Ref.TH.3260-CERN

SCALING VIOLATION IN $\gamma\gamma$ SCATTERING

E. Etim ^{*)} and E. Massó ⁺⁾

CERN - Geneva

A B S T R A C T

An unorthodox model is examined which predicts that scaling violations in structure functions are related to properties of the photon vacuum polarization amplitude. The relationship is approximate, but very useful. It reproduces the leading QCD prediction in all cases, and allows, as for instance in $\gamma\gamma$ scattering, to fill up on what QCD cannot say by itself. There is a simple formula for the total $\gamma\gamma$ cross-section, valid for all values of the photon mass. Data from e^+e^- annihilation can be fed into it to obtain a fit to the measured photon structure function. The PLUTO data are not at all difficult to reproduce.

^{*)} On leave of absence from Laboratori Nazionali INFN, Frascati, Italy.

⁺⁾ On leave of absence from Departament de Física Teòrica, Universitat Autònoma de Barcelona, Bellaterra, Spain.

Ref.TH.3260-CERN

10 March 1982

1. - INTRODUCTION

The "compositeness" of the photon, as supposedly revealed by its structure functions, should be understood in context. The photon is after all still elementary. The context is that of the coupling of photons to hadrons. In this regard the photon has shown to avail itself of two possibilities : from deep inelastic lepton-nucleon scattering and e^+e^- annihilation into hadrons, large mass photons are assumed to couple to hadrons predominantly through quark-antiquark pairs ; from photoproduction, meson and nucleon form factors at small momentum transfers and from vector meson resonance excitations in e^+e^- annihilation, low mass photons have long been known to couple to hadrons through vector mesons. It would thus seem that the simplest picture of photon-hadron interaction is that these two kinds of couplings are present at all momenta. But it is not. Even without the problem of confinement such a picture would imply a serious incompleteness of QCD and hence its inability to describe the structure of matter in terms of a few basic constituents.

This two-mode coupling model ^{1),2)} was used to provide a highly successful fit to the results of a recent PLUTO experiment ³⁾ on the inclusive process

$$e^+ + e^- \rightarrow e^+ + e^- + X \quad (1)$$

This reaction is usually interpreted as hadron production in the collision of two photons

$$\gamma(q) + \gamma(k) \rightarrow X \quad (2)$$

and hence as the probing of the structure of one photon by the other. In the PLUTO experiment one of the photons is arranged to be nearly on-shell ($k^2 \approx 0$) and the other, with a large virtual mass $Q = \sqrt{-q^2}$, is supposed to probe the structure of the quasi-real photon in a deep inelastic limit, i.e., $Q^2, k \cdot q \rightarrow \infty$, with $x = Q^2/2k \cdot q$ finite. The measured structure function does not scale.

This behaviour is explained in the two-component model. The direct coupling of the projectile photon to $q\bar{q}$ pairs in the target leads, through the box diagram, to a $\log(Q^2)$ dependence of the structure function ⁴⁾. In addition it is argued that a quasi-real photon has enough time to convert into a vector meson, whose structure is then probed by the projectile. On the basis of this argument a hadron-like scaling piece to the photon structure function is expected ^{1),2)}.

Experimentally such a contribution is necessary for small values of x ³⁾. By the same kind of reasoning, the Q^2 dependence of the structure function can be explained³⁾ by combining QCD with the vector meson dominance model (VMD).

In this paper we would like to take a different point of view, namely that the use of QCD and VMD in this simple additive manner amounts to double counting. In other words one has to work consistently, coupling photons to hadrons either through $q\bar{q}$ pairs or through vector mesons. Our main idea is that, at least in an approximate way, VMD constitutes an analytic continuation of QCD to low energies. The immediate consequence of this is that the leading asymptotic behaviour in Q^2 of any structure function, as given by QCD, can be reconstructed by coupling photons to hadrons through vector mesons only. This fact, sometimes referred to as Q^2 -duality⁵⁾ follows from the well-known theorem that the Taylor series expansion of an analytic function also allows the determination of its asymptotic expansion. We shall apply this result here to the investigation of the Q^2 dependence of the photon structure function. To set up the corresponding Taylor series expansion (the VMD model), the contributions of the known vector mesons alone are not sufficient. The continuum, although it contributes in the order of 20-30% to photo-production and the photon-photon total cross-section at $Q^2 = 0$, is nevertheless important in reconstructing the asymptotic expansions for large Q^2 . It is customary, though not always explicitly stated, to approximate the continuum by an infinite number of closely-spaced vector meson resonances. We shall adopt this approximation here too but, unlike other authors^{5),6)}, we find that it is not necessary to specify the mass spectrum of these resonances nor of their couplings to the photon. The model we are considering defines itself completely through the physics of e^+e^- annihilation into hadrons. By this we mean that, independently of the form of the vector meson mass spectrum and of their couplings to the photon, cross-sections of photon-induced hadronic processes are predicted to be simple functions or functionals of the photon vacuum polarization amplitude. The relationship is through a Hilbert-type transform. Apart from allowing us to dispense with assumptions on the infinite set of resonances (essentially the unknown continuum), it provides a new interpretation of scaling and of its breaking in terms of the properties of the photon probe. It is important that this alternative interpretation is available, for in the case of $\gamma\gamma$ scattering it would have been awkward to be able to describe the behaviour of the cross-section only in terms of the "compositeness" of the target.

