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SIMULATION OF THE BEAM-BEAM EFFECT FOR e^+e^- STORAGE RINGS

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Summary

The effect of the beam-beam interaction ('strong-strong') in e^+e^- storage rings is simulated by tracking many superparticles per beam, for many turns, through the accelerating system, the accelerator lattice and the beam-beam interaction. The motion of each superparticle is treated in six-dimensional phase space and the effects of quantum excitation and radiation damping are included. Momentum dispersions at the RF cavities and the interaction regions allow computation of single beam and beam-beam synchro-betatron resonances. The longitudinal motion of each superparticle produces a modulation of the betatron phase advance between beam-beam collisions. This modulation also drives beam-beam synchro-betatron resonances. The effects of perturbations on the superperiodicity (errors) and chromaticity are included in the simulation. The beam-beam force is calculated for an elliptical beam with Gaussian charge density distribution in three dimensions. This report is comprised of two main parts; in the first part the simulation technique is described and in the second part a compendium of results obtained from the simulation is presented. The simulation results presented here pertain to the LEP design.

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Part A. The Beam-Beam Simulation Program

A1 Introduction

The ultimate performance limitation for all electron-positron storage rings results from the highly non-linear forces which bunches from one beam exert on particles in the other beam. An analytical treatment of this effect (called the 'beam-beam limit') does not yet exist although many attempts have been made recently at computer simulation^{1,2,3}). The total computing time required by such simulations is given by the product of the number of beam-beam kicks and the computer time required to calculate one such kick. For simulation of smaller storage rings (where the number of beam-beam kicks per transverse damping time is high) this imposed a severe limitation. Consequently such simulations tended to be over-simplified and could therefore not be expected to produce realistic results. More recently, with the advent of larger circumference storage rings and faster computers the problem has been alleviated to some extent. However in order to perform a realistic simulation of the strong-strong interaction a faster technique was required for evaluating the beam-beam kick. The technique here involves interpolation between tabulated values of the complex error function, and allows evaluation of the beam-beam strength from a single set of tables irrespective of the transverse beam dimensions.

A2 Description of Simulation Technique

A simplified flow diagram of the simulation program is shown below. Initialisation is performed as follows. The equilibrium rms bunch size at the interaction point (in the absence of beam-beam effects) in six-dimensional phase space is evaluated from the input values of the radiation integrals⁴), the damping partition numbers, the betatron amplitude functions at the interaction region (β^*) and the RF parameters. The radiation integrals include the effect of wiggler magnets, and the damping partition numbers include deviations from the central orbit. A random number generator is then used to set up the initial phase space positions of a large number of superparticles with Gaussian distributions and the required variances. The initialisation having been performed, the particles are then launched into the simulated machine as described in the simplified flow chart shown below.

Simplified Flow Chart of the Simulation Program

Each superparticle is tracked through a 'sector' which consists of an accelerating cavity, a beam-beam interaction and a normal machine arc (lattice). At the end of each sector the position of each particle is compared with aperture limits; those particles which fall outside the limits are counted and excluded from further tracking. In the transverse phase plane the aperture limits are given by the machine acceptance whereas in the longitudinal phase plane the limits are imposed by the acceptance of the RF accelerating 'bucket'.

After each turn the new beam parameters (such as the beam size, the intensity) are computed and used to recalculate the beam-beam force parameters.

The simulation is usually allowed to run for around one damping time after the steady state equilibrium has been reached. During this time binning of various quantities is performed in order to reduce statistical fluctuations.

A2.1 RF cavity traversal

The energy gain in traversing an RF accelerating cavity is

$$\Delta E_{RF} = \frac{e V(\phi, \phi_s)}{n_{RF}} \quad (1)$$

where $V(\phi, \phi_s)$ is the RF voltage per turn
 ϕ_s is the stable phase angle
and n_{RF} is the number of RF stations.

In the case of a normal RF system

$$V(\phi, \phi_s) = \hat{V} \sin(\phi + \phi_s) \quad (2)$$

However the voltage function may be changed to allow for potential well deformation by higher harmonic cavities or even due to space charge effects. Each RF station may have different values of $\hat{V}$ and ϕ_s .

In the normal mode of operation of the program the change in betatron motion due to the traversal of an RF station is given by

$$\Delta y = - D_{yRF} \Delta E_{RF} / E_s \quad (3)$$

$$\Delta y' = 0$$

where y refers to either horizontal or vertical displacement
 D_{yRF} is the momentum dispersion (horizontal or vertical) at the azimuthal position of the cavity and may be different for each RF station
and E_s is the energy of the synchronous particle.

It is clear from equation (3) that no betatron damping is incurred due to traversal of an RF cavity in the normal mode of operation of the program. In this case the damping is included in the lattice. However an option does exist whereby the damping is included in the cavity traversal. The equation for the change in y' is then

$$\Delta y' = - \frac{\Delta E_{RF}}{E_s} \cdot y' \quad (4)$$

This option has been used for the single beam study⁵⁾ of the effect of wiggler magnets which change the damping partition numbers and the dependence of these numbers on the particle energy deviation.

2.2 Beam-beam traversal

For a bunch of particles with Gaussian charge distribution in all three dimensions the potential is given by⁶⁾

$$\psi(x, z, s) = \frac{Ne}{4\pi^2 \epsilon_0} \int_0^\infty \frac{\exp \left\{ - \frac{x^2}{(2a^2+q)} - \frac{z^2}{(2b^2+q)} - \frac{s^2}{(2c^2+q)} \right\} dq}{[(2a^2+q)(2b^2+q)(2c^2+q)]^{\frac{1}{2}}} \quad (5)$$

where a, b, c are the standard deviations in x, z and s
 Ne is the total charge in the bunch

The integrated transverse angular deflection received by a particle crossing a charged beam is given by

$$\Delta\left(\frac{dy}{ds}\right) = \frac{e}{\gamma m_0 c^2} \int_{-\infty}^{\infty} E_y ds = - \frac{e}{\gamma m_0 c^2} \int_{-\infty}^{\infty} \frac{\delta\psi}{\delta y} \cdot ds \quad (6)$$

where s is the longitudinal co-ordinate
 y refers to the transverse co-ordinate (x or z)
 γ is the relativistic parameter
and E_y is the electric field due to the charged beam.

