UNCERTAINTIES INDUCED BY MULTIPLE SCATTERING IN UPSTREAM DETECTORS OF THE DIRAC SETUP
(Part I: Vertex Position Uncertainty)

M. Pentia, S. Constantinescu
National Institute for Physics and Nuclear Engineering, Bucharest, P.O.Box MG-6, RO-76900, ROMANIA.

Abstract

A statistical treatment of the Multiple Scattering (MS) in upstream detector tracking system of the DIRAC setup is done. The analytical dependence of the vertex position uncertainty \( \sigma_{xy}^{(vertex)} \) on the coordinate error matrix, including MS correlated uncertainties and individual detector resolution, has been used to study some possible configurations of the upstream detector tracking system. It is discussed a background rejection method using track reconstructed vertex point position uncertainty.
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1 Introduction

In the present work we are studying the particle transport within upstream detector tracking system of the DIRAC setup (Figure 1). In this experiment the errors induced on the particle track due to MS in the upstream detector elements need to be carefully studied, especially if we try to find the track origin, the double track separation, the close opening angle and the relative momentum.

In this first paper we studied the errors due to MS on the reconstructed vertex position and have estimated the target region extension where the tracks can come from. To separate the good and background tracks coming from the target, it is necessary to select a detector configuration which permits to minimize the extension of this region, in other words to minimize the uncertainty in the reconstructed vertex point position.

Using a linear track reconstruction procedure, with a nondiagonal error matrix, we expressed the track parameters and their errors. In such a way it was possible to study the propagation of coordinate errors to the vertex position uncertainty. The calculations have been done for the present day coordinate detector system configuration (see Figure 1) and also for some combinations of SciFi, MSGC and Si detectors.

The propagation of coordinate errors leads, for example, to a vertex position uncertainty which could be minimized by a proper tracking configuration.

Figure 1:
2 The particle track position errors due to multiple scattering

When a charged particle is crossing the detector elements of a tracking system, it is subject to small deviations of the track due to MS. The effect is usually described by the theory of Molière (see for example Ref. [1]), which shows that, by crossing the detector material, thickness $s$, the particle is subject to successive small-angle deflections, symmetrically distributed around the incident direction. Applying the central limit theorem of statistics to a large number of independent scattering events, the Molière distribution of the scattering angle can be approximated by a Gaussian one [2].

\[ \frac{1}{\sqrt{2\pi \sigma_{x_i}^2}} \exp \left[ -\frac{x_i^2}{2\sigma_{x_i}^2} \right] dx_i \quad (3) \]

with the mean square deviation (distribution width) equals to the squares sum of the $(i-1)$ preceding distribution widths projected onto the $i$-th detector plane

\[ \sigma_{x_i}^2 = \sum_{k=1}^{i-1} \theta_{0k}^2 (z_i - z_k)^2 \quad (4) \]

$\sigma_{x_i}$ assigns the track position error measured on the $i$-th detector plane. The angular distribution is translated to a coordinate distribution by particle fly onto each detector plane (see Figure 2), and the more the intersected planes are the larger the distribution width (see Figure 3). The coordinate distribution is defined by statistical spread due to MS, and depends on the number and position of the intersected detector elements (see Table 1). It has the same form as angular distribution

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$\sigma_{x_i}$ assigns the track position error measured on the $i$-th detector plane. In Figure 2 the scatter-plot and in Figure 3 the plan projected distribution of the $x$-coordinate points on the first four detector planes (MSGC) in the DIRAC setup are presented for 2000 MeV/c pions.

MS produces errors correlated between one layer and the following ones. Obviously a scattering in layer 1 produces correlated position errors in layers 3, 4 and so on. The proper error matrix is non-diagonal [5, 6]. The elements of this matrix are [6]

\[ V_{ij} = \sum_{k=1}^{i-1} \theta_{0k}^2 (z_i - z_k)(z_j - z_k) \quad (5) \]

The uncorrelated position errors (detector resolution) $\sigma_{\text{det}}^2$ have to be added in squares in the diagonal terms of the error matrix $V$. 

\[ \sigma_{x_i}^2 = \sigma_{\text{det}}^2 + \sigma_{x_i}^2 \]

where $\sigma_{\text{det}}^2$ is the detector resolution. 

\[ \theta_0 = \frac{13.6 MeV}{p\beta c} z_c \sqrt{s} \frac{1 + 0.038 \ln \left( \frac{s}{X_L} \right)}{X_L} \quad (1) \]