2. - THE PHOTON STRUCTURE FUNCTION

Our aim is to compute the total cross-section, $\sigma_{\gamma\gamma}(\nu; Q^2)$, for $\gamma\gamma$ scattering in a vector meson dominance model and to show that for $Q^2 \rightarrow \infty$ it behaves as predicted by QCD. The variable ν is defined by

$$\nu = (k + q)^2 = Q^2 \left(\frac{1}{x} - 1 \right).$$

We start by defining the model. Consider the photon vacuum polarization amplitude, due to coupling to hadrons

$$\Pi(q^2) = \frac{q^2}{\pi} \int_0^\infty ds \frac{\text{Im } \Pi(s)}{s(s - q^2)}. \quad (3)$$

We assume that its imaginary part is saturated by an infinite number of vector mesons

$$\text{Im } \Pi(s) = \pi \sum_{n=0}^\infty \frac{e^2 m_n^2}{f_n^2} \delta(s - m_n^2). \quad (4)$$

The coupling of the photon to the n^{th} vector meson of mass m_n is em_n^2/f_n , where e is the electric charge. We shall eliminate this coupling constant in favour of $\text{Im } \Pi(m_n^2)$. From Eq. (4), in fact, we have

$$\frac{e^2 m_n^2}{f_n^2} = \frac{1}{\pi} \int_{m_n^2 - \frac{\Delta m_n^2}{2}}^{m_n^2 + \frac{\Delta m_n^2}{2}} ds \text{Im } \Pi(s). \quad (5)$$

With the help of the definition of the Riemann integral, Eq. (5) allows us to convert all discrete sums over vector meson states into integrals. For $\sigma_{\gamma\gamma}(\nu; Q^2)$ we therefore have, in the so-called diagonal approximation (i.e., essentially dominance of diffractive scattering),

$$\sigma_{\gamma\gamma}(\nu; Q^2) = \frac{1}{\pi} \int_0^\infty ds \frac{\text{Im } \Pi(s)}{(s + Q^2)^2} s \sigma_{\gamma V}(\nu; s) \quad (6)$$

where

$$\sigma_{\gamma V}(\nu; s) = \frac{1}{\pi} \int_0^\infty \frac{ds'}{s'} \text{Im } \Pi(s') \sigma_{VV}(\nu; s, s') \quad (7)$$

is the cross-section for the scattering of a vector meson of mass $\sqrt{s}$ on a real photon and $\sigma_{VV}(\nu; s, s')$ is the cross-section for vector meson-vector meson scattering.

From the experience with the application of VMD to deep inelastic electron-nucleon scattering, the way to proceed from Eqs. (6) and (7) would be to make scaling Ansätze for the cross-sections $\sigma_{\gamma V}(\nu; s)$ and $\sigma_{\gamma V}(\nu; s, s')$. Seen only in terms of the end result, these assumptions appear to be ad hoc. It is therefore important to understand their necessity and naturalness in terms of the structure of the model. To this end, first observe that Eq. (7) and the analogous equation obtained from Eq. (6) by setting $Q^2 = 0$, i.e.,

$$\sigma_{\gamma\gamma}(\nu) = \frac{1}{\pi} \int_0^\infty \frac{ds}{s} \text{Im} \Pi(s) \sigma_{\gamma V}(\nu; s) \quad (8)$$

together with the convergence conditions for the e^+e^- (SVZ or "magic") moments ⁷⁾

$$C_\lambda = \frac{1}{\pi} \int_0^\infty ds \frac{\text{Im} \Pi(s)}{s^{\lambda+1}} \quad (9)$$

