- and β^+ G.T. strengths in nuclei extended to baryonic excitations, in particular the Δ resonance. They establish a link between the low energy and high energy (Δ) parts of the axial strength. As an illustration we have used a schematic model. There, the coupling between the N and Δ sectors through a repulsive spin-isospin particle-hole interaction de-populates both the low energy (L.L. effect) and the high energy domains. The quark sum rule is, however, respected by a delicate balance between the $\beta^\pm$ branches. Sum rules for the G.T. strengths (i.e., taken on the line $\vec{q} = 0$) have been proved only for specific models, among which the quark models. In addition their practical application is limited by the absence of experimental information on the Δ sector at zero transfer. However, they illuminate the meaning of the model independent sum rule, the A.W. relation (a consequence of chiral symmetry) which is taken along the soft pion line ($|\vec{q}| = \omega$). Hence the high energy sector is accessible through a mild extrapolation of physical pion-nucleus cross-sections. From the A.W. sum rule, the shadowing of these cross-sections implies the quenching of the low energy strength.

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Intermediate state	N	Δ	Total (N+ Δ)
β^- branch	$\frac{25}{9}$ (proton)	$\frac{8}{9}$ (Δ^+)	$\frac{11}{3}$
β^+ branch	0	$\frac{8}{3}$ (Δ^-)	$\frac{8}{3}$
difference ($\beta^- - \beta^+$)	$\frac{25}{9}$ ($= g_A^2$)	$-\frac{16}{9}$	1

Table 1 - Saturation of the quark sum rule (3) with nucleon and Δ states in the case of the neutron [the coupling constants follow from SU(6), e.g., $g_A = 5/3$].

		N sector (with total Pauli blocking and $\epsilon_A^2 = \frac{25}{9}$)	Δ Sector	Total (N+ Δ)
Without Interaction	S_{β}^-	$\frac{25}{9} (N-Z) = 122.2$ ($\epsilon_{ph} = 9.5$ MeV)	$\frac{8}{9} (N+3Z) = 330.7$ ($\epsilon_{\Delta h} = 300$ MeV)	$\frac{1}{9} (33N-Z) = 452.9$
	S_{β}^+	0	$\frac{8}{9} (3N+Z) = 408.9$ ($\epsilon_{\Delta h} = 300$ MeV)	$\frac{8}{9} (3N+Z) = 408.9$
	$S_{\beta}^- - S_{\beta}^+$	$\frac{25}{9} (N-Z) = 122.2$	$-\frac{16}{9} (N-Z) = -78.2$	(N-Z) = 44
With Interaction	S_{β}^-	79.1 ($\epsilon_{GT} = 19.2$ MeV)	313.7 ($\epsilon_{\Delta} = 331.8$ MeV)	392.8
	S_{β}^+	0	348.8 ($\epsilon_{\Delta} = 337.1$ MeV)	348.8
	$S_{\beta}^- - S_{\beta}^+$	79.1	-35.1	(N-Z) = 44

Table 2 - The axial strengths $S_{\beta}^{\pm}$ and energies of the nucleon and Δ sectors in the schematic model. The upper and lower parts of the Table refer respectively to the unperturbed and perturbed situations [ϵ_{GT} and ϵ_{Δ} denote the positions of the Gamow-Teller and Δ resonances ; again the couplings are those of SU(6)].

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