Ref.TH.3254-CERN

POWER CORRECTIONS TO THE PARTON MODEL IN QCD

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A B S T R A C T

We calculate the power corrections to deep inelastic lepton-hadron scattering in QCD in terms of Feynman graphs. The method is applicable to other hard scattering processes. We stress the relationship with the naïve parton model and give a precise meaning to the intrinsic transverse momentum in QCD.

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Ref.TH.3254-CERN

4 March 1982

It has become increasingly clear that reliable quantitative tests of QCD will require an understanding of the corrections which fall like inverse powers of the large energy scale Q . Nowhere is this truer than in deep inelastic scattering, where a sensitive measurement of the logarithmic variation of the α constant forces one to probe the region of low Q^2 , in which the power corrections may be important. For deep inelastic scattering the apparatus of the operator product expansion allows us to describe these power corrections as matrix elements of local operators of twist higher than two. However, the problem of power corrections is common to all hard scattering processes in QCD and, as has been emphasized especially by Politzer¹⁾, a graphical method which describes the power corrections to inclusive processes is urgently needed.

In this paper we present the results of such a graphical analysis. For definiteness we will consider only the $1/Q^2$ corrections to lepton-hadron deep inelastic scattering. The extension to other processes will be presented elsewhere²⁾. At all points we try to reconcile the concepts of the naive parton model with our more complete QCD analysis. In particular we give a precise meaning to the average intrinsic transverse momentum in QCD. We limit ourselves to the so-called dynamical power corrections, controlled by the scale of the strong interactions Λ , rather than the kinematical power corrections associated with the masses of quarks and hadrons. We set the masses of all quarks and hadrons to zero. Kinematical power corrections are taken into account by ξ scaling³⁾.

In deep inelastic scattering the operators of twist two provide a description of the terms which scale (modulo logarithms). In the parton model these operators are related to parton distribution functions in the infinite momentum frame. To parametrize the frame dependence of the simple parton description we introduce a light-like vector n^μ in terms of which we can decompose the parton momentum

$$k_\mu = x p_\mu + \frac{k_\perp^2}{2x} n_\mu + k_{T\mu}, \quad p^2 = n^2 = p k_T = n k_T = 0, \quad n p = 1. \quad (1)$$

The vector p is the four-momentum of the hadron whose structure we wish to describe. The vector n is extraneous to the hadron but having chosen it we are able to specify the light-cone fraction x and the transverse momentum k_T . Since we already need the vector n at leading twist level to define the light-cone fraction x , it is natural to ask whether the next-to-leading twist corrections are described in terms of k_T .

We therefore consider a model for the handbag diagram (Fig. 1a), in which the incoming line is put on mass-shell but has non-zero transverse momentum. Denoting the parton distribution by f , our expression for $W^{\alpha\beta}$ (the standard hadronic tensor of deep inelastic scattering ⁴⁾), is

$$W^{\alpha\beta}(q, p) = \int \frac{dx}{x} d^2k_T \left[\hat{S}^{\alpha\beta}(\dot{k}) \dot{k} \right] f(x, k_T^2). \quad (2)$$

Square brackets everywhere indicate that a trace is to be taken over the spinor indices (shown implicitly in $\hat{S}$ by the hat). $\hat{S}$ is the s channel discontinuity of the photon quark forward scattering amplitude divided by (8π) . Here and elsewhere a dot on a four-vector indicates its on-shell part ($\dot{k}^2 = 0$),

$$\dot{k}_\mu = k_\mu - \frac{k^2}{2x} n_\mu. \quad (3)$$

In lowest order $\hat{S}$ is given by

$$\left[\hat{S}^{\alpha\beta}(\dot{k}) \dot{k} \right] = \frac{1}{4} \left[\gamma^\alpha (\dot{k} + \not{q}) \gamma^\beta \dot{k} \right] \delta^{(+)}((\dot{k} + q)^2) \quad (4)$$

but the arguments which we give below are valid to all orders.

The terms contributing in each order in $1/Q^2$ are isolated by performing a series expansion about the collinear direction,

$$\hat{S}^{\alpha\beta}(\dot{k}) = \hat{S}^{\alpha\beta}(xp) + \frac{\partial \hat{S}^{\alpha\beta}}{\partial \dot{k}^\mu} \Big|_{\dot{k}=xp} (\dot{k} - xp)^\mu + \frac{1}{2} \frac{\partial^2 \hat{S}^{\alpha\beta}}{\partial \dot{k}^\mu \partial \dot{k}^\nu} \Big|_{\dot{k}=xp} (\dot{k} - xp)^\mu (\dot{k} - xp)^\nu + \dots \quad (5)$$

After averaging over angles in the transverse plane and dropping higher order terms in k_T^2 we obtain

$$W^{\alpha\beta}(q, p) = \int dx d^2k_T \left(a_0^{\alpha\beta}(x) + k_T^2 \sigma^{\alpha\beta}(x) + \dots \right) f(x, k_T^2) \quad (6)$$

where we have defined (the virtual photon indices α, β are suppressed),

$$\sigma(x) = \frac{1}{2} \left\{ \frac{a_1(x)}{x^2} - \frac{1}{x} \left(b_1(x, x) - b_2(x, x) \right) - c_1(x, x, x) \right\}. \quad (7)$$

