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COMPUTER SIMULATION OF BUNCH LENGTHENING IN SPEAR

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Abstract: A technique is developed for computer simulation of bunch lengthening in $e^+ - e^-$ storage rings. This technique is applied to SPEAR, and the results are in agreement with data.

The "natural" values of the bunch length and energy spread in an electron storage ring are determined by fluctuations in the emission of synchrotron radiation by the individual particles.¹ At low beam currents collective effects are unimportant, and the bunch length and energy spread should be equal to their natural values. However, at high currents this is not the case, and collective effects cause increases above the natural values. In this paper we report the results of a simulation of collective effects for SPEAR II. The results on the current dependence of bunch length and energy spread are in agreement with data.²

The phase space distribution of the beam is represented by M randomly chosen test particles. The motion in phase space of each of the test particles is given by³:

$$\begin{aligned}
 t_m(n) &= t_m(n-1) + \frac{\alpha T_0}{E_0} \epsilon_m(n) \\
 \epsilon_m(n) &= \epsilon_m(n-1) - \frac{2T_0}{\tau_E} \epsilon_m(n-1) - \frac{2T_0}{\tau_D} \bar{\epsilon}(n-1) \quad (1) \\
 &\quad - U_0(1 - \cos(\omega_{rf} t_m(n-1))) - \hat{U} \sin(\omega_{rf} t_m(n-1)) \sin \phi_s \\
 &\quad + 2\sigma_{E0} \sqrt{\frac{T_0}{\tau_E}} R_m(n) + V_m(n) .
 \end{aligned}$$

In these equations $m=1, \dots, M$ specifies the test particle number, and the argument n specifies the number of revolutions the particles have made in the storage ring. The parameters in the equation are defined in Table I. Collective effects are contained in the wakefield and Robinson damping terms. Dipole oscillations of the bunch are damped by Robinson damping³ which is treated by using the measured damping times⁴. The wakefield gives the effect on test particle m from all particles which precede it in the bunch. The new contribution in this paper is the treatment of the wakefield.

In principle the wakefield on test particle m could be determined by considering all the test particles ahead of this one; then

$$V_m = \left(\frac{Q}{M}\right) \sum_{j(t_m > t_j)} W(t_m - t_j) \quad (2)$$

where Q is the total charge of the bunch, and $W(\tau)$ is the wakefield at time τ after the passage of a unit-charged, δ -function particle. $W(\tau)$ is proportional to the Fourier transform of the accelerator impedance. Since the real and imaginary parts of the impedance are related by the Hilbert transformation⁵,

$$W(\tau) = -\frac{2}{\pi} \int_0^{\infty} d\omega Z_R(\omega) \cos \omega \tau. \quad (3)$$

For SPEAR II the impedance was estimated from measurements of bunch lengthening and energy loss². It is shown in Fig. 1a, and the resultant wakefield is shown in Fig. 1b. It is important to note that $W(\tau) \rightarrow \infty$ as $\tau \rightarrow 0$. Since arbitrarily small values of $t_m - t_j$ are possible, arbitrarily large values of V_m are possible. Truncating $W(\tau)$ at some small τ (or equivalently $Z_R(\omega)$ at some large ω) can reduce the effects of small values of $t_m - t_j$. But, if this is done, the threshold for bunch lengthening becomes a function of the (arbitrarily chosen) truncation procedure.

An alternative method can be used to calculate the wakefield from the test particles. Let $\rho(t)dt$ be the charge of the bunch between t and $t + dt$. This charge density can be represented as an expansion in orthogonal polynomials. The coefficients of the expansion are determined by the phase space coordinates of all of the test particles. The contribution at time t to the wakefield from a given polynomial comes from an integral of the product of the polynomial, the weight function for the polynomial and the δ -function wakefield $W(\tau)$. These integrals do not have the divergence of $W(\tau)$.

The charge density $\rho(t)$ is normalized as

$$Q = \int_{-\infty}^{\infty} \rho(t) dt \quad (4)$$

where Q is the charge of the bunch. It is natural to write $\rho(t)$ as an expansion in Hermite polynomials because the weight function for Hermite polynomials is a Gaussian

$$\rho(t) = \exp\left(-\frac{(t - \bar{t})^2}{2\sigma^2}\right) \sum_{\ell=0}^L a_{\ell} H_{\ell}\left(\frac{t - \bar{t}}{\sigma}\right) \quad (5)$$

In this equation $\bar{t}$ is the mean time displacement, σ is the bunch length, a_ℓ are the expansion coefficients and $H_{e\ell}$ are Hermite polynomials⁶.