Combining equations (5) and (6) gives the integrated transverse 'kick' (due to a beam-beam crossing). For the vertical plane

$$\Delta z' = \frac{2Nr_e}{\gamma} \frac{z}{\sqrt{\pi}} \int_{-\infty}^{\infty} \int_0^{\infty} \exp \left\{ -\frac{x^2}{(2a^2+q)} - \frac{z^2}{(2b^2+q)} - \frac{s^2}{(2c^2+q)} \right\} \frac{dt ds}{(2a^2+q)^{\frac{1}{2}} (2b^2+q)^{\frac{3}{2}} (2c^2+q)^{\frac{1}{2}}} \quad (7)$$

where r_e is the classical electron radius ($e^2/4\pi\epsilon_0 m_0 c^2$)

Equation (7) and its counterpart for the horizontal kick were evaluated numerically for a range of values in x , z , s , a , b and c . These results were compared with the two-dimensional equation⁷⁾

$$\Delta z' = \frac{Nr_e}{\gamma} \sqrt{\frac{2\pi}{a^2-b^2}} R \left[w \left(\frac{x + iz}{\sqrt{2(a^2-b^2)}} \right) \exp \left\{ \frac{-x^2}{2a^2} - \frac{z^2}{2b^2} \right\} \cdot \right. \\ \left. \cdot w \left(\frac{x \frac{b}{a} + iz \frac{a}{b}}{\sqrt{2(a^2-b^2)}} \right) \right] \quad (8)$$

where $w(A+iB)$ is the complex error function.

It was found that for any reasonable storage ring parameters equation (8) was accurate to around 0.2%.

Using an accurate algorithm for the complex error function⁸⁾ it was found that the transverse kicks computed with equation (8) consumed much less computer time than those computed with equation (7). In fact the complex error function calculation is significantly faster than the two-dimensional equivalent of equation (7). However the real advantage of equation (8) becomes more apparent when one wishes to set up tables of functions and use interpolation for evaluation of the beam-beam kick. This procedure produces an enormous reduction in the computing time required. It is clear from equation (8) that, once tables of the complex error function have been set up then changes in a and b are easily accommodated by simply changing the arguments of the tables.

For these reasons the beam-beam kick is evaluated by linear interpolation (six-point) of tabulated values of the complex error function in order to evaluate equation (8). A fast interpolation subroutine has been written⁹⁾ for this purpose which has been tested to an accuracy of better than 10^{-3} .

In equations (7) and (8) the x and z are the real displacements between the position of the particle and the centre of gravity of the 'other' bunch, and are given by

$$u = y \pm y_s + D_{yi} \frac{\delta E}{E_s} \quad (9)$$

where y is the betatron displacement (horizontal or vertical)
 y_s is the separation between the centres of the colliding bunches and may be different at different interaction points
and D_{yi} is the dispersion at the interaction point and may be different at each interaction point

Separations between the beam centres may come about by electrostatic kickers (in a controlled way) or by orbit differences between the electrons and positrons¹⁰⁾ (in an uncontrolled way). For this reason y_s can consist of a given value plus a random fluctuation at each interaction point. The value of y_s may also be made time-dependent in

order to investigate the importance of the speed with which beams are brought into collision.

In addition to the change in y' the beam-beam traversal can also produce an energy loss due to synchrotron radiation. This energy loss can be related to the beam-beam kick. For the LEP machine this effect has been found to be extremely small and is not included in the 'standard' version of the program.

A2.3 Lattice Traversal

In the absence of synchrotron motion the transfer matrix between interaction regions has the usual form

$$\begin{bmatrix} \cos\mu - \frac{\beta' \sin\mu}{2} & \beta \sin\mu \\ -\frac{(1+(\beta')^2/4) \sin\mu}{\beta} & \cos\mu + \frac{\beta' \sin\mu}{2} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \quad (10)$$

where μ is the unperturbed betatron phase advance between interaction points
 β is the amplitude function
and β' is $\frac{d\beta}{ds}$

In the normal mode where the betatron damping is included in the lattice the matrix elements are multiplied by a damping factor

$$\exp \{-t_s/\tau_y\}$$

where t_s is the time between collisions
and τ_y is the transverse damping time.

The value of μ may be different for each lattice arc.

In the presence of synchrotron motion μ may be modulated at the synchrotron frequency by a non-zero chromaticity (Q')

$$Q' = \frac{\Delta Q}{\frac{\Delta E}{E_s}} \quad (11)$$

In this case it is clear that each superparticle, having a different $\frac{\Delta E}{E}$ will also have a different $\Delta\mu$.

In the presence of synchrotron motion the point of interaction between an individual particle and the centre of the other bunch is displaced longitudinally from the nominal interaction point by

$$\Delta\ell_n = \Delta\ell_{n-1} - \frac{C}{4\gamma_t^2 k_b} \frac{\Delta E}{E_s} \quad (12)$$

where n denotes the number of the collision
 C is the circumference of the accelerator
 γ_t is the transition energy of the accelerator
 k_b is the number of bunches

In this case the transfer matrix between collisions is given by

$$\begin{bmatrix} T_{11} + \Delta\ell_n T_{21} & T_{12} - \Delta\ell_{n-1} T_{11} + \Delta\ell_n \{T_{22} - \Delta\ell_{n-1} T_{21}\} \\ T_{21} & T_{22} - \Delta\ell_{n-1} T_{21} \end{bmatrix} \quad (13)$$

It should also be noted here that, due to the same longitudinal modulation, the beam size seen by the particle (and hence the strength of the beam-beam force) is also modified due to the β^* variation along the length of the interaction region. This effect was initially included and found to be insignificant. In the 'normal' mode this effect is not included.

In the longitudinal phase plane the traversal between collision points is accompanied by an energy loss and a phase change given by

$$\Delta E_{\text{rad}} = a_0 + a_1 \delta E + a_2 \delta E^2 + a_3 \delta E^3 + \dots \quad (14)$$

$$\Delta\phi = \frac{\pi h}{\gamma_{tb}^2 k_b} \frac{\delta E}{E_s} \quad (15)$$

where h is the harmonic number of the RF wave.

In the normal mode the energy loss polynomial of equation (14) contains only terms a_0 and a_1 . However for the study of the effect of higher order wiggler magnets⁵⁾ other coefficients may be evaluated and 'switched on'.

A2.3.1 Quantum excitation

The simulation of quantum excitation is performed by adding a random value to the phase plane position of each superparticle during the traversal of a lattice arc. The random number comes from a Gaussian distribution with a given variance and a mean of zero. The variances for the distribution are given by

$$\sigma^2 = 4 \sigma_{ss}^2 \frac{t_s}{\tau_y} \quad (16)$$

where τ_y is the damping time in the relative plane
and σ_{ss} is the equilibrium beam size (unperturbed) in x , z or δE as calculated during the initialisation phase.