already imply constraints on the mass dependence of $\sigma_{\gamma V}(\nu; s, s')$ and $\sigma_{\gamma V}(\nu; s)$, respectively. For instance, while terms proportional to $1/(s+s')$ and $1/ss'$ may occur in $\sigma_{\gamma V}(\nu; s, s')$, a term proportional to $(1/s+1/s')$ is not allowed since C_λ is divergent for $\lambda = 0$. Secondly we note that the idea that the photon behaves very much like a hadron may be given a precise formulation valid for all Q^2 , only if the cross-sections have simple power dependences in the masses. This is so because Eq. (6) is essentially a Hilbert transform ⁸⁾, and this transform in our case constitutes an integral representation of the generator of scale transformations, $D = Q^2(d/dQ^2)$, in momentum space. In fact Eq. (6), or more generally the cross-section for virtual Compton scattering off a target A,

$$\sigma_{\gamma A}(\nu; Q^2) = \frac{1}{\pi} \int_0^\infty ds \frac{\text{Im} \Pi(s)}{(s+Q^2)^2} s \sigma_{VA}(\nu; s) \quad (10)$$

may be put in the form

$$\sigma_{\gamma A}(\nu; Q^2) = (1+D) \frac{1}{\pi} \int_0^\infty ds \frac{\text{Im} \Pi(s)}{s+Q^2} \sigma_{VA}(\nu; s). \quad (10')$$

In the space of functions $f_\lambda(s) = \text{const. } s^{\lambda-1}$, $0 < |\lambda| < 1$, the Hilbert transform ($Q^2 > 0$) ⁸⁾

$$\int_0^\infty ds \frac{f_\lambda(s)}{s+Q^2} = \Gamma(\lambda) \Gamma(1-\lambda) f_\lambda(Q^2) \equiv H f_\lambda(Q^2) \quad (11)$$

acts just like the scale operator D . Consequently, if the integrand in Eq. (10') were of the special form of $f_\lambda(s)$ ^{*)}, i.e.,

$$\frac{1}{\pi} \text{Im} \Pi(s) \sigma_{VA}(\nu; s) \equiv f_\lambda(\nu; s) = a_\lambda(\nu) s^{\lambda-1} \quad (12)$$

then $\sigma_{VA}(\nu; Q^2)$ would be of the same form

$$\sigma_{VA}(\nu; Q^2) = \Gamma(1+\lambda) \Gamma(1-\lambda) f_\lambda(\nu; Q^2). \quad (13)$$

The photon would then indeed be "behaving" as a hadron. Since for $s \rightarrow \infty$

$$\text{Im} \Pi(s) \xrightarrow{s \rightarrow \infty} \frac{e^2}{4\pi} \sum_i q_i^2 \quad (14)$$

with q_i the charge of the quark of flavour i , one has, for the special case $\lambda = 0$ (exact scaling of the cross-sections),

$$\sigma_{VA}(\nu; Q^2) \xrightarrow{Q^2 \rightarrow \infty} \frac{\alpha}{\pi} \sigma_{VA}(\nu; Q^2) \sum_i q_i^2. \quad (15)$$

The point that is being made here is that the representation in Eqs. (6) and (10), rather than requiring ad hoc assumptions in order to be predictive, is a rigorous formalism in which to study properties of scale invariance and of its breaking. In fact, Eq. (15) considered as a leading asymptotic behaviour can be derived on slightly more general grounds than from Eq. (12). A variant of Eq. (12) for $\lambda = 0$, that is

$$s \sigma_{VA}(\nu; s) = a_0(\nu) \quad (16)$$

indeed allows the reduction of Eq. (10) to the form

$$\sigma_{VA}(\nu; Q^2) = -a_0(\nu) \frac{d\Pi(-Q^2)}{dQ^2} \quad (17)$$

^{*)} Due to the presence of $\text{Im} \Pi(s)$ there is obviously a threshold limitation in Eq. (12). This is easily taken care of by considering leading and non-leading terms. One achieves this already with the help of Eq. (16).

wherein a mechanism for the breaking of scaling is implicit. Equation (15) is the leading term in the asymptotic expansion of (17). Equation (17) generalizes a previous result obtained less straightforwardly in the case $A \equiv$ a nucleon ⁵⁾. This equation provides an alternative description of scaling and its violation in terms of the properties of the photon.