The values of the a_ℓ 's, $\bar{t}$ and σ are determined from the test particle distribution. $\bar{t}$ is approximated by the mean value of the time displacements of the test particles

$$\bar{t}(n) = \frac{1}{M} \sum_{m=1}^M t_m(n) \quad (6)$$

On any particular revolution the rms spread in the t_m 's will have contributions from all the Hermite polynomials in the expansion, but averaged over several synchrotron oscillation cycles only H_{e0} can contribute to the rms. An "exponential average" is used to determine σ on turn n from the rms of the time distribution on that turn and earlier turns

$$\sigma(n) = 0.994\sigma(n-1) + 0.006 \left[\frac{1}{M} \sum_{m=1}^M (t_m(n) - \bar{t}(n))^2 \right]^{1/2}. \quad (7)$$

In practice the σ actually used is the value calculated in (7) but binned in 10 ps bins to allow the use of a look-up table for the wakefield.

The orthogonality of the polynomials and the normalization condition (Eq. 4) determine the a_ℓ 's

$$a_\ell = \frac{1}{\sqrt{2\pi\ell!}} \int_{-\infty}^{\infty} \rho(t) H_{e\ell} \left(\frac{t - \bar{t}}{\sigma} \right) \frac{dt}{\sigma} \approx \frac{1}{\sqrt{2\pi\ell!}\sigma} \frac{Q}{M} \sum_{m=1}^M H_{e\ell} \left(\frac{t_m - \bar{t}}{\sigma} \right) \quad (8)$$

The sum is the approximation of the integral by the (finite) test particle distribution.

The wakefield is given by

$$V(t) = \sum_{\ell=0}^L a_\ell V_\ell(t - \bar{t}) \quad (9)$$

where $V_\ell(t)$ is the contribution of the ℓ -th Hermite polynomial to the wakefield and is given by

$$V_\ell(t) = \frac{2}{\pi} \int_0^\infty d\tau \int_0^\infty d\omega \exp \left(-\frac{(t - \tau)^2}{2\sigma^2} \right) H_{e\ell} \left(\frac{t - \tau}{\sigma} \right) Z_R(\omega) \cos \omega \tau \quad (10)$$

These double integrals can be performed by interchanging the orders of integration, performing the τ integral analytically, and performing the ω integral numerically. Some results for different values of ℓ and σ are shown in Fig. 2; the impedance in Fig. 1a is used in these calculations.

To obtain the results for SPEAR a table of wakefields was generated for $\ell=0, \dots, 6$ and $\sigma=40\text{pS}, 50\text{pS}, \dots, 330\text{pS}$. For each ℓ and σ the table contained 225 entries covering from -4σ to $+5\sigma$. Going to polynomials of order greater than 6 proved to be difficult and unnecessary as discussed below.

One thousand test particles were tracked using eq. (1) to calculate the phase space motion. The accelerator parameters are listed in Table II. The initial distributions in both time and energy were Gaussian with σ 's equal to the natural values. After three damping times, the mean time, the bunch length, the mean energy and the energy spread were calculated by

$$\begin{aligned}\bar{t} &= \frac{1}{N} \sum_{n=1}^N \frac{1}{M} \sum_{m=1}^M t_m(n), \\ \sigma_t &= \left[\frac{1}{N} \sum_{n=1}^N \frac{1}{M} \sum_{m=1}^M (t_m(n) - \bar{t})^2 \right]^{1/2}, \\ \bar{\epsilon} &= \frac{1}{N} \sum_{n=1}^N \frac{1}{M} \sum_{m=1}^M \epsilon_m(n), \\ \sigma_\epsilon &= \left[\frac{1}{N} \sum_{n=1}^N \frac{1}{M} \sum_{m=1}^M (\epsilon_m(n) - \bar{\epsilon})^2 \right]^{1/2}.\end{aligned}\tag{11}$$

The summation over m is a sum over particles, and the summation over n is a sum over turns. This latter sum was performed over approximately one damping time which began after the initial three damping times.

Figures 3-5 show the results; these are compared with the data from SPEAR². In all cases the mean energy $\bar{\epsilon}$ was consistent with zero, and it is not plotted. The mean time is presented as a phase shift to allow comparison with the measurements

$$\Delta\phi = -\omega_{rf}\bar{t}\tag{12}$$

The bunch lengthening threshold calculated from the simulation is in good agreement with the observed threshold. At all energies the current dependence of the bunch length is in good agreement with measurement up to bunch lengths of twice the natural length. The current dependence of the energy spread shows a systematic difference between the results of the simulation and the measurements. The measurements have a systematic uncertainty associated with knowledge of the β and η functions at the emission point of the synchrotron light used in these measurements². It is not known whether this uncertainty is sufficient to explain the differences between measurement and simulation. The phase shift results from the simulation have approximately the same functional dependence on current as do the measurements. However, above the bunch lengthening threshold the detailed agreement is not good.

Checks were performed to ensure that the results were independent of the number of test particles and the number of Hermite polynomials. In addition the simulation was repeated at several currents to give an estimate of the fluctuations in the results. The results from these repeated trials are included in Figs. 3 and 4.