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$$
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$$

where $p, \beta c$ and $z_c$ are the momentum, velocity and charge number of the incident particle, and $X_L$ is the radiation length of the scattering medium. That is, the plan projected angle $\theta_{\text{plane}, x}$ or $\theta_{\text{plane}, y}$ of the deflection angle $\theta$, onto the $xOz$ and $yOz$ planes, show an approximately Gaussian angular distribution.

$$
\frac{1}{\sqrt{2\pi \theta_0^2}} \exp \left[ -\frac{\theta_{\text{plane}}^2}{2\theta_0^2} \right] d\theta_{\text{plane}} \quad (2)
$$

Deflections into $\theta_{\text{plane}, x}$ and $\theta_{\text{plane}, y}$ are independent, identically distributed, and

$$
\theta_{\text{space}}^2 = \theta_{\text{plane}, x}^2 + \theta_{\text{plane}, y}^2.
$$
The possibility to do an independent description of the MS data on $x$ and $y$ axis, allow a separate fitting by a linear relation

$$x = x_0 + \alpha_x z$$
$$y = y_0 + \alpha_y z$$

(6)

With the coordinate and error data $(x_i \pm \sigma_{x_i})$, $(y_i \pm \sigma_{y_i})$, $z_i$, together with the corresponding error matrix $V_{ij}$ as input data, it is possible to use the least squares procedure. For $x$-data set, the $\chi^2$ in the matrix form is  

$$\chi^2 = (X - HA_x)^T V^{-1} (X - HA_x)$$

(7)

where

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}; H = \begin{pmatrix} 1 & z_1 \\ 1 & z_2 \\ \vdots & \vdots \\ 1 & z_n \end{pmatrix}; A_x = \begin{pmatrix} x_0 \\ \alpha_x \end{pmatrix}$$

(8)

Then the least squares criterion imposes

$$\frac{\partial \chi^2}{\partial A_x} = 0 \quad \text{or} \quad H^T V^{-1} (X - HA_x) = 0$$

(9)

By solving the linear system with respect to $A_x$ we get the fit parameters

$$A_x = \begin{pmatrix} x_0 \\ \alpha_x \end{pmatrix} = (H^T V^{-1} H)^{-1} (H^T V^{-1} X)$$

(10)

and the error of these parameters

$$E_{A_x} = \begin{pmatrix} \sigma^2_{x_0} & \sigma_{x_0} \sigma_{\alpha_x} \\ \sigma_{\alpha_x} \sigma_{x_0} & \sigma^2_{\alpha_x} \end{pmatrix} = (H^T V^{-1} H)^{-1}$$

(11)

It must be pointed out that the track reconstructed parameter errors (11) do not depend
on the particular track coordinate values \((x_i, y_i)\),
the errors depend only on the \(z_i\) layer position
and on the \(\theta_{0i}\) mean scattering values, of the
\(H\) and \(V\) matrices.

We applied the same procedure for \(y\) coordinate,
in order to find the best fit parameters
and their errors. Immediately we have the \(x\)
and \(y\) vertex coordinate

\[(x_0 \pm \sigma_{x0}, y_0 \pm \sigma_{y0}) \tag{12}\]

and the \(\sigma_{xy}^{\text{vertex}}\) vertex position uncertainty

\[\sigma_{xy}^{\text{vertex}} = \sqrt{\sigma_{x0}^2 + \sigma_{y0}^2} \tag{13}\]

Because \(\sigma_{x0} = \sigma_{y0}\), then \(\sigma_{xy}^{\text{vertex}} = \sqrt{2}\sigma_{x0}\),
and represents the mean radius of the reconstructed vertex pattern on the target plane.
It depends exclusively on the tracking configuration \((z_i\) detector positions) and on MS strength \((\theta_{0i})\).

4 Vertex position uncertainty

With the track reconstruction procedure described earlier, we could evaluate the vertex position resolution (uncertainty) \(\sigma_{xy}^{\text{vertex}}\) \((13)\)
by means of the \(\sigma_{x0}^2\) value as the first element
of the \(E_{Ax}\) matrix \((11)\) of the track parameter errors.

For MS evaluation we had in view all the materials in the present day DIRAC configuration (see Figure 1), in the space between target and magnet. They are presented in Table 1.

A Monte-Carlo study of the particle transport within upstream part of the detector system has been done. The track reconstruction vertex points distribution are presented in Figure 4.