This new point of view allows us in the case $A \equiv$ a photon, to avoid the contradiction of considering the photon at the same time as elementary (probe) and composite (target). In general, the function $f_\lambda(v;s)$ with $\lambda = 0$ plays a very special rôle in determining the asymptotic behaviour of $\sigma_{\gamma A}(v;Q^2)$ for all A . To say that this function, that is, the fixed point of the transformation in Eq. (10), dominates the asymptotics is equivalent to saying that the photon behaves essentially as a hadron. To see this, sum the right-hand sides of Eqs. (12) and (13) over λ and then perform an analytic continuation in λ . This is equivalent to performing the inverse Mellin transform ($\equiv$ expansion in terms of the simultaneous eigenfunctions of H and D)

$$\frac{1}{\pi} \text{Im} \Pi(s) \sigma_{\gamma A}(v;s) = \frac{1}{2\pi i} \int_{\lambda_0 - i\infty}^{\lambda_0 + i\infty} d\lambda a_\lambda(v) s^{\lambda-1} \quad (18)$$

and then substituting in Eq. (10) to obtain

$$\begin{aligned} \sigma_{\gamma A}(v;Q^2) &= -\frac{1}{2\pi i} \int_{\lambda_0 - i\infty}^{\lambda_0 + i\infty} d\lambda a_\lambda(v) \Gamma(1+\lambda) \Gamma(1-\lambda) (Q^2)^{\lambda-1} \\ &\equiv \sum_{n=0}^{\infty} (-Q^2)^n (n+1) a_{n+1}(v). \end{aligned} \quad (19)$$

The contour of integration is to the right of all singularities of $a_\lambda(v)$. From the definition

$$a_\lambda(v) = \frac{1}{\pi} \int_0^\infty ds \frac{\text{Im} \Pi(s)}{s^\lambda} \sigma_{\gamma A}(v;s) \quad (20)$$

and on account of Eqs. (8) and (19) $0 < \lambda_0 < 1$. If $\lambda = 0$ were a regular point of $a_\lambda(v)$, $\sigma_{\gamma A}(v;Q^2)$ would, according to Eq. (19), fall off at least as fast as $\ln(Q^2)/Q^4$, and $\sigma_{\gamma A}(v;s)$ at least as fast as $1/s^2$, if the next singularity is a simple pole at $\lambda = -1$. In this case it will never be possible to match the asymptotic behaviour of both cross-sections. Only a singularity of $a_\lambda(v)$ at $\lambda = 0$ allows for this matching, as is evident from the comparison of Eqs. (14) and (19). The behaviour of $\text{Im} \Pi(s)$ in Eq. (14) is very important in preserving these asymptotics.

In general an N^{th} order pole at $\lambda = 0$ leads to the asymptotic behaviour

$$\sigma_{\gamma A}(\nu; Q^2) \sim \sigma_{VA}(\nu; Q^2) \xrightarrow{Q^2 \rightarrow \infty} \frac{(\ln Q^2)^{N-1}}{Q^2}. \quad (21)$$

For $N = 1$ (a simple pole) we get the scaling result previously discussed. For $\gamma\gamma$ scattering, $N = 2$ (a double pole) would produce the QCD result

$$\sigma_{\gamma\gamma}(\nu; Q^2) \xrightarrow{Q^2 \rightarrow \infty} \frac{\ln Q^2}{Q^2}. \quad (22)$$

An important point to note is that the characterization of the fixed point of the transformation in Eq. (10) is so unique that the generation of the higher order poles involves no more complication than the appearance of ever new mass scales. For instance, in the case of $\gamma\gamma$ scattering, one has naturally, from $\sigma_{VV}(\nu; s, s')$ [cf. Eq. (7)], the new mass scale $s + s'$, so that, instead of Eq. (16), one may now assume

$$(s + s') \sigma_{VV}(\nu; s, s') = b_0(\nu) + b_1(\nu) \left(\frac{1}{s} + \frac{1}{s'} \right) + b_2(\nu) \frac{s'}{s + s'} + \dots \quad (23)$$

The first term on the right-hand side of (23) gives rise to a double pole of $a_\lambda(\nu)$ at $\lambda = 0$, and the other two terms to a simple pole each. The asymptotic behaviour associated with them shows this even more directly, and confirms the generation of higher order poles of $a_\lambda(\nu)$ at $\lambda = 0$ through changes in mass scale. Substituting Eq. (23) into (7), and making use of the fixed point property, gives

$$\sigma_{\gamma\gamma}(\nu; Q^2) \sim \sigma_{\gamma V}(\nu; Q^2) = -b_0(\nu) \frac{\pi(-Q^2)}{Q^2} + b_1(\nu) \frac{C_1}{Q^2} - b_2(\nu) \frac{d\pi(-Q^2)}{dQ^2} + \dots \quad (24)$$