A3 Aperture Limitations

The aperture limits are set in the program by input values of the machine acceptance in the two transverse planes and by the calculated RF bucket size in the longitudinal plane. For storage rings which do not yet exist (such as LEP) the machine acceptance is assumed to be given by the mechanical aperture, whereas in existing machines the measured dynamic aperture is of course more relevant.

After each accelerator section the phase plane location of each particle is compared with its calculated aperture limit and each particle which falls outside is eliminated from further calculations. A count of

the number of such 'lost' particles is of course performed for both beams. This information is used to reduce in proportion the actual number of particles, thereby reducing the strength of the beam-beam force.

A4 Calculation of Distribution Functions

The maximum number of superparticles which can be tracked by a simulation program is limited by the amount of computing time available. Consequently at any given time the statistical fluctuations associated with the bunch of superparticles are rather large. In particular it is very difficult to obtain reliable information concerning the form of the distribution or about the tails of the distribution. However once an apparent steady state distribution has been reached in the presence of the beam-beam forces then the statistical fluctuations may be reduced by examining the positions of the superparticles over many turns. Due to computer space limitation this is best performed by successively binning the positions into histograms after the traversal of each machine sector. As an option this technique has been applied for the computation of the superparticle distributions in betatron and synchrotron tune (Q_x , Q_z , Q_s) and in betatron and synchrotron displacement (x , z , s).

The Q distributions allow calculation of the average tune shift and the tune spread for one beam-beam crossing as well as for all beam-beam crossings, while the betatron distributions allow a more accurate evaluation of the quantum lifetime.

Part B. A Compendium of Beam-Beam Simulation Results

B1 Introduction

For beams having Gaussian charge distributions in all three dimensions the luminosity is given by

$$\mathcal{L} = \frac{N_+ N_- f_{\text{rev}} k_b}{2\pi \sqrt{(\sigma_{x+}^{*2} + \sigma_{x-}^{*2})(\sigma_{z+}^{*2} + \sigma_{z-}^{*2})}} \exp \left\{ - \frac{(\bar{x}_+ - \bar{x}_-)^2}{2(\sigma_{x+}^{*2} + \sigma_{x-}^{*2})} - \frac{(\bar{z}_+ - \bar{z}_-)^2}{2(\sigma_{z+}^{*2} + \sigma_{z-}^{*2})} \right\} \quad (17)$$

where the subscripts +, - refer to the positrons and electrons respectively.

If the intensities and transverse dimensions for electrons and positrons are equal and the beams are colliding with zero separations (x and z) then the luminosity is given by the simpler equation

$$\mathcal{L} = \frac{N^2 f_{\text{rev}} k_b}{4\pi \sigma_x^* \sigma_z^*} = \frac{k_b i_b^2}{4\pi e^2 f_{\text{rev}} \sigma_x^* \sigma_z^*} \quad (18)$$

The 'beam-beam strength parameter' (ξ) is given by

$$\xi_y = - \frac{\beta_y^*}{4\pi} \frac{\Delta y'}{y_0} \quad (19)$$

where $\Delta y'$ is the kick received by a particle (at a very small displacement y_0) due to the other beam and is given by equation (8).

Using the same assumptions as for equation (18) it can be shown that the horizontal beam-beam strength parameter ξ_x is given by

$$\xi_x = \frac{r_{e i_b}}{2\pi \gamma e f_{\text{rev}}} \frac{1}{\epsilon_x \left(1 + \frac{\sigma_z^*}{\sigma_x^*} \right)} \quad (20)$$

and the vertical ξ_z by

$$\xi_z = \frac{r_e i_b}{2\pi\gamma e f_{rev}} \frac{1}{\epsilon_x \left(1 + \frac{\sigma_z^*}{\sigma_x^*}\right)} \frac{1}{K \frac{\sigma_x^*}{\sigma_z^*}} \quad (21)$$

where K is the emittance ratio ϵ_z/ϵ_x

When $\sigma_x/\sigma_z \gg 1$, as is usually the case for large modern e^+e^- storage rings, equation (21) can be written as

$$\xi_z = \frac{r_e i_b}{2\pi\gamma e f_{rev}} \frac{\beta_z^*}{\sigma_x^* \sigma_z^*} \quad (22)$$

Combining equations (18) and (22) the luminosity may be written as

$$\mathcal{L} = \frac{\gamma}{2r_e e} \frac{\xi_z k_b i_b}{\beta_z^*} \quad (23)$$

In the design of e^+e^- storage rings ξ_x and ξ_z are usually equal. It is clear from (20) and (21) that this condition is fulfilled when

$$K = K_{opt} = \frac{\sigma_z^*}{\sigma_x^*} = \frac{\beta_z^*}{\beta_x^*} = \frac{\epsilon_z}{\epsilon_x} \quad (24)$$

B2 Parameters Relevant to Simulation

The most recent version of the LEP design¹¹⁾ has been used as a model for the simulation in order to estimate the performance of this storage ring. The machine parameters are given in Appendix 1. The aperture of the LEP machine has been taken to be the mechanical aperture minus some estimated closed orbit distortion. It was found during the course of simulating the beam-beam effect that certain 'machine errors' were rather important input parameters. The most important of these

errors turn out to be (i) variations in the phase advance per interaction region, (ii) beam separations in the interaction regions and (iii) dispersion in the interaction regions.

Estimates of these errors have been made using other computer simulation programs¹²⁾. In the beam-beam simulation the error in each interaction point is a random value with a given variance (as input) and mean of exactly zero.

B3 Simulation Results for LEP

B3.1 Optimum vertical tune range

The vertical tune value plays an important part in determining the maximum luminosity of a storage ring. Due to the numerology of resonances the integer part of the tune value can be important. For LEP with four bunches per beam and eight interaction regions the search for the optimum Q value must be performed over a range of eight integers so as to cover one integer in the Q advance per beam-beam crossing. Fig. 1 shows the calculated luminosity ($\text{cm}^{-2}\text{s}^{-1}$) per crossing point, evaluated after around two transverse damping times, plotted against the unperturbed vertical tune value Q_{z0} . All of the large and many of the small dips in luminosity can be shown to correspond to resonances of order up to around 6.

It is also evident from Fig. 1 that the luminosity behaviour repeats itself every four integers (with a few exceptions). This indicates that it is sufficient to cover a half integer in Q advance per crossing. The choice of the best Q_{z0} range seems to be between 94 to 95 and 97 to 98. However from the plot a slight preference is given to the former range.