These quantities are defined in terms of $\hat{S}$ as follows :

$$\begin{aligned} a_0(x) &= [\hat{S}(xp) \not{x}] , & a_1(x) &= [\hat{S}(xp) \not{x}] , \\ b_1(x, x) &= d_{\mu\nu} \left[\frac{\partial \hat{S}}{\partial k_\mu} \gamma^\nu \right] \Big|_{k=xp} , & b_2(x, x) &= m_\mu \left[\frac{\partial \hat{S}}{\partial k_\mu} \not{x} \right] \Big|_{k=xp} , \\ c_1(x, x) &= d^{\mu\nu} \left[\frac{1}{2} \frac{\partial \hat{S}}{\partial k^\mu \partial k^\nu} \not{x} \right] \Big|_{k=xp} . \end{aligned} \quad (8)$$

The meaning of the repeated arguments for b and c will become clear later. Throughout this paper we use the following notation for the two transverse tensors in gluon indices,

$$d^{\mu\nu} = g^{\mu\nu} - p^\mu n^\nu - n^\mu p^\nu , \quad e^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} m_\alpha p_\beta . \quad (9)$$

Using the parametrization

$$m^\mu = \frac{1}{\nu} (x_B p^\mu + q^\mu) , \quad x_B = \frac{Q^2}{2\nu} , \quad \nu = p q , \quad Q^2 = -q^2 ,$$

we obtain the result for the lowest order

$$a_0(x) = e_T \delta(x - x_B) , \quad (10)$$

$$b(x) = \frac{1}{Q^2} \left\{ 4(e_T + e_L) \delta(x - x_B) - e_T x_B \delta'(x - x_B) \right\} . \quad (11)$$

The virtual photon indices are carried by the transverse and longitudinal tensors,

$$e_T^{\alpha\beta} = \frac{x_B}{\nu} p^\alpha p^\beta + \frac{1}{2\nu} (p^\alpha q^\beta + p^\beta q^\alpha) - \frac{1}{2} g^{\alpha\beta} , \quad (12)$$

$$e_L^{\alpha\beta} = \frac{1}{2} \left(g^{\alpha\beta} - \frac{q^\alpha q^\beta}{q^2} \right) . \quad (13)$$

The results for the structure functions in our naïve parton model are,

$$W^{\alpha\beta}(q, p) = e_L^{\alpha\beta} F_L(x_B, \frac{\Lambda^2}{Q^2}) + e_T^{\alpha\beta} F_T(x_B, \frac{\Lambda^2}{Q^2}) \quad (14)$$

where,

$$F_T(x_B, \frac{\Lambda^2}{Q^2}) = \int d^2 k_T f(x_B, k_T^2) + \frac{1}{Q^2} \int d^2 k_T k_T^2 \left\{ 4f(x_B, k_T^2) + x_B \frac{\partial}{\partial x_B} f(x_B, k_T^2) \right\}, \quad (15)$$

$$F_L(x_B, \frac{\Lambda^2}{Q^2}) = \frac{4}{Q^2} \int d^2 k_T k_T^2 f(x_B, k_T^2) = \frac{4 \langle k_T^2 \rangle}{Q^2}. \quad (16)$$

F_T and F_L are related to the standard structure functions by $F_T = (F_2/x_B)$, $F_L = (\frac{1}{x_B} F_2 - 2F_1)$. The presence of the intrinsic transverse momentum k_T has generated a longitudinal structure function ⁵⁾.

Our simple model has several deficiencies which are related. From Eq. (15), we see that the Adler sum rule (exact to all orders in $1/Q^2$) is not satisfied, except for special choices of the function f . This is a consequence of the form of our parton model ansatz, Eq. (2). In general $f(x, k_T^2)$ depends on the vector n , and the Lorentz invariance is mutilated. If we restore the Lorentz invariance (with vanishing target mass $M = 0$) the model becomes trivial ($k_T \equiv 0$). The model is non-trivial only for the case $M \neq 0$; Lorentz invariance now requires that f have the form,

$$f(x, k_T^2) = \frac{1}{M^2} \Phi\left(\frac{2pk}{M^2}\right) \Theta((p-k)^2) = \frac{1}{M^2} \Phi\left(x + \frac{k_T^2}{xM^2}\right) \Theta(x(1-x)M^2 - k_T^2). \quad (17)$$

Working out the kinematics exactly we see that this form of f reproduces ξ scaling ⁶⁾. However, the real point of the model is that in the interacting theory, even for $M = 0$, we find that the transverse degrees of freedom are important. The transverse momentum, which we inserted in Eq. (2) in an ad hoc fashion, will be constructed dynamically from the transverse components of the gluon field. In QCD, gauge invariance provides another argument that the model with k_T alone and no interactions cannot be correct. The transverse momentum is not a gauge covariant quantity, so it is natural to consider gluon fields in association with it. We maintain the similarity between transverse momentum and additional gluon fields by working in the light-like axial gauge $n \cdot A = 0$ and choosing the same vector n to fix the gauge and to define the transverse direction.

Examples of interaction of the struck quark with the gluon field are given in Fig. 1b,c. An attractive feature of the light-like axial gauge is that the number of partons which can contribute at level $(\Lambda/Q)^T$ is limited. By a

simple power counting argument ⁷⁾ we find that the hadronic blobs with n_F quarks and n_G gluons contribute in this gauge with τ given by,

$$\tau \geq n_F + n_G - 2 + \frac{1}{2} \left\{ 1 - (-1)^{n_G} \right\}. \quad (18)$$

In accordance with the remarks made above about the similarity of momenta and gluon fields, n_G is to be interpreted as the number of vector indices in the lower blob. These may come either from gluon fields or from powers of the momentum $(k-xp)^\mu$ which are produced in the Taylor expansion. For the $(\Lambda/Q)^2$ corrections in non-singlet deep inelastic scattering we may limit ourselves to the combinations,

$$(n_F, n_G) = (2, 0), (2, 1), (2, 2), (4, 0). \quad (19)$$

All lowest order diagrams are shown in Figs. 1, 2. The contribution to deep inelastic scattering coming from diagrams containing four quarks (Fig. 2) have been considered elsewhere ^{8),9)}. We will not consider them further in this paper. Since we focus our attention on power corrections, we will also postpone the question of logarithmic radiative corrections. These latter effects are calculable and give slow logarithmic variations on top of the power law behaviour discussed here. In our gauge they come from generalized ladder graphs. We therefore limit our discussion to tree diagrams.