C_1 is an e^+e^- moment [cf. Eq. (9)]. From Eq. (17) the last term in Eq. (24)

*) That is, besides s or s' separately. Note that the term with coefficient b_2 in Eqs. (23) and (24), included to illustrate the origin of a simple pole singularity and its consequences, is really unnecessary. Its effects are already contained in the term with coefficient b_0 since for $Q^2 \rightarrow 0$, $\pi(-Q^2)/Q^2 \rightarrow -d\pi(-Q^2)/dQ^2$.

may be interpreted as the piece corresponding to a hadron-like structure function. This contribution and the one corresponding to the leading QCD contribution can thus be obtained consistently from a unified framework.

Here is a summary of our procedure. We start from a Taylor series expansion about $Q^2 = 0$ [cf. Eq. (19)] which we assume to agree with the VMD model. We then place conditions on the analytic continuation of the coefficients of this series such that the asymptotic expansion determined by them, for large Q^2 , agrees with that given by QCD, at least in leading order. Mathematically, such a problem always admits a solution. Physically, what we have shown is that the QCD asymptotics can be matched provided the asymptotic expansion is made about a scale invariant fixed point. This requirement is the formulation of the statement that the photon behaves very much like a hadron. This leads to a preservation of asymptotics even when scale invariance is broken. A naïve (canonical) scaling assumption at the hadronic level is sufficient to generate the type of singularities of the analytically continued Taylor series coefficients appropriate to the photon-induced process under study.

3. - COMPARISON WITH EXPERIMENT

The model presented in the last section purports to be a consistent description of photon-induced hadronic processes, valid for all values of the photon mass. The PLUTO data ³⁾ on the total cross-section of $\gamma\gamma$ scattering constitute an important test of this claim. The data indicate a pronounced change in the behaviour of $\sigma_{\gamma\gamma}(\nu; Q^2)$ as a function of Q^2 , as one goes from $Q^2 = 0$ to about 10 GeV^2 . The change corresponds to a departure of the cross-section from one dominated by one or a few vector mesons to one governed by QCD. The transition, which occurs around $Q^2 = 1 \text{ GeV}^2$, is dramatic enough to suggest a breakdown of the simple VMD description in going from small to large values of Q^2 . The success of QCD would lead one to stop there. However, the PLUTO data are consistent with QCD only in the trend but not in detail, especially in the behaviour of the structure function

$$F_2(x, Q^2) = \frac{1}{4\pi^2\alpha} Q^2 \sigma_{\gamma\gamma}(\nu; Q^2) \quad (25)$$

for small values of x . There are therefore good reasons for examining models which are consistent with both VMD and QCD and are predictive for all values of Q^2 .

We shall specialize Eq. (23) in order to have simple formulae with which to compare with experiment, but we will not present any detailed fits. Putting Eqs. (6) and (7) together and using only the first term on the right-hand side of (23), we have

$$\sigma_{\gamma\gamma}(\nu; Q^2) = \frac{2m_0^2 \sigma_0(\nu)}{\pi^2} \int_{m_0^2}^{\infty} \int_{m_0^2}^{\infty} ds ds' \frac{\text{Im } \Pi(s) \text{Im } \Pi(s') s}{s' (s+s') (s+Q^2)^2}. \quad (26)$$

We have written $b_0(\nu)$ as $2m_0^2 \sigma_0(\nu)$, with m_0 the mass of the lowest vector meson state ^{*)}. As a first approximation we replace $\text{Im } \Pi(s)$ by its asymptotic expression according to Eq. (14) ^{**)}, and obtain from Eq. (26)

$$\sigma_{\gamma\gamma}(\nu; Q^2) = \left(\frac{\alpha q^2}{\pi}\right)^2 m_0^2 \sigma_0(\nu) \left[\frac{1}{Q^2 - m_0^2} \ln \left(\frac{m_0^2 + Q^2}{2m_0^2} \right) + \frac{\ln 2}{m_0^2 + Q^2} \right]. \quad (27)$$

Such an approximation is not expected to be very good for small Q^2 . Here is how it extrapolates to $Q^2 = 0$,

$$\sigma_{\gamma\gamma}(\nu; Q^2=0) \equiv \sigma_{\gamma\gamma}(\nu) = 2 \ln 2 \left(\frac{\alpha q^2}{\pi}\right)^2 \sigma_0. \quad (28)$$