### Table 1.

<table>
<thead>
<tr>
<th>Ndet</th>
<th>Detector material</th>
<th>(z) position (cm)</th>
<th>Thickness (µm)</th>
<th>XL (cm)</th>
<th>(\theta_{0i}) plane (\theta_0) (mrad)</th>
<th>Detector resolution (\sigma_{x0}) (µm)</th>
<th>MS resolution (µm)</th>
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<td>1</td>
<td>Mylar</td>
<td>229</td>
<td>250</td>
<td>28.7</td>
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<td>2</td>
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<td>6500</td>
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<td>24.59</td>
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<tr>
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<td>6500</td>
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<td>0.40974</td>
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<tr>
<td>6</td>
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<td>0.42132</td>
<td>125</td>
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<tr>
<td>7</td>
<td>SciFi</td>
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<td>2500</td>
<td>42.4</td>
<td>0.42132</td>
<td>125</td>
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<td>2000</td>
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<td>9</td>
<td>IH B</td>
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<td>-</td>
<td>612.53</td>
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<tr>
<td>10</td>
<td>Mylar</td>
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<td>250</td>
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Table 2.

<table>
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<tr>
<th>Ndet</th>
<th>$x(\mu m)$</th>
<th>$\sigma^{det}_{ii}(\mu m)$</th>
<th>$\sigma^{MS}_{ii}(\mu m)$</th>
<th>$y(\mu m)$</th>
<th>$\sigma^{det}_{ii}(\mu m)$</th>
<th>$\sigma^{MS}_{ii}(\mu m)$</th>
<th>$z(cm)$</th>
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<td>1</td>
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<td>0.000</td>
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<td>0.000</td>
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<td>612.533</td>
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<td>612.533</td>
<td>311.60</td>
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<td>-1200.304</td>
<td>-</td>
<td>842.229</td>
<td>1865.362</td>
<td>-</td>
<td>842.229</td>
<td>334.40</td>
</tr>
</tbody>
</table>

With given (measured) coordinates and associated errors (including intrinsic detector resolution $\sigma^{det}_{ii}$ and MS error $\sigma^{MS}_{ii}$), as are given in Table 2, the track reconstruction procedure uses the symmetric nondiagonal correlation matrix, which for actual configuration looks as follows

$$
\begin{pmatrix}
.100E+01 & .000E+00 & .000E+00 & .000E+00 & .000E+00 & .000E+00 & .000E+00 & .000E+00 & .000E+00 & .000E+00 \\
.000E+00 & .100E+01 & .103E+00 & .111E+00 & .607E-01 & .604E-01 & .343E-13 & .357E-13 & .455E-13 & .555E-13 \\
.000E+00 & .103E+00 & .100E+01 & .197E+00 & .222E+00 & .169E-00 & .168E+00 & .976E-13 & .102E-12 & .132E-12 \\
.000E+00 & .111E+00 & .197E+00 & .100E+01 & .335E+00 & .311E+00 & .311E+00 & .182E-12 & .190E-12 & .248E-12 \\
.000E+00 & .113E+00 & .222E+00 & .335E+00 & .100E+01 & .508E+00 & .509E+00 & .300E-12 & .313E-12 & .409E-12 \\
.000E+00 & .607E-01 & .169E+00 & .311E+00 & .508E+00 & .100E+01 & .914E-00 & .545E-12 & .570E-12 & .751E-12 \\
.000E+00 & .604E-01 & .168E+00 & .311E+00 & .509E+00 & .914E-00 & .100E+01 & .549E-12 & .575E-12 & .759E-12 \\
.000E+00 & .343E-13 & .976E-13 & .182E-12 & .300E-12 & .545E-12 & .549E-12 & .100E+01 & .357E-24 & .481E-24 \\
.000E+00 & .357E-13 & .102E-12 & .190E-12 & .313E-12 & .570E-12 & .575E-12 & .357E-24 & .100E+01 & .508E-24 \\
.000E+00 & .455E-13 & .132E-12 & .248E-12 & .409E-12 & .751E-12 & .759E-12 & .481E-24 & .508E-24 & .100E+01 \\
\end{pmatrix}
$$

Finally the linear track reconstructed data are

$$
x_0 = 1973.896 \pm 1829.597 \mu m \quad ; \quad \alpha_x = -0.00082 \pm 0.00077 \quad (14)
$$

$$
y_0 = -3944.988 \pm 1829.597 \mu m \quad ; \quad \alpha_y = 0.00164 \pm 0.00077
$$

and the vertex position uncertainty, according to (13) and (14) is

$$
\sigma^{(vertex)}_{xy} = 2587.4 \mu m
$$

and represents the mean radius of the reconstructed vertex point pattern on the target plane, for 2000 MeV/c pions.
5 Analysis of some possible configurations

Our study has been focused on the dependence of the vertex position uncertainty \( \sigma_{xy}^{(\text{vertex})} \) on first detector pair position \( (z_1) \).