Thus in QCD we calculate the structure function from the diagrams of Fig. 1a,b,c.

$$W(2, p) = \left[\hat{S}(k) \hat{\Gamma}(k) + \hat{S}_\mu(k_2, k_1) \hat{\Gamma}^\mu(k_2, k_1) + \hat{S}_{\mu\nu}(k_2, k, k_1) \hat{\Gamma}^{\mu\nu}(k_2, k, k_1) \right]_{(20)}$$

Integration over repeated four momenta k_1, k_2 and k is understood in this and subsequent formulae. The quantities $\hat{S}$, $\hat{S}_\mu$ and $\hat{S}_{\mu\nu}$ are discontinuities in x_B of the parton-photon amplitudes of Fig. 1. Parton propagators, the QCD coupling constant and SU(3) colour matrices are all included in the respective hadronic blobs Γ . This means that $\hat{S}$, $\hat{S}_\mu$ and $\hat{S}_{\mu\nu}$ are all of zero order in the strong coupling constant g and they coincide with the QED expressions. Neglecting generalized ladder diagrams means to order Λ^2/Q^2 that the Γ 's should be four-particle irreducible ⁷⁾. For simplicity we will take the Γ 's to be full Green's functions (with 2, 3 and 4 parton states included) and turn a blind eye to the logarithmic ultra-violet divergences which are consequently generated in the Taylor expansion of Eq. (20).

We now proceed to factorize Eq. (20), emulating as closely as possible the naive parton model form Eq. (2). After completion of the factorization our final formula like Eq. (2) will be expressed in terms of amplitudes for on-shell partons with only physical polarizations.

As a tentative first step we perform a general invariant decomposition of $\hat{\Gamma}$,

$$\hat{\Gamma}(k) = \not{k} \Gamma_1(k) + \not{\epsilon} \Gamma_2(k) + \not{k}^2 \Gamma_3(k). \quad (21)$$

The first term corresponds to the sum over physical polarizations as in Eq. (2), but we also obtain the unwanted terms Γ_2, Γ_3 . In a similar way the gluon contributions (Fig. 1b,c) contain sums over unphysical polarizations because the gluons are off-shell. The systematic way to eliminate the unwanted polarizations is to relate the off-shellness to more partons using the equations of motion ^{10),1)}. Technically this is achieved by inserting the right-hand side of the identity,

$$1 = \frac{1}{2x} (\not{x} \not{k} + \not{k} \not{x}) \quad (22)$$

on both sides of the spinor function Γ in Eq. (20). In the second and third terms in Eq. (20) we also insert the right-hand side of Eq. (22) with a judicious choice of the momenta k_1, k_2 . When the $k(k_1, k_2)$ acts directly on the hadronic blob it is reduced to a blob containing more fields as a consequence of the quark equation of motion,

$$\begin{aligned} (2\pi)^4 \not{k} \hat{\Gamma}(k) &= - \int d^4 k_2 d^4 k_1 \not{\epsilon}^\mu \hat{\Gamma}_\mu(k_2, k_1) \delta^4(k - k_2), \\ (2\pi)^4 \hat{\Gamma}(k) \not{k} &= - \int d^4 k_2 d^4 k_1 \hat{\Gamma}_\mu(k_2, k_1) \not{\epsilon}^\mu \delta^4(k - k_1), \\ (2\pi)^4 \not{k}_1' \hat{\Gamma}_\mu(k_2', k_1') &= - \int d^4 k_2 d^4 k d^4 k_1 \not{\epsilon}^\nu \hat{\Gamma}_{\mu\nu}(k_2, k, k_1) \delta^4(k_1' - k) \delta^4(k_2' - k_2), \\ (2\pi)^4 \hat{\Gamma}_\nu(k_2', k_1') \not{k}_2' &= - \int d^4 k_2 d^4 k d^4 k_1 \hat{\Gamma}_{\mu\nu}(k_2, k, k_1) \not{\epsilon}^\mu \delta^4(k_1' - k_1) \delta^4(k_2' - k), \\ (2\pi)^8 \not{k} \hat{\Gamma}(k) \not{k} &= \int d^4 k_2 d^4 k_1 \not{\epsilon}^\nu \hat{\Gamma}_{\mu\nu}(k_2, k, k_1) \not{\epsilon}^\mu. \end{aligned} \quad (23)$$