Fig. 6 a-c shows the phase space distributions of the test particles for currents below (2 mA), just above (5 mA), and well above (12 mA) the bunch lengthening threshold. Comparing these distributions one sees a core of roughly the same size at all three currents. The striking difference is the population of the halo outside the core. The increase in bunch length is due to an increase in the number of particles in the halo. In Fig 6 d the phase space distributions at 12 mA for fifty successive turns are plotted. Phase space trajectories show departures from circular paths, and the distribution in time clearly is not Gaussian.

Fig. 7 and 8 show the bunch length vs turn number at currents below and well above the bunch lengthening threshold.* Below threshold the dominant oscillation of the bunch length is at twice the synchrotron oscillation frequency. The beats which are visible in Fig. 7 are evidence for additional oscillation frequencies. Above threshold (Fig. 8) there is significant spectral content at several frequencies which are multiples of the synchrotron oscillation frequency.

* On turn n , the mean time for that turn (Eq. 7) was used to calculate the bunch length.

To determine whether the oscillations in bunch length are a consequence of the finite number of test particles, a spectral analysis was performed for different numbers of test particles. A beam energy of 3 GeV and currents of 16 and 40 mA were used; the 3 GeV beam energy was chosen to allow the spectrum analysis to be performed over several radiation damping times without requiring excessive computer time. Below the threshold (16 mA) oscillation amplitudes scaled with the number of test particles, M , and $1/\sqrt{M}$. It is concluded that the oscillations below threshold are a consequence of the finite number of test particles.

Above the threshold there are no systematic dependences on M . Therefore, the oscillations cannot be associated directly with the number of test particles, and more complicated phenomena are implicated. Among the possible explanations are coherent oscillations of the bunch or an equilibrium state determined by fluctuations early in the evaluation of the bunch. More detailed systematic studies are necessary before quantitative information can be obtained from the spectrum of oscillations.

Bunch lengthening calculations are underway for existing and proposed storage rings. In addition the technique presented in this paper is easily generalized for transverse calculations.

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TABLE I: Definitions of Symbols used in Equation (1)

t_m :	deviation from the zero current phase stable time,
ϵ_m :	Deviation from energy equilibrium,
α :	momentum compaction,
T_0 :	beam revolution period,
E_0 :	beam energy,
τ_E :	radiation damping time for energy oscillations,
τ_D :	Robinson damping time,
$\bar{\epsilon}$:	mean energy displacement $(\frac{1}{M} \sum_{m=1}^M \epsilon_m)$,
U_0 :	average synchrotron radiation energy loss per turn
$\hat{U}$:	peak energy gain from rf
ω_{rf} :	angular frequency of the rf
ϕ_s :	zero current synchronous phase angle $(= \cos^{-1}(U_0/\hat{U}))$
$\sigma_{\epsilon 0}$:	natural energy spread of the beam
R_m :	a Gaussian distributed random number with mean = 0 and rms = 1
V_m :	wakefield

TABLE II: Accelerator Parameters

Energy (GeV)	1.55	2.21	3.00
Peak of voltage (MV)	0.91	1.30	2.90
Synchrotron tune	0.033	0.033	0.042
Energy loss/ turn (MeV)	0.040	0.17	0.56
Radiation damping time (mS)	30.4	10.5	4.2
Natural bunch length (pS)	58.4	83.4	88.8
Natural energy spread (MeV)	0.576	1.17	2.16

rf frequency (MHz)	358.5
Revolution period (μ s)	0.781
Momentum compaction	0.0418
Robinson damping time (mS)	0.6 / I(mA)

The bunch lengthening calculation was done for the three energies above. The quantities in the second part of the table were taken as energy independent. Most of the parameters are from Ref. 4.

Figure Captions

1. a) The real part of the impedance used in this calculation (from Ref. 2).
b) The wakefield resulting from this impedance.
2. The wakefield resulting from Eq. (10) for two different Hermite polynomials.
3. Bunch length, core energy spread and phase shift vs current for $E_0 = 1.55$ GeV. The data which are represented by the solid points and the curves come from Ref. 2.
4. Bunch length, core energy spread and phase shift vs current for $E_0 = 2.21$ GeV. The data are from Ref. 2.
5. Bunch length, core energy spread and phase shift vs current for $E_0 = 3.00$ GeV. The data from Ref. 2 are represented by the solid points and the curves.
6. Phase space distributions from 1000 test particles at $E_0 = 1.55$ GeV. Time displacement is plotted on the abscissa and energy displacement on the ordinate. These are plotted with divisions equal to twice the natural value of bunch length or energy spread. The origin of each plot has been displaced to be at the center of bunch. The currents for the plots are: a) 2 mA, b) 5 mA, c) 12 mA, d) 12 mA. Plots a)-c) are for a single turn; plot d) is for 50 successive turns.
7. Bunch length vs turn number for a bunch current of 2mA. Turn number 0 occurs after 3 damping times, and the plot covers 0.077 of a damping time. The insert shows the bunch length for the first 125 turns of the main plot.
8. Bunch length vs turn number for a bunch current of 12mA. The comments in the caption for Fig. 7 apply.

References

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