We took in this study two types of tracking systems. The first one with two pairs of SciFi coordinate detectors and the second one with three pairs of MSGC and SciFi detector pair combinations. In both of these cases we compared the results with a Si microstrip and SciFi combination.

The most affecting vertex resolution (uncertainty) is the closest to the target coordinate detector. As close this detector can be placed, the best vertex position resolution can be obtained. This is why we studied the vertex position uncertainty as a function of the first detector position. In such a way we considered the first detector pair as a moving one and are looking for the vertex position uncertainty variation. The last detector pair in both of the cases have been considered a SciFi placed right before the magnet (about 6 m from the target). The results are presented in Figure 5 and Figure 6.

In Figure 5 there are presented a lot of combination of SciFi detector pairs. The first pair (moving) are 100\( \mu m \) and 125\( \mu m \) intrinsic resolution as long as the last one are 125\( \mu m \), 175\( \mu m \), 250\( \mu m \) and 500\( \mu m \). All other elements in the upstream channel have been included in the particle transport study. They are the mylar foils and Ionisation Hodoscopes (IH) as are presented in the Table 1. From
Figure 5 it can be seen that there are a small difference between all kind of SciFi combinations. Nevertheless, they are separated in 2 groups, one with 100 $\mu$m and the other with 125 $\mu$m as moving coordinate detector pair. For all of them the vertex resolution varies uniformly with increasing $z_1$ distance between moving detector and the target. When $z_1$ becomes greater than 310 cm (the moving detector passes the IH’s, the most important scatterer) and closes to the fixed SciFi pair, the vertex resolution becomes worse.

In the same Figure 5 it can be seen the vertex resolution variation for a silicon microstrip detector pair (10 $\mu$m coordinate resolution). This one is very sensitive to any changement in the scattering environment. At about $z_1 = 240$ cm, it "sees" the changement in the scattering due to first mylar foil. The same changement is seen at about $z_1 = 310$ cm, position of IH’s, and at about $z_1 = 335$ cm, position of the second mylar foil.

The SciFi detectors do not "see" the passage through the mylar foil, because their intrinsic resolution ($\approx 100\mu$m) is larger than the MS resolution contribution of the mylar, and they are added in squares.

In Figure 6 there is a more complicated variation of the vertex position resolution. Here we considered three coordinate detector pairs. In this case the vertex resolution do not shows a so large variation as in previous case. This is due to the fact that three detector pairs now keep the linear track in a more rigide situation. Nevertheless, when the moving detector becomes closer to a fixed one, the vertex resolution increases very large. This is due to the fact that the "error bar" constrains (detector resolution) of the two closest detectors becomes very permissibile for reconstructed tracks.

In the case of MSGC/SciFi combination, the moving detector pair shows a large transparence for reconstructed tracks at about $z_1 = 240$ cm, when it meets the second MSGC/SciFi coordinate detector pair. The same effect is present in the case of moving MSGC pair through the second coordinate detector pair (SciFi) at about $z_1 = 290$ cm. After passing the second coordinate detector pair, the vertex resolution is not so affected, because now the closest coordinate detector pair relative to target, is another one, the first fixed pair.

In the Figure 6 is presented also the same combination of silicon microstrip detector as in Figure 5. Here it can be seen that even the silicon detector in a two pair combination can present a worse vertex position resolution than a three pair MSGC-SciFi combination.

### 6 Conclusion

For the background and good particle track separation it is possible to identify the track intersection points with the target plane, and so to find the source coordinates (vertex position) of the detected particles. After track reconstruction within the present day tracking detector system the pointlike source for pions 2000 MeV/c, becomes, due to MS, a spot of mean radius 2587 $\mu$m. Within the domain specified by this radius there is not possible to separate the background and good particle tracks, but for outside this region, the background can easely be rejected.

The present paper shows the possibilities to reduce the reconstructed vertex point spread for interesting particles, using a proper tracking detector configuration for the upstream DIRAC setup.
References