For simplicity, we shall neglect the dependence of the cross-sections on ν . To be consistent we should then compare Eq. (27), and other similar equations, only with the leading ν independent terms in the experimental fits. The constant $b_0 = 2m_0^2 \sigma_0$ is determined by comparing Eq. (27) with the QCD prediction. From the coefficient of the $\ln(Q^2)/Q^2$ term one finds

$$m_0^2 \sigma_0 = 2\pi^3. \quad (29)$$

Equation (27) is compared with data in Fig. 1 (dashed curve) using the known vector meson families. The corresponding value of q^4 for the ρ and ω families is

^{*)} From Eq. (23), $\sigma_0(\nu) = \sigma_{\gamma\gamma}(\nu; m_0^2, m_0^2)$ at this order of approximation. b_0 is a known constant as follows from Eq. (29). From this equation the magnitude of σ_0 is of the order of a typical hadronic cross-section.

^{**)} We consider now just one vector meson family corresponding to the quark with charge q . A sum over families will be carried out at the end.

$$q^4 = \frac{1}{2} (q_e^4 + 6 q_e^2 q_w^2 + q_w^4) = q_u^4 + q_d^4 \quad (30)$$

where $(q_p, q_w) = (1/\sqrt{2}, 1/3\sqrt{2})$ and $(q_u, q_d) = (\frac{2}{3}, -\frac{1}{3})$. The value of $\sigma_{\gamma\gamma}(Q^2=0)$ is 130 nb, to be compared with the standard fit 9)

$$\sigma_{\gamma\gamma}(\nu) = \left(240 + \frac{270 \text{ GeV}}{\sqrt{\nu}} \right) \text{ nb.} \quad (31)$$

Note that the last term in Eq. (27) is just the leading asymptotic behaviour of the "hadron-like" contribution ¹⁾⁻³⁾ $-(d\Pi(-Q^2)/dQ^2)$. This contribution dominates the cross-section for small values of Q^2 . For small Q^2 the dashed curve is essentially the extrapolation of the asymptotic behaviour of $-d\Pi(-Q^2)/dQ^2$ into this region. For these values of Q^2 , $-(d\Pi(-Q^2)/dQ^2)$ is much larger than its asymptotic extrapolation.

As a second approximation we first integrate over s' in Eq. (26) and have

$$\sigma_{\gamma\gamma}(Q^2) = -4\pi^2 \int_{m_0^2}^{\infty} ds \frac{\text{Im } \Pi(s) \Pi(-s)}{(s + Q^2)^2} \quad (32)$$

and use an explicit spectrum of vector mesons ^{*)}. $\text{Im } \Pi(s)$ is given by Eq. (4). For m_n^2 and f_n^2 we take

$$m_n^2 = m_0^2 (1 + b n),$$

$$\frac{m_n^2}{f_n^2} = \frac{m_0^2}{f_0^2} = \text{const.} ; \quad n = 0, 1, 2, \dots \quad (33)$$

$\Pi(-s)$ is given by

$$\Pi(-s) = \frac{e^2}{b f_0^2} \left[\Psi(1/b) - \Psi(1/b + s/b m_0^2) \right] \quad (34)$$

where $\psi(z)$ is the psi function defined by

*) The integral in Eq. (32) can also be done approximately numerically using data from e^+e^- annihilation into hadrons.

$$\Psi(z) = -\gamma + \sum_{n=0}^{\infty} \left(\frac{1}{n+1} - \frac{1}{n+z} \right). \quad (35)$$

$\gamma \approx 0.577$ is Euler's constant. Substituting from Eq. (34) into (32) gives

$$\begin{aligned} \sigma_{\gamma\gamma}(Q^2) &= \frac{4\pi^3 e^2}{b f_0^2} \Psi(1/b) \frac{d\Pi(-Q^2)}{dQ^2} + \\ &+ \frac{4\pi^2 e^2}{b f_0^2} \int_{m_0^2}^{\infty} ds \Psi(1/b + s/b m_0^2) \frac{\text{Im } \Pi(s)}{(s + Q^2)^2}. \end{aligned} \quad (36)$$