When the k in Eq. (22) acts on $\hat{S}$ we replace it by $\dot{k}$ (since it is adjacent to $\not{n}$). The $\dot{k}$ now acts as a physical polarizer for the upper part $\hat{S}$. The only remaining dependence on the off-shellness k^2 is contained in $\hat{S}$. We make this dependence manifest by Taylor expansion,

$$\hat{S}(k) = \hat{S}(\dot{k}) + \frac{1}{2X} m^\mu \left. \frac{\partial \hat{S}(k)}{\partial k^\mu} \right|_{k=\dot{k}} k^2 + \dots \quad (24)$$

and reduce this additional $k^2 (= KK)$ by the equations of motion, Eq. (23). After eliminating the off-shellness we decouple the spinor trace over the lower parts (Γ) from the trace over the rest, by use of the identity (valid for massless quarks),

$$\frac{1}{2} [\hat{\sigma} \hat{\Gamma}] = [\hat{\sigma}_L \gamma_5] \langle \gamma^5 \gamma_L \hat{\Gamma} \rangle + [\hat{\sigma}_R \gamma_5] \langle \gamma^5 \gamma_R \hat{\Gamma} \rangle \quad (25)$$

where $\gamma_L = \frac{1}{2}(1-\gamma_5)$, $\gamma_R = \frac{1}{2}(1+\gamma_5)$ and the brackets $\langle \rangle \equiv \frac{1}{4} \text{Tr} \{ \}$.

After performing all these manipulations Eq. (20) becomes,

$$\begin{aligned} W(q, p) = & \frac{1}{X} [\hat{S}(\dot{k}) \dot{k}] \langle \not{n} \hat{T}(k) \rangle \\ & + \frac{1}{x_2 x_1} \left\{ [\hat{\sigma}_\mu(k_2, k_1) \gamma_L \dot{k}_1 \not{n} \dot{k}_2] \langle \not{n} \gamma_L \hat{T}_\mu(k_2, k_1) \rangle + (L \leftrightarrow R) \right\} \\ & + 2 \left\{ [\hat{\sigma}_{\mu\nu}(x_2 p, y p, x_1 p) \gamma_L \not{p}] \langle \not{n} \gamma_L \hat{T}_{\mu\nu}(k_2, k, k_1) \rangle + (L \leftrightarrow R) \right\}. \end{aligned} \quad (26)$$

In this equation x_1, x_2 and y are the light-cone fractions of k_1, k_2 and k respectively [$x_1 = (k_1 \cdot n / p \cdot n)$, etc.] and

$$\begin{aligned} \hat{\sigma}^\mu(k_2, k_1) = & \hat{S}^\mu(k_2, k_1) \\ & - \frac{1}{4x_2^2} \left\{ \not{n} \dot{k}_2 \hat{S}(k_2) \not{n} \gamma^\mu + \frac{1}{2} \not{n} \dot{k}_2 m^\alpha \left. \frac{\partial \hat{S}(k_2)}{\partial k_2^\alpha} \right|_{k_2=\dot{k}_2} \dot{k}_2 \gamma^\mu \right\} \\ & - \frac{1}{4x_1^2} \left\{ \gamma^\mu \not{n} \hat{S}(k_1) \dot{k}_1 \not{n} + \frac{1}{2} \gamma^\mu \dot{k}_1 m^\alpha \left. \frac{\partial \hat{S}(k_1)}{\partial k_1^\alpha} \right|_{k_1=\dot{k}_1} \dot{k}_1 \not{n} \right\}, \end{aligned}$$

$$\begin{aligned} \hat{\sigma}^{\mu\nu}(k_2, k, k_1) &= \hat{S}^{\mu\nu}(k_2, k, k_1) + \frac{1}{4y_2} \{ \gamma^\mu \hat{S}(k) \gamma^\nu \} \\ &\quad - \frac{1}{4yx_2} \{ \gamma^\mu \hat{\sigma}(k_2, k) \gamma^\nu \} - \frac{1}{4yx_1} \{ \gamma^\mu \hat{\sigma}^\nu(k, k_1) \gamma^\nu \}. \end{aligned} \quad (27)$$

The first term in Eq. (26) now resembles the naïve parton model formula, Eq. (2). Since all the k^2 dependence is in $\hat{\Gamma}(k)$ the correspondence can be made exact by setting,

$$f(x, k_T^2) = \frac{1}{2x(2\pi)^4} \int dk^2 \langle \gamma \hat{T}(k) \rangle. \quad (28)$$

The terms contributing in a given order of Λ^2/Q^2 are obtained by appropriate expansion of $[\hat{S}(k)\hat{k}]$ in k_T about the direction $\hat{k} = x\hat{p}$. The expansion of this term to order $1/Q^2$ has already been given in Eqs. (5), (6). The isolation of the other $1/Q^2$ terms in Eq. (26) is similarly given by expansion about the collinear direction $k_i = x_i\hat{p}$. The resulting derivatives of the functions $\hat{S}, \hat{S}_\mu$, can be related in QCD to the soft insertions of extra gluons because of the Ward identities,

$$\begin{aligned} \frac{\partial \hat{S}(k)}{\partial k^\mu} &= \hat{S}_\mu(k, k), \quad \frac{1}{2} \frac{\partial^2 \hat{S}(k)}{\partial k^\mu \partial k^\nu} = \hat{S}_{\mu\nu}(k, k, k), \\ \frac{\partial \hat{S}_\mu(k_2, k_1)}{\partial k_1^\nu} &= \hat{S}_{\mu\nu}(k_2, k_1, k_1), \quad \frac{\partial \hat{S}_\nu(k_2, k_1)}{\partial k_2^\mu} = \hat{S}_{\mu\nu}(k_2, k_2, k_1). \end{aligned} \quad (29)$$

(We use QED expressions since the colour degrees of freedom are included in the Γ 's).

