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MEASUREMENT OF THE MAGNETIC BIREFRINGENCE
IN OXYGEN AND NITROGEN GASES

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ABSTRACT

The Cotton-Mouton constant for oxygen and nitrogen has been measured at room temperature and pressure using a laser with $\lambda = 514.5$ nm.

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1. INTRODUCTION

We present here the results of a measurement of the Cotton-Mouton constant C_{CM} for oxygen and nitrogen at room temperature ($17^\circ\text{C} \pm 0.5$) and ambient pressure (1017 ± 2 mb) with light of wavelength $\lambda = 514.5$ nm. The data are compared with the available published values obtained at high pressure (100 abs atm) and in white light [1,2].

The Cotton-Mouton effect is a birefringence induced in matter by a magnetic field. It is measured by the constant C_{CM} defined by the relation

$$\Delta n = n_L - n_T = C_{CM} \lambda B^2, \quad (1)$$

where B is the component of the magnetic field orthogonal to the light propagation vector, and n_L and n_T are the indices of refraction for light which is polarized linearly in a direction parallel and orthogonal, respectively, to that of the magnetic field direction.

The measurements have been performed in the course of the development of an ellipsometer able to detect the magnetic birefringence of vacuum [3-5]. A heterodyne technique [6] has been used by modulating both the polarization of the light and the applied magnetic field. The sensitivity so far obtained in the detection of small ellipticities is 5×10^{-9} in 1000 s of running time.

2. EXPERIMENTAL APPARATUS

The experimental set-up is schematically shown in fig. 1. The light source is a cw argon ion laser tuned for $\lambda = 514.5$ nm and for a power output $W_L = 600$ mW. The Glan-Thompson prism P introduces a well-defined linear polarization on the light beam. The Faraday cell FCM rotates the plane of polarization around the propagation axis by the angle

$$\phi(t) = \phi_0 \sin(2\pi f_F t),$$

where $f_F = 1562.500$ Hz. The quarter wave-plate QW is mounted with its fast axis aligned with the axis of the prism P . The combination of the two elements FCM and QW introduces a modulated ellipticity $\phi(t)$ on the light beam. The gas under

study is contained in a stainless-steel cylindrical vessel of 4 cm diameter with two BK7 1 cm thick windows. The central region of the vessel is traversed over a length $\ell \approx 100$ cm by a magnetic field $B(t)$ perpendicular to the propagation direction of the light and at 45° with respect to the axis of the plate QW. The magnetic field is produced by a CERN standard MNP18 dipole and is time modulated so that

$$B(t) = B_1 + B_2 \cos(2\pi f_M t) ,$$

with $B_1 \gtrsim B_2 \approx 1500$ G and $f_M = 0.397$ Hz. The ellipticity $\Psi(t)$ due to the magnetic birefringence of the gas according to expression (1) is given by

$$\Psi(t) = \pi \Delta n (\ell/\lambda) = \pi C_{CM} \int B^2(t, x) dx ,$$

where the integral is taken along the light path to include the end effects. Particular care was taken to confine the magnetic field to the central region of the vessel, by placing two soft iron plates S at the sides of the dipole yoke in order to shunt the stray field away from the beam axis. The Glan-Thompson analyser A is oriented for maximum extinction of the transmitted ray. In the first approximation, the light power of the transmitted and the reflected rays is given, respectively, by

$$W_T(t) = W_0 \left\{ \sigma^2 + [\beta + \Phi(t) + \Psi(t)]^2 \right\} \quad (2)$$

and

$$W_R = W_0 - W_T \approx W_0 ,$$

where

W_0 is the beam power prior to the prism A (100 mW);

σ^2 is the overall extinction factor (10^{-6});

β is the misalignment between the axes of the prism P and the plate QW.

This misalignment is kept within 10^{-5} rad by a microprocessor-controlled feedback loop acting on the Faraday cell FCC. Both Faraday cells FCM and FCC use 1 cm thick BK7 optical flats.

From eq. (2) it is easy to see that, without the modulation $\Phi(t)$, the useful signals on W_T containing $\Psi(t)$ have a frequency not exceeding $4f_M$. Since f_M in the iron-yoke dipole cannot be pushed beyond 1 Hz, the signals would be lost in the low-frequency laser noise [5]. The modulation $\Phi(t)$ at frequency f_F acts as a carrier of $\Psi(t)$, which can be analysed in a less noisy frequency region by using lock-in techniques.

The detector PD is a PIN photodiode of 2.5 mm diameter used in photovoltaic mode, with a quantum efficiency of $\eta = 0.5$.

The signal is processed by two lock-in amplifiers H and K, with reference frequencies of $2f_F$ and f_F , respectively, and output time constants of 0.3 s. The outputs from the lock-in amplifiers are sampled at a frequency of 20 Hz and written on magnetic tape.

The following quantities are also continuously monitored and written on tape:

- a) the magnetic field $B(t)$, by measuring it with a Hall probe;
- b) the modulation $\Phi(t)$, by measuring the driving current to the cell FCM;
- c) the light power W_0 , by measuring W_R with a photodiode.

All frequencies of the signals used in the measurement were controlled by a 10 MHz master clock, temperature-controlled, with a stability of 1 part in 10^8 .

3. SENSITIVITY OF THE APPARATUS

According to eq. (2) the Fourier amplitude of the photodiode current proportional to Ψ is

$$i_s = 2W_0 \frac{e\lambda}{hc} \eta \Psi_0 \Phi_0, \quad (3)$$

where e is the electron charge, h is the Planck constant, and c is the velocity of light. This signal has to be compared with the main sources of noise.