The remaining integral in Eq. (36) is actually a discrete sum but it cannot be done exactly. Using Eq. (33) [i.e., $(dm_n^2/dn) = b m_0^2$] it can still be approximated by an integral

$$\begin{aligned} J(Q^2) &= \int_{m_0^2}^{\infty} ds \frac{\Psi(1/b + s/b m_0^2)}{(s + Q^2)^2} \\ &= \frac{-\ln \Gamma(2/b)}{(m_0^2 + Q^2)^2} + 2 \int_{m_0^2}^{\infty} ds \frac{\ln \Gamma(1/b + s/b m_0^2)}{(s + Q^2)^3}. \end{aligned} \quad (37)$$

$\Gamma(z)$ is the usual gamma function. With the help of Stirling's formula, the last integral in (37) can be done. The final result for $\sigma_{\gamma\gamma}(Q^2)$ is

$$\begin{aligned} \sigma_{\gamma\gamma}(Q^2) &= \frac{4\pi^3 \alpha^2}{b^2} \left(\frac{4\pi}{f_0^2} \right)^2 \left\{ \frac{-1}{b m_0^2} \Psi(1/b) \Psi'(1/b + Q^2/b m_0^2) + \right. \\ &+ \frac{1}{Q^2 - m_0^2} \left[\ln \left(\frac{m_0^2 + Q^2}{2 m_0^2} \right) + \frac{Q^2 - m_0^2}{Q^2 + m_0^2} \right] + \frac{\ln(2/b e)}{m_0^2 + Q^2} - \\ &- \frac{b m_0^2}{(Q^2 - m_0^2)^2} \left[\ln \left(\frac{m_0^2 + Q^2}{2 m_0^2} \right) - \frac{Q^2 - m_0^2}{Q^2 + m_0^2} \right] + \\ &\left. + \frac{2 b m_0^2}{(m_0^2 + Q^2)^2} \left[\frac{1}{4} \ln(2\pi) + \left(\frac{1}{b} - \frac{1}{4} \right) \ln \left(\frac{2}{b} \right) - \frac{1}{b} - \frac{1}{2} \ln \Gamma(2/b) \right] \right\}. \end{aligned} \quad (38)$$

The parameter $4\pi^2/bf_0^2$ in Eq. (38) is related to the square of the quark charge. On comparing Eq. (34) with the result of the quark model for $s \rightarrow \infty$, one obtains

$$q^2 = 4\pi^2/b f_0^2. \quad (39)$$

This equation is the consequence of the so-called Q^2 -duality and can be generalized to the case of non-linear mass spectra. Whatever the spectrum it ensures, as can easily be checked from Eq. (38), that the coefficient of the $\log(Q^2)/Q^2$ term agrees with the QCD result. We use Eq. (39) for this purpose, but not to determine b from the known values of q^2 and $4\pi/f_0^2$. The reason is that, for the linear mass spectrum, the value of b determined from Eq. (39) is not consistent with that from the experimental mass spectrum. For a non-linear spectrum this problem does not exist¹⁰⁾, but we shall not worry about it here since we use the simpler linear spectrum only to obtain an approximation for the contributions of the higher lying states and of the continuum. From the masses of $\rho(0.776)$, $\rho'(1.25)$ and $\rho'(1.66)$ one gets a value of $b = 1.6$. This is the value used in Eq. (38) after having eliminated $4\pi^2/bf_0^2$ in favour of q^2 . The comparison of Eq. (38) with data is shown as a full curve in Figs. 1 and 2. We call attention to the fact, noted earlier, that the hadronic piece, proportional to $(d\Pi(-Q^2)/dQ^2)$ dominates the cross-section $\sigma_{\gamma\gamma}(Q^2)$ overwhelmingly for low Q^2 . Without this contribution the extrapolation of the QCD prediction to low Q^2 falls far below the experimental cross-section. We are able to accommodate the data over the entire range of Q^2 because, even with the simple-minded mass spectrum or the very rough approximation in Eq. (27), the two essential components of the cross-section, the "soft" hadronic and the "hard" asymptotically dominating pieces, are present. We conclude therefore that it is indeed possible to interpolate smoothly between the VMD and QCD régimes in a consistent way, using only the couplings of photons to vector mesons.