The details of the behaviour of relevant beam parameters in this range is plotted in Fig. 2. In cases where there are two plots, each plot represents the behaviour of one of the beams (either electrons or positrons). In beam-beam collision it is evident that for lifetime reasons neither of the beams should be greatly enlarged in transverse dimensions. In fact the optimum conditions are obtained when both beams

are equally enlarged¹⁵⁾ (see later). In this Q range this condition is reasonably well fulfilled at $Q_{z0} = 94.30 + 94.35$. It is also clear from Fig. 2 that neither the horizontal beam-beam strength parameter (ξ_x) nor the horizontal beam size is dependent on the vertical tune value.

It can also be seen from these plots that the actual beam-beam strength parameters are much less than the unperturbed ones (0.06) due to the enlargement of the transverse beam dimensions.

B3.2 The 'beam-beam limit' in the optimum tune range

It has been observed in nearly all storage rings that increasing the beam intensity increases the luminosity quadratically up to a certain intensity level, further increase of the intensity causes the luminosity to rise less rapidly and finally at a high intensity the lifetime of one or other of the beams becomes disastrously short.

In the previous section it was seen that the maximum luminosity at a given beam intensity is strongly dependent on the vertical tune value. It is also reasonable to expect that the optimum tune value is dependent on the intensity level since the intensity changes the total (perturbed) tune value and the tune spread.

Fig. 3 shows the variation of luminosity with unperturbed vertical tune value for five different intensity levels. It is rather surprising to find that each curve is rather similar in character and that an optimum tune value for one intensity level is close to the optimum for other levels. From Fig. 3 a Q_{z0} of 94.30 appears reasonable for all intensity levels. Based on this observation a simulation run with fixed Q_{z0} (94.30) and variable intensity was performed. The results, shown in Fig. 4, indicate that at low intensities the luminosity increases quadratically [as indicated by equation (18)], but that as the intensity increases further the luminosity increases nearly linearly, consistent with equation (23) but with constant value for ξ_z . The saturation of ξ_z (and ξ_x) is also observed from the simulation results of Fig. 4.

One of the problems in comparing computed results with existing experience is the definition of terms. In most operating storage rings the beam-beam tune shift is evaluated from luminosity measurements and is thus a single value independent of beam. In reality the beam-beam tune shift (or strength parameter) is different for each beam as shown by the simulation results. For this reason it is useful to define a new parameter ($\bar{\xi}_z$) which relates measurement with prediction

$$\bar{\xi}_z = \frac{L}{L_{\max}} \xi_{z0}$$

where L is the actual luminosity
 $L_{\max}$ is the luminosity calculated in the absence of beam-beam 'blow-up'
 ξ_{z0} is the beam-beam strength parameter in the absence of beam-beam 'blow-up'.

Figure 5 shows a new plot of the results of Fig. 4; however in this case the independent variable is chosen to be ξ_0 which is of course linearly related to the beam intensity by equation (22). Of particular interest is the middle plot of $\bar{\xi}_z$ against ξ_0 . It is clear that at lower values of ξ_0 , $\bar{\xi}_z$ increases linearly whereas at higher values of ξ_0 there is saturation in $\bar{\xi}_z$. A maximum value for $\bar{\xi}_z$ of $\sim 0.03 \pm 0.035$ is clearly predicted. This is the beam-beam limit ($\hat{\xi}_z$) for this particular set of LEP parameters.

B3.3 Dependence of the beam-beam limit on machine parameters

Experimental evidence has shown that the beam-beam limit ($\hat{\xi}_z$) varies from storage ring to storage ring and can even change from day to day on the same storage ring¹⁵⁾. It is therefore interesting to examine the dependence of $\hat{\xi}_z$ on various accelerator parameters. It has already been seen that $\hat{\xi}_z$ is dependent on the unperturbed tune value. In the next subsections the influence of other machine parameters is examined.

B3.3.1 The number of bunches

Changing the number of bunches k_b in a given storage ring causes changes in the following parameters:

- (i) the perturbed tune value via the total beam-beam tune shift (and of course the total beam-beam tune spread)
- (ii) the betatron phase advance per crossing
- (iii) the numerology for resonance excitation
- (iv) the number of beam-beam kicks per revolution (and hence per transverse damping time)

Since the first three of these phenomena are tune-dependent it was decided to examine the dependence of $\bar{\xi}_z$ on the number of bunches over a range in Q_{z0} . From Fig. 1 it was seen that the tune dependence repeats itself over an integer range modulo the number of bunches (i.e. every half integer change in the Q advance per crossing). It was therefore decided to vary Q_{z0} over a range which covered from the half integer to the integer in Q advance per crossing. Fig. 6 shows the computed results of luminosity and $\bar{\xi}_z$ against Q_{z0} for 1, 2 and 4 bunches: a strong Q dependence is evident. It is also clear that the maximum value of $\bar{\xi}_z$ decreases as the number of bunches increases. These results are summarized in Table 1 below.

B3.3.2 The vertical betatron function β_z^*

It appears from equation (23) that, for a fixed value for the beam-beam limit and a fixed value for the maximum intensity, the luminosity may be increased by decreasing β_z^* . In this section the variation of $\bar{\xi}_z$ with β_z^* is examined for a constant value of 0.06 for ξ_0 . The longitudinal displacement of the collision point between a particle of one beam and the centre of gravity of the other beam [denoted by Δl_n in equation (12)] is directly related to the bunch length, i.e. if the bunch length is increased due to collective effects the Δl_n will also be increased. The modulation of the Q advance per crossing is given by (for constant β_z^* along the interaction region)

$$\Delta Q_{\text{mod}} \approx \frac{(\Delta l_n - \Delta l_{n-1})}{2\pi \beta_z^*}$$

Table 1. $\bar{\xi}$ as a function of k_b

		Optimum values of specified parameters		
		best	2nd best	3rd best
$k_b = 1$	Q_{zo}	93.062+93.125	93.662-93.687	-
	$\mathcal{L}(10^{31})$	0.90	0.80	-
	$\bar{\xi}_z$	0.045	0.040	-
$k_b = 2$	Q_{zo}	95.05	94.150+94.175	94.35+94.40
	$\mathcal{L}(10^{31})$	1.6	1.45	1.35
	$\bar{\xi}_z$	0.040	0.036	0.033
$k_b = 4$	Q_{zo}	94.30+94.35	94.05	93.50+93.55
	$\mathcal{L}(10^{31})$	2.45	2.4	2.4
	$\bar{\xi}_z$	0.031	0.030	0.030

It is therefore apparent that the Q modulation is increased with increasing bunch length and decreasing β^* .