- i) The photodiode current exhibits a shot noise given by

$$\langle i_{\text{shot}} \rangle = \sqrt{2e i_{\text{dc}} \Delta\nu}$$

where $\Delta\nu$ is the inverse of the running time τ .

ii) The Johnson noise present on the load resistor R of the current amplifier is

$$\langle i_J \rangle = \sqrt{\frac{4kT\Delta\nu}{R}} .$$

iii) The laser noise exceeding the shot noise, according to fig. 7 of ref. 5, can be expressed as

$$\langle i_L \rangle = i_{dc} F \sqrt{\Delta\nu} ,$$

with $F = 10^{-5}/\sqrt{\text{Hz}}$ for the laser we used, in the frequency region near f_F .

The operation of the apparatus is subjected to two somewhat conflicting conditions:

- a) the light noise on the detector approaches the limiting shot noise only for a sufficiently low light power;
- b) the signal i_s increases linearly with the power W_0 prior to the analyser [see eq. (3)].

Thus a good sensitivity requires a very low extinction factor σ^2 . We have consistently obtained $\sigma^2 = 1.5 \times 10^{-6}$ throughout the measurements.

The other parameters were set as follows:

$$\Phi_0^2 = 2\sigma^2 , \quad W_0 = 100 \text{ mW} , \quad R = 1 \text{ M}\Omega .$$

Under these conditions one has (for $\tau = 1 \text{ s}$)

$$\begin{aligned} \langle i_{\text{shot}} \rangle &= 1.4 \times 10^{-13} \text{ A} \\ \langle i_J \rangle &= 1.3 \times 10^{-13} \text{ A} \\ \langle i_L \rangle &= 7.8 \times 10^{-13} \text{ A} . \end{aligned}$$

Therefore the limiting sensitivity due to the previous noise contributions is

$$\psi_{\min} = 1.6 \times 10^{-8} \quad (\tau = 1 \text{ s}) \quad (4)$$

and it scales proportionally to $\tau^{-1/2}$. However, this sensitivity was not reached in the present measurements because of the noise introduced mainly by thermal gradients

and birefringence in the vessel windows and Faraday cells. In fact, the effective sensitivity in the running conditions was

$$\Psi_{\text{exp}} = 1.5 \times 10^{-7} \quad (\tau = 1 \text{ s}) .$$

Work is in progress in order to reach the limiting sensitivity [see eq. (4)].

4. DATA ANALYSIS AND CONCLUSIONS

The Fourier transform of the output of the lock-in amplifier K (see fig. 1) at the frequency f_M gives a signal proportional to the product $(W_0 \Phi_0 \Psi_0)$ [see eq. (3)], where

$$\Psi_0 = 2\pi C_{\text{CM}} \int B_1(x) B_2(x) dx . \quad (5)$$

To obtain Ψ_0 and, consequently, C_{CM} we fold in the quantity $(W_0 \Phi_0^2)$, as given by the lock-in amplifier H, and Φ_0 derived from the current driving the modulator FCM. The integral appearing in expression (5) has been evaluated by measuring the magnetic field with a Hall probe all along the light path. The estimated error in this integral is $\pm 2\%$.

We have performed 53 oxygen runs and 32 nitrogen runs. The running time of each measurement has been 1000 s in order to obtain a frequency resolution of 1 mHz. The mean values of the Cotton-Mouton constant obtained in the oxygen and nitrogen runs are

$$C_{\text{CM}}(\text{O}_2) = -(4.89 \pm 0.11) \times 10^{-16} \text{ G}^{-2} \text{ cm}^{-1} \quad (6)$$

and

$$C_{\text{CM}}(\text{N}_2) = -(4.98 \pm 0.26) \times 10^{-17} \text{ G}^{-2} \text{ cm}^{-1} . \quad (7)$$

The scatter plots of the deviations from the mean of the measured values $C_{\text{CM}}(\text{O}_2)$ and $C_{\text{CM}}(\text{N}_2)$ are shown in fig. 2. We should point out that the error quoted in eq. (6) is mainly systematic, owing to the uncertainty in the magnetic field integral.

As a cross check, we have performed several runs with the vessel evacuated or filled with hydrogen. In both cases, we have found no detectable ellipticity at f_M . The results of the measurements in H_2 are in agreement with those of Bizette and Tsai Belling [7].

Moreover, since the Verdet constant for hydrogen is roughly the same as for oxygen and nitrogen [8], we conclude that any contribution from the Faraday effect to our measurements was below the experimental sensitivity.

The results of the present experiment together with the values quoted in refs. 1 and 2 are summarized in table 1.

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Table 1

Summary of experimental data for the Cotton-Mouton constant
in O₂ and N₂ gases

Gas	Pressure (abs atm)	Light wavelength (nm)	Detector	Measured Cotton-Mouton constant (G ⁻² cm ⁻¹)	Ref.
O ₂	96.78	white light	eye	-4.8×10^{-14}	1
O ₂	100	white light	eye	-4.05×10^{-14}	2
N ₂	100	white light	eye	-0.33×10^{-14}	2
O ₂	1.00	514.5	photodiode	$-(4.89 \pm 0.11) \times 10^{-16}$	present paper
N ₂	1.00	514.5	photodiode	$-(4.98 \pm 0.26) \times 10^{-17}$	present paper

Figure captions

- Fig. 1 : Schematic view of the experimental apparatus.
- Fig. 2 : Scatter plots of the deviations ξ from the mean values of $C_{CM}(O_2)$ and $C_{CM}(N_2)$ in units of $10^{-20} \text{ G}^{-2} \text{ cm}^{-1}$:
- a) for the 53 runs in oxygen,
 - b) for the 32 runs in nitrogen.

Fig. 2