4. - CONCLUSIONS

The model discussed in this paper is seriously incomplete. There is no simple way, within this scheme, to include a non-trivial x dependence of the structure function. The reason for this failure may be important for latter developments. The x dependence is supposed to characterize the composite structure of the target in terms of its quark constituents. The point of view of this paper, on the other hand, leads to an interpretation of scaling and its violation in terms of the properties of the photon probe, considered to be elementary. It thus seems that the two points of view will have to be considered together in order to obtain a more satisfactory description. QCD goes a long way

in providing such a description, but, as previously mentioned, the low x behaviour of the photon structure functions shows that it has not gone far enough. As a contribution towards the solution of this problem, a lot has still to be learnt from the fact that what we call properties of the photon probe are really due to hadrons. Although we deal with vector mesons we use the asymptotic quark model structure of $\Pi(s)$ to define our model. Since we deal with asymptotic expansions about a scale invariant theory, a way must be found to introduce v also as a mass scale in the cross-sections $\sigma_{VA}(v;s)$, $\sigma_{VV}(v;s,s')$, etc., if the x dependence is to be incorporated in a non-trivial way. One would then probably lose the simple connection between $\sigma_{VA}(v;Q^2)$ and $\Pi(-Q^2)$ but not completely. For instance, if $\Pi(-s)$ in Eq. (32) say, were replaced by $\Pi(-v) = \Pi(-s(\frac{1}{x}-1))$, a non-trivial x dependence would have been introduced, but we do not know how to do this with more than just words.

The problem to which the approach developed in this paper is most suited is the investigation of the Q^2 dependence of structure functions for all values of Q^2 . It is an enormous, but interesting, simplification that scaling violations in all structure functions are predicted to be related, and reducible to the study of the properties of just one function, $\Pi(-Q^2)$. This is obviously an approximation, but at least the extend of its validity can easily be tested.

The problem of the photon structure function has revealed that, even with the undeniable success of QCD, its predictions will have to be supplemented by simple phenomenologies based on the vector dominance model. We have tried to show that the so-called generalized vector meson dominance model is less ad hoc than applications of it have tended to imply. The kinship between the photon and vector mesons naturally leads to a representation which allows us to interpret this relationship also in terms of a symmetry. It is remarkable that the symmetry, scale invariance, is the one relevant to the analysis of the large Q^2 behaviour of structure functions.

ACKNOWLEDGEMENTS

We would like to thank Ch. Berger for discussions on the PLUTO data and how they were fitted. We are also grateful to G. Veneziano for discussions on the use of QCD as a constraint in constructing models of the type presented in this paper and for reading the manuscript.

One of us (E.M.) would like to thank the "Caixa de Barcelona" for financial support.

REFERENCES

- 1) C. Peterson, T.F. Walsh and P.M. Zerwas - Nucl.Phys. B174 (1980) 424.
- 2) See the talks of S.J. Brodsky and P.M. Zerwas in Fourth International Colloquium on $\gamma\text{-}\gamma$ Interactions, ed. G.W. London, World Scientific Publishing Co., Singapore (1981).
- 3) PLUTO Collaboration - Phys.Letters 99B (1981) 287 ; 107B (1981) 168 ; Ch. Berger - CERN Preprint EP/81-154 (November 1981).
- 4) T.F. Walsh and P.M. Zerwas - Phys.Letters 44B (1973) 195 ; E. Witten - Nucl.Phys. B120 (1977) 189.
- 5) A. Bramon, E. Etim and M. Greco - Phys.Letters 41B (1972) 609 ; J.J. Sakurai - Phys.Letters 46B (1973) 207.
- 6) For a review and references, see : A. Donnachie and G. Shaw - in "Electromagnetic Interactions", vol. 2, pp. 169-194, eds. A. Donnachie and G. Shaw, Plenum Press (1978).
- 7) M.A. Shifman, A.I. Vainshtain and V.I. Zakharov - Nucl.Phys. B147 (1979) 385 ; J.S. Bell and R.A. Bertlmann - Nucl.Phys. B177 (1981) 218.
- 8) A. Erdélyi (ed.) - Tables of Integral Transforms. Bateman Manuscript Project, Vol. II, McGraw Hill, New York (1954).
- 9) S.J. Brodsky - J.Physique C-2 Suppl. 3 (1974) 69.
- 10) E. Etim and L. Schülke - Z.Phys. C9 (1981) 229.

FIGURE CAPTIONS

Figure 1 Total $\gamma\gamma$ cross-section, $\sigma_{\gamma\gamma}(Q^2)$, as a function of Q^2 . Data points are from Ref. 3). The dashed curve is the prediction of Eq. (27). The full curve is given by Eq. (38).

Figure 2 The photon structure function, $F_2(Q^2)$, defined by Eq. (25) (averaged over x) as a function of $Q^2 > 1 \text{ GeV}^2$. Data points are from Ref. 3). The full curve is the prediction of Eq. (38).

Fig. 1

Fig. 2