The bunch lengthening factor due to turbulence in LEP has been estimated at around a factor of four¹³⁾. Figures 7 and 8 show the computed variation of $\bar{\xi}_z$ and luminosity with β_z^* for bunch lengthening factors of 1 through to 5. For these results β_x^* was also varied so as to maintain optimum coupling and hence constant ξ_0 (0.06).

Several observations may be made from Figs. 7 and 8.

- (i) For large values of β_z^* the $\bar{\xi}_z$ is maximum (~ 0.04) and independent of bunch length over the range of values examined.

- (ii) Low values of β_z^* produce an increase in luminosity only when the bunch length is short compared with the value of β_z^* .
- (iii) At very low values of β_z^* the lifetime becomes dramatically low (indicated by $\bar{\xi}_z = \mathcal{L} = 0$). For the simulation this simply means that some of the superparticles were 'lost' against the aperture limitations. In reality this would indicate an extremely short quantum lifetime of several damping times. However it should be emphasised that in cases where the luminosity and $\bar{\xi}_z$ are plotted (at low values of β_z^*) it does not necessarily imply that the quantum lifetime is adequate but rather that the quantum lifetime is greater than a few transverse damping times. It will be seen later that the quantum lifetime may be estimated by examining the transverse charge distributions of the beams in collision.
- (iv) A lower limit on the minimum value of β_z^*/σ_s appears to be in the range 1.5 to 2. If β_z^*/σ_s is decreased below this limit the quantum lifetime appears to decrease rapidly.
- (v) If the previously stated limit for β_z^*/σ_s is not violated an increase in luminosity always occurs when β_z^* is decreased even though the $\bar{\xi}_z$ value tends to decrease with decreasing β_z^* .

B3.3.3 The transverse damping time

In electron storage rings the transverse damping coefficient ($\alpha = 1/\tau$) is a strong function of the beam energy; e.g. in the presence of wiggler magnets α varies as E^3 . For LEP Phase I with the wiggler magnets powered the vertical damping time of 0.04 s at 51.5 GeV is increased to 0.32 s at 22 GeV. Experimental evidence indicates that the beam-beam limit is a function of the transverse damping time¹⁵⁾. In this section the effect of variation of the transverse damping is simulated for LEP at 22 GeV. For these runs the quantum excitation was varied as well as the damping rate, so as to maintain the beam size constant in all dimensions.

Figure 9 shows the simulation results for the nominal LEP design at 22 GeV. It is clear that the luminosity and the beam-beam limit are greatly reduced by increasing the damping time. In fact for the results shown superparticles were lost at all damping times greater than 0.12 s. The design value of $\tau_z = 0.32$ s proved to be disastrous in that nearly all of the particles had been lost after two damping times. It is clear from these results that physics data taking at 22 GeV would require a damping time much shorter than the design value or a substantial reduction in ξ_0 . The behaviour of the plot of vertical blow-up is also interesting in that it shows that one of the beams becomes badly blown up while the other suffers no blow-up whatever. This situation (referred to as the 'flip-flop' later) occurs when the beam-beam limit is grossly exceeded.

Intuitively it would appear more appropriate to examine a parameter which relates the amount of damping between beam-beam collisions rather than the amount of damping per machine revolution. In this case a reasonable parameter for comparison with different machines would be related to the product $k_b \tau_z f_{\text{rev}}$. Figures 10 and 11 show the variation of relevant beam parameters as a function of damping time but for one bunch and two bunches per beam respectively. For each case the optimum Q_{z0} values found previously were used for the results. Figure 10 shows clearly that with one bunch per beam the reduction in the beam-beam limit with increasing τ_z is much less dramatic. However one bunch per beam is unacceptable for the LEP machine since for Phase 1 at least four experimental areas are foreseen thereby requiring at least two bunches per beam. Figure 11 shows that with two bunches the design value for τ_z of 0.32 s is still unacceptable but that the slope in $\bar{\xi}_z$ is less steep than with four bunches.

From these results it appears that operation of LEP at low energy at maximum luminosity requires:

- (i) a reduction of at least around a factor of two in the transverse damping time,
- and (ii) either to reduce the number of bunches to two or in case of operation with four bunches, to separate the beams in the interaction areas where there are no experiments. This second situation has been simulated³⁾ and produced a ξ_z almost identical to that of Fig. 11.

B3.3.4 The residual chromaticity (Q')

For large e^+e^- storage rings correction of the natural chromaticity (defined here as $Q' = E \, dQ/dE$) is a major problem in itself. In this section the effect on the beam-beam limit of the residual chromaticity after correction is examined. Figures 12 and 13 show the dependence of relevant machine parameters on the horizontal and vertical chromaticity. It is apparent that even up to the relatively large value of $Q' = 20$ the beam-beam limit (and of course the luminosity) remains constant. This is an indication that the Q modulation due to the chromaticity and the energy oscillations are not sufficiently large to provoke additional blow-up of the transverse beam dimensions.

B3.3.5 Machine 'errors'

As described previously (section B2) the importance of machine imperfections on the beam-beam limit became apparent during the early stages of the simulation. For all results presented up to this point the machine imperfections ('errors') have been held constant at the estimated values (given in Appendix 2). In this section an attempt is made to highlight the importance of each machine error in relation to beam-beam effects. To this end, all errors are initially set to zero and subsequently the value of one error is increased in order to examine its effect on the beam-beam limit. Figures 14 to 19 show the computed results for variation (i) in the tune error per interaction region, (ii) spurious beam separations in the interaction regions, and (iii) momentum dispersion in the interaction regions. Figure 20 shows a summary of these results by simply plotting $\bar{\xi}_Z$ as a function of each error. It is clear that each of these parameters has an important influence on the beam-beam limit. In particular a relatively small increase from zero of the horizontal phase error per crossing or the horizontal dispersion produces a significant reduction in $\bar{\xi}_Z$.

B4 Operating at the Beam-Beam Limit

It is clear from equation (23) that for a given maximum beam intensity (limited for example by beam instabilities) the maximum luminosity is only obtained at the maximum value of the beam-beam

strength parameter ξ_z , i.e. at the beam-beam limit. It has already been shown that the beam-beam limit is reached at a value of around 0.06 for the unperturbed beam-beam strength parameter [given by equation (21)]. Hence in any given storage ring, in order to achieve the maximum luminosity, the maximum current should be determined and then the transverse beam dimensions adjusted so as to ensure a ξ_{z0} of around 0.06. In LEP this possibility is foreseen by using wiggler magnets in order to modify the horizontal emittance.