All formulae can now be expressed in terms of on-shell, collinear parton quark amplitudes $a_0(x), a_1(x)$ [Eq. (8)] and,

$$\begin{aligned} b_1(x_2, x_1) &= d^{\mu\nu} [\hat{S}_\mu(x_2 p, x_1 p) \gamma_\nu], \quad \tilde{b}_1(x_2, x_1) = i\epsilon^{\nu\mu} [\hat{S}_\mu(x_2 p, x_1 p) \gamma_5 \gamma_\nu], \\ b_2(x_2, x_1) &= m^\mu [\hat{S}_\mu(x_2 p, x_1 p) \not{p}], \\ c_1(x_2, y, x_1) &= d^{\mu\nu} [\hat{S}_{\mu\nu}(x_2 p, y p, x_1 p) \not{p}], \quad \tilde{c}_1(x_2, y, x_1) = i\epsilon^{\mu\nu} [\hat{S}_{\mu\nu}(x_2 p, y p, x_1 p) \gamma_5 \not{p}]. \end{aligned} \quad (30)$$

The notation used in Eq. (8) can now be seen to follow from Eqs. (29) and (30).
The result for the hadronic tensor W is,

$$\begin{aligned}
 W(q,p) = & A_0(x) \langle \mathcal{H} \hat{T}(x) \rangle \\
 & + \sigma(x,x,x) \langle \mathcal{H} k_T^2 \hat{T}(x) \rangle \\
 & + \sigma(x_2, x_2, x_1) d_{\mu\nu} \langle -\mathcal{H} k_{T2}^\mu \hat{T}^\nu(x_2, x_1) \rangle + \tilde{\sigma}(x_2, x_2, x_1) i \epsilon_{\mu\nu} \langle \mathcal{H} \delta_5 k_{T2}^\mu \hat{T}^{\nu}(x_2, x_1) \rangle \\
 & + \sigma(x_2, x_1, x_1) d_{\mu\nu} \langle -\mathcal{H} \hat{T}^\mu(x_2, x_1) k_{T1}^\nu \rangle + \tilde{\sigma}(x_2, x_1, x_1) i \epsilon_{\mu\nu} \langle \mathcal{H} \delta_5 \hat{T}^\mu(x_2, x_1) k_{T1}^\nu \rangle \\
 & + \sigma(x_2, y, x_1) d_{\mu\nu} \langle -\mathcal{H} \hat{T}^{\mu\nu}(x_2, y, x_1) \rangle + \tilde{\sigma}(x_2, y, x_1) i \epsilon_{\mu\nu} \langle \mathcal{H} \delta_5 \hat{T}^{\mu\nu}(x_2, y, x_1) \rangle \\
 & + \left[b_2(x_2, x_1) - \frac{1}{2} b_2(x_2, x_2) - \frac{1}{2} b_2(x_1, x_1) \right] p_\mu \langle \mathcal{H} \hat{T}^\mu(x_2, x_1) \rangle,
 \end{aligned}
 \tag{31}$$

where,

$$\begin{aligned}
 \sigma(x_2, y, x_1) = & \frac{1}{2} \left\{ \frac{a_1(y)}{y^2} - \frac{1}{2y} (b_1(x_2, y) + b_1(y, x_1) + \tilde{b}_1(x_2, y) - \tilde{b}_1(y, x_1) - b_2(x_2, x_2) - b_2(x_1, x_1)) \right. \\
 & \left. - c_1(x_2, y, x_1) \right\}, \\
 \sigma(x_2, y, x_1) + \tilde{\sigma}(x_2, y, x_1) = & -\frac{1}{2} \left\{ c_1(x_2, y, x_1) + \tilde{c}_1(x_2, y, x_1) \right\}.
 \end{aligned}
 \tag{32}$$

The functions in brackets in Eq. (31) are derived from the corresponding quantities in Eq. (26) after integration over all components of the four-momenta, k_1 , k_2 and k except for their light-cone fractions, x_1 , x_2 and y . For example,

$$\langle -\mathcal{H} \hat{T}^\mu(x_2, x_1) k_{T1}^\nu \rangle = \int \frac{d^4 k_2}{(2\pi)^4} \frac{d^4 k_1}{(2\pi)^4} \delta\left(x_2 - \frac{k_2}{p^+}\right) \delta\left(x_1 - \frac{k_1}{p^+}\right) \langle -\mathcal{H} \hat{T}^\mu(k_2, k_1) k_{T1}^\nu \rangle.
 \tag{33}$$

The coefficients of the terms in Eq. (31) are correlated because of colour gauge invariance. To write the result in an explicitly gauge covariant form we define,

$$A_0(x) = \langle \mathcal{H} \hat{T}(x) \rangle,
 \tag{34}$$

$$C_1(x_2, y, x_1) = \frac{1}{2} d_{\mu\nu} \left\{ \delta(x_2 - y) \langle \not{x} \not{k}_T^\mu \hat{T}(y) \not{k}_T^\nu \rangle \delta(x_1 - y) \right. \\ \left. + \langle \not{x} \hat{T}^\mu(x_2, y) \not{k}_T^\nu \rangle \delta(x_1 - y) + \delta(x_2 - y) \langle \not{x} \not{k}_T^\mu \hat{T}^\nu(y, x_1) \rangle \right. \\ \left. + \langle \not{x} \hat{T}^{\mu\nu}(x_2, y, x_1) \rangle \right\},$$

$$\tilde{C}_1(x_2, y, x_1) = \frac{1}{2i} \epsilon_{\mu\nu} \left\{ \langle \not{x} \gamma_5 \hat{T}^\mu(x_2, y) \not{k}_T^\nu \rangle \delta(x_1 - y) + \delta(x_2 - y) \langle \not{x} \gamma_5 \not{k}_T^\mu \hat{T}^\nu(y, x_1) \rangle \right. \\ \left. + \langle \not{x} \gamma_5 \hat{T}^{\mu\nu}(x_2, y, x_1) \rangle \right\},$$

(35)

$$B_2(x_2, x_1) = p_\mu \left\{ \langle \not{x} \not{k}^\mu \hat{T}(x_1) \rangle \delta(x_2 - x_1) + \langle \not{x} \hat{T}^\mu(x_2, x_1) \rangle \right\}.$$

(36)

The full expression for the structure function can now be written in a compact form,

$$W(Q, p) = A_0(x) A_0(x) + \left\{ b_2(x_2, x_1) - \frac{1}{2} b_2(x_2, x_2) - \frac{1}{2} b_2(x_1, x_1) \right\} B_2(x_2, x_1) \\ - 2 \left\{ \sigma(x_2, y, x_1) C_1(x_2, y, x_1) + \tilde{\sigma}(x_2, y, x_1) \tilde{C}_1(x_2, y, x_1) \right\}.$$

(37)

[Note that the first term in Eq. (36) does not contribute in Eq. (37)].

The parton correlation functions C_1 , $\tilde{C}_1$ and B_2 are gauge invariant. This is visible if we write them as matrix elements of products of fields on the light cone.

$$A_0(x) = \frac{1}{4} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P | T \{ \bar{\Psi}(0) \not{x} \Psi(\lambda) \} | P \rangle,$$

(38)

$$C_1(x_2, y, x_1) = \frac{1}{2} d_{\mu\nu} \frac{1}{4} \int \frac{d\omega d\eta d\lambda}{(2\pi)^3} e^{i\omega(x_2 - y)} e^{i\eta(y - x_1)} e^{i\lambda x_1} \langle P | T \{ \bar{\Psi}(0) \not{x} D^\mu(\omega) D^\nu(\eta) \Psi(\lambda) \} | P \rangle,$$

(39)

$$\tilde{C}_1(x_2, y, x_1) = \frac{1}{2i} \epsilon_{\mu\nu} \frac{1}{4} \int \frac{d\omega d\eta d\lambda}{(2\pi)^3} e^{i\omega(x_2 - y)} e^{i\eta(y - x_1)} e^{i\lambda x_1} \langle P | T \{ \bar{\Psi}(0) \not{x} \gamma_5 D^\mu(\omega) D^\nu(\eta) \Psi(\lambda) \} | P \rangle,$$

(40)

$$B_2(x_2, x_1) = \frac{1}{4} p_\mu \int \frac{d\eta d\lambda}{(\eta)^2} e^{i\eta(x_2-x_1)} e^{i\lambda x_1} \langle P | T \{ \bar{\Psi}(0) \not{D}(\eta) \Psi(\lambda) \} | P \rangle. \quad (41)$$

In the above expressions we have defined

$$\Psi(\lambda) \equiv \psi(\lambda n), \quad D_\mu(\lambda) = i\partial_\mu - gA_\mu(\lambda n). \quad (42)$$

We use a matrix notation for the colour degrees of freedom of the gluon field and the sums over colour indices are implicit. Since we calculate the incoming quark-photon amplitudes, only one time ordering should be taken from Eqs. (38)-(41). The other ordering of the fermion fields corresponds to antiquarks. To make the local gauge invariance of Eqs. (38)-(41) explicit we may formally insert line integrals of the gluon field, which, however, give no contribution when taken along the straight line on the light cone. For example,

$$\bar{\Psi}(0) \not{D} \Psi(\lambda) \equiv \bar{\Psi}(0) \not{P} \left\{ \exp \left(ig \int_0^\lambda d\eta \, n^\mu A_\mu(\eta) \right) \right\} \Psi(\lambda), \quad (43)$$

where P is the path ordering operator. Explicit lowest order calculation for the electromagnetic current scattering off a quark of unit charge gives,

$$\begin{aligned} Q_0(x) &= \delta(x-x_B) e_T \\ \sigma(x_2, y, x_1) &= \frac{4}{Q^2} \delta(y-x_B) (e_\perp + e_T) - \frac{x_B}{Q^2} \left\{ \frac{\delta(x_2-x_B) - \delta(x_1-x_B)}{x_2-x_1} \right\} e_T, \\ \tilde{\sigma}(x_2, y, x_1) &= -\frac{4}{Q^2} \delta(y-x_B) (e_\perp + e_T) - \frac{x_B}{Q^2} \left\{ \frac{\delta(x_2-x_B) - \delta(x_1-x_B)}{x_2-x_1} \right\} e_T, \\ b_2(x_2, x_1) &= 0. \end{aligned} \quad (44)$$

Our final result for the structure function then is

$$F_\perp(x_B, \frac{\Lambda^2}{Q^2}) = \frac{4\Lambda^2}{Q^2} T_1(x_B) + O\left(\frac{1}{Q^4}\right), \quad (45)$$

$$\begin{aligned} F_T(x_B, \frac{\Lambda^2}{Q^2}) &= A_0(x) + \\ &+ \frac{\Lambda^2}{Q^2} \left\{ 4T_1(x_B) - x_B \int dx_2 dx_1 \frac{\delta(x_2 - x_B) - \delta(x_1 - x_B)}{x_2 - x_1} T_2(x_2, x_1) \right\} + O\left(\frac{1}{Q^4}\right), \end{aligned} \quad (46)$$

where

$$\Lambda^2 T_1(x) = -2 \int dx_2 dx_1 \left\{ C_1(x_2, x, x_1) - \tilde{C}_1(x_2, x, x_1) \right\}, \quad (47)$$

$$\Lambda^2 T_2(x_2, x_1) = -2 \int dx \left\{ C_1(x_2, x, x_1) + \tilde{C}_1(x_2, x, x_1) \right\}. \quad (48)$$

The dimensions of C_1 and $\tilde{C}_1$ are carried by the strong interaction scale parameter Λ , so that the functions T_1 and T_2 are dimensionless.