The simulation program was modified so as to make the horizontal emittance linearly dependent on the beam intensity (the constant of proportionality was chosen so as to ensure $\xi_0 = 0.06$). The coupling factor K was maintained at its 'optimum' value so that the beam-beam unperturbed strength parameter was identical in the horizontal and vertical planes. Figure 21 shows the results obtained. It is clear that the luminosity is in fact a fairly linear function of the intensity [as predicted by equation (23)] and that the beam-beam limit is constant at around 0.032 over a wide range in beam intensity. However for the lowest intensity the beam-beam limit falls to a value of around 0.02. This is due to the fact that at low intensity the transverse beam dimensions become very small in order to maintain ξ_0 constant [see equation (23)]. Consequently machine imperfections such as beam separations and momentum dispersion become relatively more important and thereby reduce the beam-beam limit as seen in section B3.5.

Figure 21 gives a clear indication that beam size wigglers will be useful for LEP, particularly in the case where the maximum intensity is less than that conceived in the design.

B5 Asymmetric Beam-Beam Strengths

In all simulation results presented thus far the unperturbed beam-beam strength of each beam was absolutely equal. However in a real storage ring with many bunches per beam it is often unavoidable that the beams may have different total intensities or at least different intensities per bunch. In this section an investigation is made of the effect of asymmetric beam strength for each beam. Two situations are chosen:

- (i) one beam with the LEP design intensity (6.6 mA) held constant and the intensity of the other beam varied over a range,
- (ii) one beam with vanishing constant intensity (weak beam) and the intensity of the strong beam varied over a range.

B5.1 Constant high-intensity beam against variable intensity

In this case the intensity of beam B was fixed at 6.6 mA ($\xi_0 = 0.06$) and the intensity of beam A was varied from 2.0 to 9.0 mA. The computed steady state values for the beam-beam strength parameter ξ_z and the relative vertical blow-up (σ_z/σ_{z0}) are shown as a function of the intensity of beam A in Fig. 22. When beam A has low intensity, beam B causes its vertical dimensions to increase, thereby reducing even further its beam-beam strength. It is interesting to note that at $i_{0A} \approx 2.0$ mA the beam-beam strength parameter is reduced from its unperturbed value of ≈ 0.02 to a perturbed value of 0.003! Of course when the beam-beam strength of beam A is so low there is practically no effect on beam B dimensions which in this case maintains a ξ_z nearly equal to its unperturbed value. However as the strength of beam A is increased (by increasing the intensity) its vertical blow-up factor becomes gradually less until at one point both beam sizes (and strengths) are approximately equal. In the absence of random machine errors this should occur at equal intensities. As the intensity of beam A is increased still further then beam A becomes dominant and causes more and more blow-up of beam B. It is also clear from Fig. 22 that small variations of the beam-beam strength around the balance point can cause the two-beam system to flip into a weak-strong situation. Such a flip-flop phenomenon has been observed experimentally¹⁴).

It is also interesting to examine the transverse charge distributions which exist in this 'flip-flop' situation. Figures 23 and 24 show the vertical charge distributions (on a logarithmic scale) and Fig. 25 the horizontal. These histograms are obtained as follows. The simulation is allowed to run until a steady state is reached. On each revolution after this steady state has been reached the transverse positions of each particle are binned in a histogram. The simulation is then run in this mode for several thousand turns in order to improve the statistical errors in the histograms and to allow inspection of the tails of the distribution.

In Figs. 23, 24 and 25 the charge distributions of the constant intensity beam are shown on the right and those of the variable intensity beam are shown on the left. From these plots it is evident that in the weak-strong case (e.g. 2.0 mA vs. 6.6 mA) the distribution of the weak beam becomes non-Gaussian in the tails. The long tails are produced by the beam-beam strength of the other beam and these tails of course reduce drastically the quantum lifetime of the weak beam. As the intensity of the weak beam is increased more and more of the beam takes up a Gaussian distribution and the 'tails' become less dense.

Figure 24 shows plots similar to Fig. 23 but in this case the intensity of beam A is increased in much smaller increments. It is clear that a reasonable balance is reached with 6.7 mA (A) against 6.6 mA (B) [(e) and (f)]. It is surprising to find that a small reduction of 0.1 mA from this 'balance' situation produces a large growth in the density of the tails for beam A [Fig. 24(c)]. These results highlight the importance of suitable diagnostics for ensuring that the strength parameters of each bunch of each beam are maintained equal.

Figure 25 illustrates that although the horizontal beam dimensions are increased the tails of the distribution fall faster than Gaussian.

B5.2 Constant low-intensity beam against variable intensity

In this case the intensity of beam A was fixed at 0.1 μ A and that of beam B was varied from 1.1 mA to 11.0 mA (ξ_0 varied from 0.01 to 0.10). Figure 26 shows the transverse blow-up factors plotted as a function of the intensity of beam B. In the vertical plane the beam maintains its natural size until the intensity is increased beyond around 3.3 mA ($\xi_0 > 0.03$). Above these values the beam size increases rather rapidly with increasing the intensity of beam B. In the horizontal plane the blow-up is significantly less.

Figures 27 and 28 show the corresponding vertical and horizontal charge distributions. In the vertical plane the tails of the distribution become large when the ξ_0 increases beyond around 0.05 ($i_0 > 5.5$ mA). In the horizontal plane the tails do become enlarged but to a much lesser extent than in the vertical plane.

Figure 29 shows the beam-beam tune shift distribution for this case. Plots (a) and (b) show a rather regular distribution with a total tune spread corresponding to approximately the product of the number of interaction regions and the beam-beam strength parameter. However as the intensity of the strong beam is increased the distribution becomes less regular. From plots (d), (e) and (f) it appears that it is not possible to have a high distribution density at values of ΔQ_{bb} greater than 0.15. Since the value of the unperturbed vertical tune is in this case 94.35 this corresponds to a perturbed tune value of

$$Q_z = 94.50$$

The strange behaviour of the distribution may then be explained qualitatively as follows. Particles at small amplitudes (large beam-beam tune shifts) are diffused towards larger amplitudes (small beam-beam tune shifts) by the beam-beam excitation and quantum excitation. However when these particles reach the half integer resonance ($Q_z = 94.50$) they are diffused even more rapidly towards larger amplitudes. Whereas particles with larger amplitudes which tend to diffuse towards smaller amplitudes (helped by radiation damping) soon reach the half integer resonance which wins over the damping and drives the particles back to larger amplitudes. This mechanism of course causes a bias in the distribution. It is important to realise that if this hypothesis is correct it is evident that the unperturbed tune value and the total beam-beam tune spread are important factors in determining the beam-beam limit. It has already been seen that the beam-beam limit is in fact Q_{z0} -dependent. The evidence from variation of the number of bunches indicates also that lower total beam-beam tune spreads produce higher beam-beam limit values.