The structure functions are defined as $F_T = \frac{1}{x} F_2^{EM}$, $F_L = \frac{1}{x} F_2^{EM} - 2F_1^{EM}$. The results for the weak V-A current scattering off a quark are

$$F_{1,2}^{V-A} = 2F_{1,2}^{EM},$$

$$F_3^{V-A} = 2F_1^{V-A}. \quad (49)$$

F_3 has the usual sign change for scattering off an antiquark.

For the longitudinal structure function F_L , Eq. (45) is the full result in the massless theory apart from logarithmic scaling violations. For the transverse part, Eq. (46), in addition to the omitted logarithmic scaling violations, there is also a contribution from the four-fermion graphs of Fig. 2. At order $1/Q^2$ these diagrams should be calculated for quarks on-shell and collinear with their parent hadron, so the parton interpretation is immediate. The results have been presented in Refs. 8), 9).

What physical meaning do the functions T_1 and T_2 have? It is instructive to rewrite Eqs. (47), (48) as time-ordered products of fields on the light cone

$$\Lambda^2 T_1(x) = \frac{1}{4} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P | T \left\{ \bar{\psi}(0) \gamma_\mu \not{x} \gamma_\nu D_T^\mu(0) D_T^\nu(\lambda) \psi(\lambda) \right\} | P \rangle, \quad (50)$$

$$\Lambda^2 T_2(x_2, x_1) = \frac{1}{4} \int \frac{d\lambda d\eta}{(2\pi)^2} e^{i\eta(x_2-x_1)} e^{i\lambda x_1} \langle P | T \{ \delta_\mu \kappa \delta_\nu D_T^\nu(\eta) D_T^\mu(\eta) \Psi(\lambda) \} | P \rangle. \quad (51)$$

D_T denotes the transverse components of the covariant derivative Eq. (42). We see that the transverse degrees of freedom alone provide the scale of the power corrections. The correspondence of T_1 and T_2 with the parton density $f(x, k_T^2)$ of the naïve model [Eq. (2)] can be established formally by taking the limit $g \rightarrow 0$ (i.e., discarding the gluon part of the covariant derivative). We obtain,

$$\Lambda^2 T_1(x) \stackrel{g \rightarrow 0}{=} \int d^2 k_T k_T^2 f(x, k_T^2) = \frac{1}{4} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P | T \{ \bar{\Psi}(0) \not{x} \Psi(\lambda) \} | P \rangle, \quad (52)$$

$$T_2(x_2, x_1) \stackrel{g \rightarrow 0}{=} \delta(x_2 - x_1) T_1(x_1). \quad (53)$$

Equations (45) and (46) reproduce the naïve parton model results, Eqs. (15), (16), in this limit. The QCD improvement of the parton model is in essence the replacement of transverse momentum by the combination of momentum and gluon field dictated by the covariant derivative. The precise statement of this simple idea is given by Eqs. (50) and (51).

The limit $g \rightarrow 0$ in Eqs. (52), (53) is given for the purposes of illustration only. As stated previously in connection with the parton model, Lorentz invariance implies that in this limit the only power law corrections are due to ξ scaling. After setting M and g to zero we find $T_1 = T_2 \equiv 0$ and all power corrections vanish. The dynamical power corrections exist only in the interacting theory. Equations (50), (51) show that they are generated by the gluon interactions in such a way that the physical (transverse) components of the gluon field simulate an intrinsic transverse momentum.

The phenomenology of the power corrections is controlled by the two functions $T_1(x)$ and $T_2(x_2, x_1)$. $T_1(x)$ is phenomenologically indistinguishable from the average intrinsic transverse momentum. It is a function of one variable and satisfies positivity constraints

$$T_1(x) = T_1^{\bar{2}}(x) \theta(x) - T_1^{\bar{2}}(-x) \theta(-x), \quad T_1^{\bar{2}}(x) > 0, T_1^{\bar{2}}(-x) > 0. \quad (54)$$

To illustrate the positivity it is useful to rewrite Eq. (50) as,

$$\Lambda^2 T_1(x) = \frac{1}{4} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle I | T \{ \bar{\Phi}(0) \not{x} \Phi(\lambda) \} | I \rangle, \quad (55)$$

where $\Phi(\lambda) = \mathbb{D}_T(\lambda)\psi(\lambda)$. Equation (54) emphasizes the similarity with the leading twist contribution Eq. (38) for which the separation into quark and antiquark distributions analogous to Eq. (54) is valid. Further support for the interpretation of $T_1(x)$ as an average transverse momentum comes from the comparison of the coefficient function of k_T^2 in the naïve model, Eq. (7), with the full QCD result, Eq. (32),

$$\sigma(x, x, x) = \sigma(x). \quad (56)$$

Both σ 's can be derived by taking the derivative over k_T^2 of the parton-photon cross-section.

The reduction of Eq. (37) to purely transverse objects $C_1, \tilde{C}_1$ appears to be an accident because $b_2(x_2, x_1)$ vanishes to lowest order [Eq. (44)]. However, one can show that the parton correlation function $B_2(x_2, x_1)$ is related to C_1 and $\tilde{C}_1$ by Lorentz and gauge invariance. A simple way of deriving these relations is to require the n independence of the final formula for $W^{\alpha\beta}$. The exact relationships will be given in Ref. 2).