B6 Conclusion

Computer simulation of the beam-beam effect predicts a value for the beam-beam limit which is in agreement with that measured on existing storage rings. For the case of the LEP storage ring the simulation predicts a beam-beam limit of around 0.035. The dependence of the beam-beam limit on many machine parameters has been investigated. The beam-beam limit decreases slightly with the number of bunches although the

luminosity continues to increase up to the design value of four bunches. The beam-beam limit decreases rapidly when the value of β^* approaches around twice the bunch length (σ_s). This will ultimately limit the minimum value of β^* for micro-beta proposals.

The beam-beam limit has also been shown to be strongly dependent on the transverse damping or more appropriately the relative change (due to damping) of the betatron amplitude between beam-beam kicks which is proportional to the product of the number of bunches, the revolution frequency and the transverse damping time. For operation of LEP at low energies a lower beam-beam limit is predicted.

The transverse tune modulation caused by the residual chromaticity has been shown to produce no significant reduction in the beam-beam limit for the LEP machine up to rather high values of chromaticity. On the other hand it has also been shown that machine 'errors' can produce a significant reduction in the beam-beam limit. In particular great care should be taken to minimise the errors in the horizontal tune per crossing and in the horizontal dispersion.

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Appendix 1. Relevant Parameters for LEP Version 11

Beam energy**	(E_S)	51.5	GeV
Machine circumference	(C)	26658.879	m
Bending radius	(ρ)	3.104	km
Number of intersections**		8	
Number of bunches per beam**	(k_b)	4	
Horizontal betatron wave number	(Q_{x0})	91.35	
Vertical betatron wave number**	(Q_{z0})	94.30	
Momentum compaction factor	(α)	1.928×10^{-4}	
Harmonic number	(h)	31320.0	
Peak RF voltage/turn**	($\hat{V}_{RF}$)	357.6	MV
Unperturbed synchrotron tune	(Q_{S0})	0.07	
Vertical betatron amplitude function at interaction	(β_z^*)	0.10	m
Horizontal betatron amplitude function at interaction	(β_x^*)	1.60	m
Horizontal damping partition number**	(J_x)	0.5	
Longitudinal damping partition number**	(J_E)	2.5	
Unperturbed beam-beam strength parameter**	(ξ_0)	0.06	
Beam current per bunch**	(i_b)	1.65	mA
Unperturbed horizontal beam size**	(σ_{x0}^*)	316.0	μm
Unperturbed vertical beam size**	(σ_{z0}^*)	20.0	μm
Natural rms energy spread	(σ_E/E)	0.00104	
Natural rms bunch length	(σ_{S0})	1.2	cm
Bunch lengthening factor	(σ_S/σ_{S0})	4.0	

** Unless otherwise specified.

Appendix II. Machine Errors

Type of Error	Mean square value of error	
	Horizontal	Vertical
Phase advance per interaction / 2π	0.0125	0.0125
Dispersion at interaction (m)	0.05	0.005
Beam separations at interaction (μm)	1.8	1.6

Figure Captions

- Fig. 1. Luminosity ($\text{cm}^{-2}\text{s}^{-1}$) as a function of unperturbed vertical tune value in search for an optimum tune range.
- Fig. 2. Relevant beam parameters as a function of Q_{z0} in the range 94 to 95. (ξ_0 constant at 0.06)
- Fig. 3. Luminosity ($\text{cm}^{-2}\text{s}^{-1}$) as a function of Q_{z0} for different beam intensities.
- Fig. 4. Relevant beam parameters as a function of beam intensity in amperes (constant Q_{z0} of 94.30)
- Fig. 5. Relevant beam parameters as a function of unperturbed beam-beam strength parameter (ξ_0). ξ_0 was varied by varying the beam intensity.
- Fig. 6. Luminosity as a function of Q_{z0} for different numbers of bunches. the Q range in each case corresponds to a phase advance per interaction region of half integer to integer.
- Fig. 7. Mean beam-beam strength parameter as a function of β_z^* for various bunch lengths.
- Fig. 8. Luminosity as a function of β_z^* for various bunch lengths.
- Fig. 9. Relevant beam parameters as a function of the transverse betatron damping time (τ_z) in seconds. The damping times (horizontal and vertical) are changed by the radiation integrals in such a way as to maintain the beam size constant in all phase planes.
- Fig. 10. Relevant beam parameters as a function of the transverse betatron damping time ($k_b = 1$).
- Fig. 11. Relevant beam parameters as a function of the transverse betatron damping time ($k_b = 2$).
- Fig. 12. Relevant beam parameters as a function of the horizontal chromaticity.
- Fig. 13. Relevant beam parameters as a function of the vertical chromaticity.
- Fig. 14. Relevant beam parameters as a function of the rms value of the horizontal phase advance 'error' per interaction region.
- Fig. 15. Relevant beam parameters as a function of the rms value of the vertical phase advance 'error' per interaction region.
- Fig. 16. Relevant beam parameters as a function of the rms value of the horizontal separation 'error' of the beams at the interaction region.
- Fig. 17. Relevant beam parameters as a function of the rms value of the vertical separation 'error' of the beams at the interaction region.

- Fig. 18. Relevant beam parameters as a function of the rms value of the horizontal momentum dispersion 'error' at the interaction region.
- Fig. 19. Relevant beam parameters as a function of the rms value of the vertical momentum dispersion 'error' at the interaction region.
- Fig. 20. Summary of the effects of the various types of machine 'errors'.
- Fig. 21. Relevant beam parameters as a function of the beam intensity. In this case wiggler magnets are used to make the beam cross-section dependent on the intensity so as to maintain ξ_0 constant at 0.06.
- Fig. 22. Asymmetric beam-beam strengths. Intensity of beam B constant at 6.6 mA and intensity of beam A varied. ($Q_{z0} = 94.35$, $J_x = 1.0$, $J_E = 2.0$).
- Fig. 23. Vertical charge density distributions for the case of asymmetric beam-beam strengths. Intensity of beam B constant at 6.6 mA and intensity of beam A varied. ($Q_{z0} = 94.35$, $J_x = 1.0$, $J_E = 2.0$).
- Fig. 24. Vertical charge density distributions for the case of asymmetric beam-beam strengths. Intensity of beam B constant at 6.6 mA and intensity of beam A varied. ($Q_{z0} = 94.35$, $J_x = 1.0$, $J_E = 2.0$).
- Fig. 25. Horizontal charge density distributions for the case of asymmetric beam-beam strengths. Intensity of beam B constant at 6.6 mA and intensity of beam A varied. ($Q_{z0} = 94.35$, $J_x = 1.0$, $J_E = 2.0$).
- Fig. 26. Transverse blow-up for the case of asymmetric beam-beam strengths. Intensity of beam A constant at vanishing intensity and intensity of beam B varied. ($Q_{z0} = 94.35$, $J_x = 1.0$, $J_E = 2.0$).
- Fig. 27. Vertical charge density distributions for asymmetric beam-beam strengths, Intensity of beam A constant at vanishing intensity and intensity of beam B varied. ($Q_{z0} = 94.35$, $J_x = 1.0$, $J_E = 2.0$).
- Fig. 28. Horizontal charge density distributions for asymmetric beam-beam strengths, Intensity of beam A constant at vanishing intensity and intensity of beam B varied. ($Q_{z0} = 94.35$, $J_x = 1.0$, $J_E = 2.0$).
- Fig. 29. Vertical tune distributions for asymmetric beam-beam strengths, Intensity of beam A constant at vanishing intensity and intensity of beam B varied. ($Q_{z0} = 94.35$, $J_x = 1.0$, $J_E = 2.0$).