These n independence relations lead to one constraint for T_1 and T_2 ,

$$4 \int dx T_1(x) = \int dx_2 dx_1 T_2(x_2, x_1). \quad (57)$$

This relationship has important consequences for the sum rules of deep inelastic scattering. Equation (57) reveals the inviolate nature of the Adler sum rule,

$$\int_0^1 dx_B \left[F_T^{\nu H}(x_B, \frac{\Lambda^2}{Q^2}) - F_T^{\bar{\nu} H}(x_B, \frac{\Lambda^2}{Q^2}) \right] = A. \quad (58)$$

A is a pure number dependent on the flavour content of the target, H . Restoring the first logarithmic corrections we may write the Gross-Llewellyn Smith sum rule as

$$\int_0^1 dx_B \left[F_3^{\nu H}(x_B, \frac{\Lambda^2}{Q^2}) + F_3^{\bar{\nu} H}(x_B, \frac{\Lambda^2}{Q^2}) \right] = L_0 \left\{ 1 - \frac{\alpha_S(Q^2)}{\pi} - \frac{\Lambda^2}{Q^2} L_1 + \dots \right\}. \quad (59)$$

L_0 is a number dependent on the quark content of the target hadron and the quantity $(L_1 L_0)$ is positive in the valence approximation as a consequence of Eq. (54)

$$L_1 L_0 = 4 \sum_{\text{flavours } f} \int dx \left\{ T_1^{2f}(x) - T_1^{\bar{2}f}(x) \right\}. \quad (60)$$

Note that the diagrams of Fig. 2 do not contribute to the first moments in Eqs. (58), (59).

The phenomenological significance of our formulae for other moments is complicated. Strictly speaking, the fit to the deep inelastic scattering data should be made with the functions T_1 , T_2 and the appropriate four-fermion correlation functions (corresponding to Fig. 2). They are determined by QCD but only as the result of a non-perturbative calculation. A more pragmatic approach which has been adopted in the past is to fit the moments of the structure functions with the simple form, suggested in Ref. 11),

$$\int dx_B x_B^{N-1} F(x_B, \frac{\Lambda^2}{Q^2}) = M_N \left(1 + \frac{aN}{Q^2} \right). \quad (61)$$

M_N is the standard leading twist result and the strength of the power corrections is determined by one parameter a .

The simple model for the functions T_1 and T_2 which reproduces the N dependence in Eq. (61) is,

$$\Lambda^2 T_1(x) = x^2 A_0(x) \quad (62)$$

$$\Lambda^2 T_2(x_2, x_1) = 4x^2 \delta(x_2 - x_1) A_0(x_1). \quad (63)$$

These formulae represent an educated guess, consistent with the information which we have about T_1 and T_2 [cf. Eq. (57)]. The underlying physical motivation for this model is that we expect the integration over the gluon momenta implicit in the definition of T_1 and T_2 to be dominated by the soft gluon region. In this approximation the x dependence of T_1 and T_2 is determined by the shape of the quark distribution function $A_0(x)$. Inserting Eqs. (62), (63) into Eqs. (45), (46), the moments of the structure functions become

$$\int dx_B x_B^{N-1} F_T(x_B, \frac{\Lambda^2}{Q^2}) = M_N \left[1 + \frac{4x^2}{Q^2} (1-N) \right], \quad (64)$$

$$\int dx_B x_B^{N-1} F_3(x_B, \frac{\Lambda^2}{Q^2}) = M_N \left[1 - \frac{4x^2}{Q^2} N \right], \quad (65)$$

$$\int dx_B x_B^{N-1} F_L(x_B, \frac{\Lambda^2}{Q^2}) = M_N \frac{4x^2}{Q^2}. \quad (66)$$

These equations [and also Eqs. (58), (59)] are written for the case of zero target mass M . To compare with the data we should use Nachtmann moments ^{3),12)} which take into account the kinematic scaling violations due to non-vanishing target mass. Our soft gluon ansatz together with the exact QCD result, gives us greater intuition about the sign and functional form of the power law corrections. The positive parameter κ^2 plays the rôle of the average transverse momentum which is generated dynamically as explained above. The factor N which enhances the $1/Q^2$ corrections to the moments follows from the assumption that the final state interactions are dominated by soft-gluon exchanges. Equations (64)-(66) do not include the four-fermion contributions. For lowest moments, the size of these contributions has been estimated in Ref. 9) and found to be small.

The power corrections discussed in this paper have also been considered in Refs. 9),13). Following Ref. 1), these authors construct a basis of operators containing only parton fields with no derivatives. We shall call this the collinear basis to contrast it with the transverse basis used in this paper. By using the Lorentz and gauge invariance of the theory these two bases can be related. The transverse momentum can be expressed by off-shellness of partons, which in turn is related to more partons via the equations of motion. In this manner we have checked that our results for the coefficient functions coincide with those of Refs. 9), 13). Full details will be given in Ref. 2).

The transverse basis of this paper and the collinear basis of the papers mentioned above are complementary descriptions of the hadron structure. The transverse basis has the advantage that the coefficient functions are simpler and that the relationship to the naïve parton model which is so rich in physical intuition is apparent. We have consolidated the concept of an intrinsic transverse momentum in QCD and explained how it is defined in the theory of interacting partons. Furthermore our graphical method opens up the exciting prospect of calculating the power corrections to all hard hadronic processes.

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- Figure 1 -

- Figure 2 -