Fig. 1. Luminosity ($\text{cm}^{-2}\text{s}^{-1}$) as a function of unperturbed vertical tune value in search for an optimum tune range.

Fig. 2. Relevant beam parameters as a function of Q_{z0} in the range 94 to 95. (ξ_0 constant at 0.06)

Fig. 3. Luminosity ($\text{cm}^{-2} \text{ s}^{-1}$) as a function of Q_{z0} for different beam intensities.

Fig. 4. Relevant beam parameters as a function of beam intensity (constant Q_{z0} of 94.30)

Fig. 5

Relevant beam parameters as a function of unperturbed beam-beam strength parameter (ξ_0). ξ_0 was varied by varying the beam intensity.

$$Q_{z0}=91.35 \quad E=51.5\text{GeV}$$

$$\xi_{z0} = \xi_z = .06$$

Fig. 6. Luminosity as a function of Q_{z0} for different numbers of bunches. the Q range in each case corresponds to a phase advance per interaction region of half integer to integer.

Fig. 8. Luminosity as a function of β_z^* for various bunch lengths.

LEP VERSION 11 $E=22.0$ $K_y=4$ $\beta_z=.10m$
 $(\xi_0=.06 \quad Q_{z0} = 94.30)$

Fig. 9. Relevant beam parameters as a function of the transverse betatron damping time (τ_z) in seconds. The damping times (horizontal and vertical) are changed by the radiation integrals in such a way as to maintain the beam size constant in all phase planes.

LEP VERSION 11 E=22.0 $K_b=1$ $\beta_z=.10m$
 $(\xi_0=.06 \quad Q_{z0}=93.06)$

Fig. 10. Relevant beam parameters as a function of the transverse betatron damping time ($k_b = 1$).

LEP VERSION 11 E=22.0 $K_b=2$ $\beta_z=.10m$
 $(\xi_0=.06 \quad Q_{z0} = 94.15)$

Fig. 11. Relevant beam parameters as a function of the transverse betatron damping time ($k_b = 2$).

LEP VERSION 11 E=51.5 $K_y=4$ $\beta_z=.10m$
 $(\xi_0=.06)$

LEP VERSION 11 $E=51.5$ $K_s=4$ $\beta_z=.10m$
 $(\xi_0=.06)$

Fig. 13. Relevant beam parameters as a function of the vertical chromaticity.

Fig. 14. Relevant beam parameters as a function of the rms value of the horizontal phase advance 'error' per interaction region.

Fig. 15. Relevant beam parameters as a function of the rms value of the vertical phase advance 'error' per interaction region.

Fig. 16. Relevant beam parameters as a function of the rms value of the horizontal separation 'error' of the beams at the interaction region.

Fig. 17. Relevant beam parameters as a function of the rms value of the vertical separation 'error' of the beams at the interaction region.

Fig. 18. Relevant beam parameters as a function of the rms value the horizontal momentum dispersion 'error' at the interaction region.

LEP VERSION 11 E=51.5 K_z=4

Fig. 20. Summary of the effects of the various types of machine 'errors'.

LEP VERSION 11 $E=51.5$ $K_p=4$ $\beta_z=.10m$
 ($\xi_0=.06$ kept constant by varying E_x by wigglers)

Fig. 21. Relevant beam parameters as a function of the beam intensity. In this case wiggler magnets are used to make the beam cross-section dependent on the intensity so as to maintain ξ_0 constant at 0.06.

Fig. 22. Asymmetric beam-beam strengths. Intensity of beam B constant at 6.6 mA and intensity of beam A varied. ($Q_{z0} = 94.35$, $J_x = 1.0$, $J_E = 2.0$).

Fig. 23. Vertical charge density distributions for the case of asymmetric beam-beam strengths. Intensity of beam B constant at 6.6 mA and intensity of beam A varied. ($Q_{\gamma 0} = 94.35$, $J_v = 1.0$, $J_r = 2.0$).

Fig. 24. Vertical charge density distributions for the case of asymmetric beam-beam strengths. Intensity of beam B constant at 6.6 mA and intensity of beam A varied.

($Q_{z0} = 94.35$, $J_x = 1.0$, $J_E = 2.0$).

Fig. 25. Horizontal charge density distributions for the case of asymmetric beam-beam strengths. Intensity of beam B constant at 6.6 mA and intensity of beam A varied.

($Q_{z0} = 94.35$, $J_y = 1.0$, $J_z = 2.0$).

Fig. 26. Transverse blow-up for the case of asymmetric beam-beam strengths. Intensity of beam A constant at vanishing intensity and intensity of beam B varied.

($Q_{z0} = 94.35$, $J_x = 1.0$, $J_E = 2.0$).

Fig. 27. Vertical charge density distributions for asymmetric beam-beam strengths, Intensity of beam A constant at vanishing intensity and intensity of beam B varied.

($Q_{z0} = 94.35$, $J_x = 1.0$, $J_E = 2.0$).

Fig. 28. Horizontal charge density distributions for asymmetric beam-beam strengths, Intensity of beam A constant at vanishing intensity and intensity of beam B varied.

$$(Q_{zn} = 94.35, J_x = 1.0, J_E = 2.0).$$

Fig. 29. Vertical tune distributions for asymmetric beam-beam strengths, Intensity of beam A constant at vanishing intensity and intensity of beam B varied.

$$(Q_{z0} = 94.35, J_x = 1.0, J_E = 2.0).$$